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Composite Implicit Random Iterates for Approximating Common Random Fixed Point for a Finite Family of Asymptotically Quasi-nonexpansive Type Random Operators

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Abstract. The goal of this paper is to establish some strong convergence theorems of composite implicit random iterates to a common random fixed point for a finite family of asymptotically quasi-nonexpansive type random operators in the setting of separable uniformly convex Banach spaces.

1 Introduction

Random approximations and random fixed point theorems are stochastic generalization of classical approximations and fixed point theorems. The study of random fixed point theorems was initiated by Prague school of probabilities in $1950's$. The interest in these problems was enhanced after the publication of the survey article of Bharucha-Reid [5]. Random fixed point theory and applications have been further developed rapidly in recent years (see [2, 3, 10, 11, 17, 19] and references therein).

Fixed point iteration schemes for nonlinear operators on Banach and Hilbert spaces have been developed and studied by many authors in recent years. The development of random fixed point iterations was first initiated by Choudhury in [6] where random Ishikawa iteration scheme was defined and its strong convergence to a random fixed point in Hilbert spaces was discussed. After that several authors have worked on random fixed point iterations (see e.g. [2, 7, 8, 12, 13, 14, 15] and references therein).

In 2005 Beg and Abbas [2] constructed and studied different random iterative algorithms for weakly contractive and asymptotically nonexpansive random operators on arbitrary Banach spaces. They also established the convergence of an implicit random iteration process to a common random fixed point for a finite family of asymptotically quasi-nonexpansive random operators. In 2007 Plubtieng et al. [12] constructed and established the convergence of an implicit random iteration process with errors to a common random fixed point for a finite family of asymptotically quasi-nonexpansive random operators in the framework of uniformly convex Banach spaces. In 2009 Beg and Thakur [4] introduced modified general composite implicit random iteration process and they gave necessary and sufficient condition for strong convergence of said iteration process to a common random fixed point of a finite family of random asymptotically nonexpansive mappings in separable Banach spaces and also they established some strong convergence theorems for said iteration scheme and mappings in separable uniformly convex Banach spaces. Recently, Benerjee and Choudhury [1] constructed and studied composite implicit random iterations for a finite family of asymptotically nonexpansive random operators.

They also established the convergence of above said iteration scheme and random operators to a common random fixed point in the setting of separable Banach spaces.

The purpose of this paper is to construct and study the composite implicit random iterations for a finite family of asymptotically quasi-nonexpansive type random operators and also establish some strong convergence theorems for above said iteration scheme and operators in the framework of separable Banach spaces. Our results extend and improve the corresponding results of [1, 4, 12] and many others from the current literature.

2 Preliminaries

Throughout this paper, (Ω, Σ) denotes a measurable space and X stands for a real Banach space. For any function $T\colon \Omega \times X \to X$ we denote the nth iterate $T(\omega, T(\omega, T(\omega, \dots, T(\omega, x)\dots,)))$ of T by $T^n(\omega, x)$.

Definition 2.1. A function $f\colon \Omega \to X$ is said to be measurable if $f^{-1}(B) \in \Sigma$ for every Borel subset B of X.

Definition 2.2. An operator $T\colon \Omega \times X \to X$ is called a random operator if $T(., x)\colon \Omega \to X$ is measurable for every $x \in X$.

Definition 2.3. A random operator $T\colon \Omega \times X \to X$ is continuous if $T(\omega, .)\colon X \to X$ is continuous for each $\omega \in \Omega$.

Definition 2.4. A measurable function $p\colon \Omega \to X$ is said to be a random fixed point of the random operator $T\colon \Omega \times X \to X$ if $T(\omega, p(\omega)) = p(\omega)$, $\forall \omega \in \Omega$. The set of all random fixed points of T is denoted by $RF(T)$.

Definition 2.5. Let C be a nonempty subset of a separable Banach space X and $T\colon \Omega \times C \to C$ be a random operator. Then T is said to be

(i) nonexpansive random operator if

$$\|T(\omega, x) - T(\omega, y)\| \leq \|x - y\|$$

for all $x, y \in X$ and for each $\omega \in \Omega$.

(ii) asymptotically nonexpansive random operator if there exists a sequence of measurable mappings $h_n\colon \Omega \to [1, \infty)$ with $\lim_{n\to\infty} h_n(\omega) = 1$, for each $\omega \in \Omega$, such that for $x, y \in C$, we have

$$\|T^n(\omega, x) - T^n(\omega, y)\| \leq h_n(\omega)\|x - y\|$$

for each $\omega \in \Omega$ and $n \in N$.

(iii) asymptotically quasi-nonexpansive random operator if for each $\omega \in \Omega$, $G(\omega) = \{x \in C : x = T(\omega, x)\} \neq \phi$ and there exists a sequence of measurable mappings $h_n\colon \Omega \to [1, \infty)$ with $\lim_{n\to\infty} h_n(\omega) = 1$, for each $\omega \in \Omega$, such that for $x \in C$ and $y \in G(\omega)$, the following inequality holds:

$$\|T^n(\omega, x) - y\| \leq h_n(\omega)\|x - y\|$$

for each $\omega \in \Omega$ and $n \in N$.

(iv) asymptotically nonexpansive type random operator if for any $x \in C$, the following inequality holds:

$$\limsup_{n\to\infty} \left\{ \|T^n(\omega, x) - T^n(\omega, y)\| - \|x - y\| : y \in C \right\} \leq 0.$$

(v) asymptotically quasi-nonexpansive type random operator if for each $\omega \in \Omega$, $G(\omega) = \{x \in C : x = T(\omega, x)\} \neq \phi$ and the following inequality holds:

$$\limsup_{n\to\infty} \left\{ \|T^n(\omega, x) - p\| - \|x - p\| : p \in G(\omega) \right\} \leq 0, \ x \in C.$$

(vi) uniformly L-Lipschitzian random operator if for any $x, y \in C$ and for each $\omega \in \Omega$

$$\|T^n(\omega, x) - T^n(\omega, y)\| \leq L\|x - y\|$$

where $n \in N$ and L is a positive constant.

(vii) semi-compact random operator if for a sequence of measurable mappings $\{\xi_n\}$ from Ω to C, with $\lim_{n\to\infty} \|\xi_n(\omega) - T(\omega, \xi_n(\omega))\| = 0$ for all $\omega \in \Omega$, we have a subsequence $\{\xi_{n_k}\}$ of $\{\xi_n\}$ such that $\xi_{n_k}(\omega) \to \xi(\omega)$ for each $\omega \in \Omega$, where ξ is a measurable function from Ω to C.

Definition 2.6. A finite family $\{T_i : i \in I\}$ of N continuous random operators from $\Omega \times C \to C$ with $F = \bigcap_{i=1}^{N} RF(T_i) \neq \emptyset$ is said to satisfy Condition (B) if there is a nondecreasing function $f\colon [0, \infty) \to [0, \infty)$ with $f(0) = 0$ and $f(r) > 0$ for all $r \in (0, \infty)$ such that for all $\omega \in \Omega$

$$f(d(x(\omega), F)) \leq \max_{1\leq i\leq N} \left\{ \|x(\omega) - T_i(\omega, x(\omega))\| \right\} \text{ for all } x,$$

where $x\colon \Omega \to C$ is a measurable function.

Lemma 2.1.[9] Let (Ω, Σ) be a measurable space, X be a separable Banach space and $T\colon \Omega \times X \to X$ be a continuous random operator. Then for any measurable function $x\colon \Omega \to X$, the function $T(\omega, x(\omega))$ is also measurable.

Definition 2.7. (Composite implicit random iteration process with errors) Let $\{T_i : i \in I = \{1, 2, \ldots, N\}\}$ be a finite family of N continuous random operators from $\Omega \times C$ to C where C be a nonempty closed convex subset of a separable Banach space X. Let $\xi_0\colon \Omega \to C$ be any measurable function. Then composite implicit random iteration scheme with errors is defined as follows:

$$
\begin{aligned}
\xi_1(\omega) &= \alpha_1\xi_0(\omega) + \beta_1 T_1(\omega, a_1\xi_1(\omega)) + b_1 T_1(\omega, \xi_1(\omega)) + c_1 g_1(\omega) + \gamma_1 f_1(\omega) \\
\xi_2(\omega) &= \alpha_2\xi_1(\omega) + \beta_2 T_2(\omega, a_2\xi_2(\omega)) + b_2 T_2(\omega, \xi_2(\omega)) + c_2 g_2(\omega) + \gamma_2 f_2(\omega) \\
&\vdots \\
\xi_N(\omega) &= \alpha_N\xi_{N-1}(\omega) + \beta_N T_N(\omega, a_N\xi_N(\omega)) + b_N T_N(\omega, \xi_N(\omega)) + c_N g_N(\omega) \\
&\quad + \gamma_N f_N(\omega) \\
\xi_{N+1}(\omega) &= \alpha_{N+1}\xi_N(\omega) + \beta_{N+1} T_1^2(\omega, a_{N+1}\xi_{N+1}(\omega)) + b_{N+1} T_1^2(\omega, \xi_{N+1}(\omega)) \\
&\quad + c_{N+1} g_{N+1}(\omega) + \gamma_{N+1} f_{N+1}(\omega) \\
&\vdots \\
\xi_{2N}(\omega) &= \alpha_{2N}\xi_{2N-1}(\omega) + \beta_{2N} T_N^2(\omega, a_{2N}\xi_{2N}(\omega)) + b_{2N} T_N^2(\omega, \xi_{2N}(\omega)) \\
&\quad + c_{2N} g_{2N}(\omega) + \gamma_{2N} f_{2N}(\omega) \\
\xi_{2N+1}(\omega) &= \alpha_{2N+1}\xi_{2N}(\omega) + \beta_{2N+1} T_1^3(\omega, a_{2N+1}\xi_{2N+1}(\omega)) + b_{2N+1} T_1^3(\omega, \xi_{2N+1}(\omega)) \\
&\quad + c_{2N+1} g_{2N+1}(\omega) + \gamma_{2N+1} f_{2N+1}(\omega) \\
&\vdots
\end{aligned}
$$

which can be written in the compact form as

$$
\begin{aligned}
\xi_n(\omega) &= \alpha_n\xi_{n-1}(\omega) + \beta_n T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) + \gamma_n f_n(\omega) \\
\eta_n(\omega) &= a_n\xi_n(\omega) + b_n T_{i(n)}^{k(n)}(\omega, \xi_n(\omega)) + c_n g_n(\omega),\ n \geq 1,\ \forall \omega \in \Omega,
\end{aligned}
\tag{2.1}
$$

where $n = (k-1)N + i$, $i = i(n) \in I$ and $k = k(n) \geq 1$ is a positive integer such that $k(n) \to \infty$ as $n \to \infty$ and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ are sequences in $[0,1]$ with $\alpha_n + \beta_n + \gamma_n = a_n + b_n + c_n = 1$ and $\{f_n\}$, $\{g_n\}$ are bounded sequences of measurable functions from Ω to C.

Remark 2.1. By Lemma 2.1 the sequence $\{\xi_n\}$ defined in (2.1) is a sequence of measurable functions.

Lemma 2.2. (see [18]) Let $\{s_n\}$ and $\{t_n\}$ be sequences of nonnegative real numbers satisfying the inequality

$$s_{n+1} \leq s_n + t_n,\ n \geq 1.$$

If $\sum_{n=1}^{\infty} t_n < \infty$, then $\lim_{n\to\infty} s_n$ exists. In particular, if $\{s_n\}$ has a subsequence converging to zero, then $\lim_{n\to\infty} s_n = 0$.

Lemma 2.3. (Schu [16]) Let E be a uniformly convex Banach space and $0 < a \leq t_n \leq b < 1$ for all $n \geq 1$. Suppose that $\{x_n\}$ and $\{y_n\}$ are sequences in E satisfying $\limsup_{n\to\infty} \|x_n\| \leq r$, $\limsup_{n\to\infty} \|y_n\| \leq r$ and $\lim_{n\to\infty} \|t_n x_n + (1-t_n) y_n\| = r$ hold for some $r \geq 0$. Then $\lim_{n\to\infty} \|x_n - y_n\| = 0$.

3 Main Results

In this section, we give the necessary and sufficient conditions of the composite implicit random iteration process with errors to converge to common random fixed point for a finite family of asymptotically quasi-nonexpansive type random operators and also discuss strong convergence of above said scheme and random operators with additional conditions.

Theorem 3.1. Let X be a separable Banach space and C be a nonempty closed convex subset of X. Let $\{T_i : i \in I = \{1, 2, \ldots, N\}\}$ be N uniformly L-Lipschitzian asymptotically quasi-nonexpansive type random operators from $\Omega \times C$ to C. Assume that $F = \cap_{i=1}^{N} RF(T_i) \neq \emptyset$. Let $\{\xi_n(\omega)\}$ be the implicit random iterative sequence with errors defined by (2.1). Put

$$\begin{aligned} A_{in}(\omega) &= \max\Big\{ \sup_{\xi(\omega)\in F,\ n\geq 1} \Big(\|T_i^n(\omega, \xi_n(\omega)) - \xi(\omega)\| - \|\xi_n(\omega) - \xi(\omega)\|\Big) \\ &\qquad \vee 0 : i \in I\Big\} \end{aligned} \tag{3.1}$$

and

$$\begin{aligned} B_{in}(\omega) &= \max\Big\{ \sup_{\xi(\omega)\in F,\ n\geq 1} \Big(\|T_i^n(\omega, \eta_n(\omega)) - \xi(\omega)\| - \|\eta_n(\omega) - \xi(\omega)\|\Big) \\ &\qquad \vee 0 : i \in I\Big\} \end{aligned} \tag{3.2}$$

where $n = (k-1)N + i$ and $i = i(n) \in I$. Assume that $\sum_{n=1}^{\infty} A_{in}(\omega) < \infty$, $\sum_{n=1}^{\infty} B_{in}(\omega) < \infty$, $0 < \alpha \leq \alpha_n, \beta_n \leq \beta < 1$ for some $\alpha, \beta \in (0, 1)$, $\sum_{n=1}^{\infty} \gamma_n < \infty$ and $\sum_{n=1}^{\infty} c_n < \infty$. Then $\{\xi_n\}$ converges strongly to a common random fixed point of the random operators $\{T_i : i \in I = \{1, 2, \ldots, N\}\}$ if and only if for all $\omega \in \Omega$, $\liminf_{n\to\infty} d(\xi_n(\omega), F) = 0$, where $d(\xi_n(\omega), F) = \inf\{\|\xi_n(\omega) - \xi(\omega)\| : \xi \in F\}$.

Proof. The necessity is obvious and so it is omitted. Now, we prove the sufficiency. Let $\xi \in F$. Since $\{f_n\}$, $\{g_n\}$ are bounded sequences of measurable functions from Ω to C, we can put for each $\omega \in \Omega$,

$$M(\omega) = \sup_{n\geq 1} \|f_n(\omega) - \xi(\omega)\| \vee \sup_{n\geq 1} \|g_n(\omega) - \xi(\omega)\|$$

obviously $M(\omega) < \infty$ for each $\omega \in \Omega$. Now for $\xi \in F$ and for each $\omega \in \Omega$ using (2.1) and (3.1), we have

$$\begin{aligned} \|\eta_n(\omega) - \xi(\omega)\| &= \Big\|a_n\xi_n(\omega) + b_n T_{i(n)}^{k(n)}(\omega, \xi_n(\omega)) + c_n g_n(\omega) - \xi(\omega)\Big\| \\ &\leq a_n \|\xi_n(\omega) - \xi(\omega)\| + b_n \Big\|T_{i(n)}^{k(n)}(\omega, \xi_n(\omega)) - \xi(\omega)\Big\| \\ &\quad + c_n \|g_n(\omega) - \xi(\omega)\| \\ &\leq a_n \|\xi_n(\omega) - \xi(\omega)\| + b_n[\|\xi_n(\omega)) - \xi(\omega)\| + A_{ik(n)}(\omega)] \\ &\quad + c_n \|g_n(\omega) - \xi(\omega)\| \\ &\leq (a_n + b_n) \|\xi_n(\omega) - \xi(\omega)\| + b_n A_{ik(n)}(\omega) + c_n M(\omega) \\ &= (1 - c_n) \|\xi_n(\omega) - \xi(\omega)\| + b_n A_{ik(n)}(\omega) + c_n M(\omega) \\ &\leq \|\xi_n(\omega) - \xi(\omega)\| + A_{ik(n)}(\omega) + c_n M(\omega). \end{aligned} \tag{3.3}$$

Again using (2.1), (3.2) and (3.3), we have,

$$\begin{aligned}
\|\xi_n(\omega)-\xi(\omega)\| &= \left\|\alpha_n\xi_{n-1}(\omega)+\beta_nT_{i(n)}^{k(n)}(\omega,\eta_n(\omega))+\gamma_nf_n(\omega)-\xi(\omega)\right\| \\
&\le \alpha_n\|\xi_{n-1}(\omega)-\xi(\omega)\|+\beta_n\left\|T_{i(n)}^{k(n)}(\omega,\eta_n(\omega))-\xi(\omega)\right\| \\
&\quad +\gamma_n\|f_n(\omega)-\xi(\omega)\| \\
&\le \alpha_n\|\xi_{n-1}(\omega)-\xi(\omega)\|+\beta_n[\|\eta_n(\omega)-\xi(\omega)\|+B_{ik(n)}(\omega)] \\
&\quad +\gamma_n\|f_n(\omega)-\xi(\omega)\| \\
&\le \alpha_n\|\xi_{n-1}(\omega)-\xi(\omega)\|+\beta_n[\|\xi_n(\omega)-\xi(\omega)\|+A_{ik(n)}(\omega) \\
&\quad +c_nM(\omega)]+\beta_nB_{ik(n)}(\omega)+\gamma_nM(\omega) \\
&\le \alpha_n\|\xi_{n-1}(\omega)-\xi(\omega)\|+\beta_n\|\xi_n(\omega)-\xi(\omega)\|+\beta_n(A_{ik(n)}(\omega) \\
&\quad +B_{ik(n)}(\omega))+(\beta_nc_n+\gamma_n)M(\omega) \\
&= \alpha_n\|\xi_{n-1}(\omega)-\xi(\omega)\|+(1-\alpha_n-\gamma_n)\|\xi_n(\omega)-\xi(\omega)\| \\
&\quad +\beta_n(A_{ik(n)}(\omega)+B_{ik(n)}(\omega))+(\beta_nc_n+\gamma_n)M(\omega) \\
&\le \alpha_n\|\xi_{n-1}(\omega)-\xi(\omega)\|+(1-\alpha_n)\|\xi_n(\omega)-\xi(\omega)\| \\
&\quad +A_{ik(n)}(\omega)+B_{ik(n)}(\omega)+(\beta_nc_n+\gamma_n)M(\omega). \qquad (3.4)
\end{aligned}$$

By rearranging both sides of (3.4) we have that

$$\begin{aligned}
\|\xi_n(\omega)-\xi(\omega)\| &\le \|\xi_{n-1}(\omega)-\xi(\omega)\|+\frac{A_{ik(n)}(\omega)+B_{ik(n)}(\omega)}{\alpha_n} \\
&\quad +\frac{\beta_nc_n+\gamma_n}{\alpha_n}M(\omega) \\
&\le \|\xi_{n-1}(\omega)-\xi(\omega)\|+\frac{A_{ik(n)}(\omega)+B_{ik(n)}(\omega)}{\alpha} \\
&\quad +\frac{\beta_nc_n+\gamma_n}{\alpha}M(\omega) \\
&= \|\xi_{n-1}(\omega)-\xi(\omega)\|+\lambda_{ik(n)}(\omega)+\sigma_n(\omega) \qquad (3.5)
\end{aligned}$$

where $\lambda_{ik(n)}(\omega)=\frac{A_{ik(n)}(\omega)+B_{ik(n)}(\omega)}{\alpha}$ and $\sigma_n(\omega)=\frac{\beta_nc_n+\gamma_n}{\alpha}M(\omega)$. Since by assumptions of the theorem $\sum_{n=1}^{\infty}A_{in}(\omega)<\infty$, $\sum_{n=1}^{\infty}B_{in}(\omega)<\infty$, $\sum_{n=1}^{\infty}\gamma_n<\infty$ and $\sum_{n=1}^{\infty}c_n<\infty$, it follows that $\sum_{k(n)=1}^{\infty}\lambda_{ik(n)}(\omega)<\infty$ and $\sum_{n=1}^{\infty}\sigma_n(\omega)<\infty$ for all $i\in I$ and for each $\omega\in\Omega$. This gives that

$$d(\xi_n(\omega),F) \le d(\xi_{n-1}(\omega),F)+\lambda_{ik(n)}(\omega)+\sigma_n(\omega). \qquad (3.6)$$

Hence by Lemma 2.2 we have $\lim_{n\to\infty}d(\xi_n(\omega),F)$ exists for each $\omega\in\Omega$. Further, by the condition of the theorem we have for all $\omega\in\Omega$,

$$\lim_{n\to\infty}d(\xi_n(\omega),F)=0. \qquad (3.7)$$

Now from (3.5) we have that

$$\begin{aligned}\|\xi_{n+m}(\omega)-\xi(\omega)\| &\leq \|\xi_{n+m-1}(\omega)-\xi(\omega)\|+\lambda_{ik(n+m)}(\omega)+\sigma_{n+m}(\omega)\\ &\vdots\\ &\leq \|\xi_n(\omega)-\xi(\omega)\|+\sum_{j=n+1}^{n+m}\lambda_{ik(j)}(\omega)+\sum_{j=n+1}^{n+m}\sigma_j(\omega)\\ &\leq \|\xi_n(\omega)-\xi(\omega)\|+\sum_{j=n+1}^{\infty}\lambda_{ik(j)}(\omega)\\ &\quad+\sum_{j=n+1}^{\infty}\sigma_j(\omega) \qquad (3.8)\end{aligned}$$

for each $\omega\in\Omega$ and for all natural numbers m,n. Therefore for any $\xi\in F$ we have that

$$\begin{aligned}\|\xi_{n+m}(\omega)-\xi_n(\omega)\| &\leq \|\xi_{n+m}(\omega)-\xi(\omega)\|+\|\xi_n(\omega)-\xi(\omega)\|\\ &\leq \|\xi_n(\omega)-\xi(\omega)\|+\sum_{j=n+1}^{\infty}\lambda_{ik(j)}(\omega)\\ &\quad+\sum_{j=n+1}^{\infty}\sigma_j(\omega)+\|\xi_n(\omega)-\xi(\omega)\|\\ &= 2\,\|\xi_n(\omega)-\xi(\omega)\|+\sum_{j=n+1}^{\infty}\lambda_{ik(j)}(\omega)\\ &\quad+\sum_{j=n+1}^{\infty}\sigma_j(\omega). \qquad (3.9)\end{aligned}$$

Since $\sum_{j=n+1}^{\infty}\lambda_{ik(j)}(\omega)<\infty$, $\sum_{j=n+1}^{\infty}\sigma_j(\omega)<\infty$ and $\lim_{n\to\infty}d(\xi_n(\omega),F)=0$, there exists $n_1\in N$ such that for all $n\geq n_1$ we have $d(\xi_n(\omega),F)<\frac{\varepsilon}{12}$, $\sum_{j=n+1}^{\infty}\lambda_{ik(j)}(\omega)<\frac{\varepsilon}{3}$ and $\sum_{j=n+1}^{\infty}\sigma_j(\omega)<\frac{\varepsilon}{3}$. So there exists $p\in F$ such that $\|\xi_n(\omega)-p(\omega)\|<\frac{\varepsilon}{6}$ for all $n\geq n_1$.

Therefore from (3.9) we have that for all $n\geq n_1$,

$$\begin{aligned}\|\xi_{n+m}(\omega)-\xi_n(\omega)\| &\leq 2\,\|\xi_n(\omega)-p(\omega)\|+\sum_{j=n+1}^{\infty}\lambda_{ik(j)}(\omega)\\ &\quad+\sum_{j=n+1}^{\infty}\sigma_j(\omega)\\ &< 2\,\frac{\varepsilon}{6}+\frac{\varepsilon}{3}+\frac{\varepsilon}{3}=\varepsilon \qquad (3.10)\end{aligned}$$

which implies that $\{\xi_n(\omega)\}$ is a Cauchy sequence for each $\omega\in\Omega$. Therefore $\xi_n(\omega)\to q(\omega)$ as $n\to\infty$ for each $\omega\in\Omega$, where $q\colon\Omega\to F$, being the limit of the sequence of measurable functions is also measurable. Now we prove that $q\in F$. Since for each $\omega\in\Omega$, $\xi_n(\omega)\to q(\omega)$ as $n\to\infty$ there exists $n_2\in N$ such that $\|\xi_n(\omega)-q(\omega)\|<\frac{\varepsilon}{2(L+1)}$ for all $n\geq n_2$. Since $\lim_{n\to\infty}d(\xi_n(\omega),F)=0$ for each $\omega\in\Omega$, there exists $n_3\in N$ such that $d(\xi_n(\omega),F)<\frac{\varepsilon}{2(L+1)}$

for all $n \geq n_3$. So there exists $q_1 \in F$ such that $\|\xi_n(\omega) - q_1(\omega)\| < \frac{\varepsilon}{2(L+1)}$ for all $n \geq n_3$. Let $n_4 = \max\{n_2, n_3\}$. Now for all $l \in I$ and for all $n \geq n_4$

$$\begin{aligned}
\|T_l(\omega, q(\omega)) - q(\omega)\| &\leq \|T_l(\omega, q(\omega)) - q_1(\omega)\| + \|q_1(\omega) - q(\omega)\| \\
&\leq \|T_l(\omega, q(\omega)) - T_l(\omega, q_1(\omega))\| + \|q_1(\omega) - q(\omega)\| \\
&\leq L\,\|q(\omega) - q_1(\omega)\| + \|q_1(\omega) - q(\omega)\| \\
&= (L+1)\,\|q_1(\omega) - q(\omega)\| \\
&\leq (L+1)\,\|q_1(\omega) - \xi_n(\omega)\| + (L+1)\,\|\xi_n(\omega) - q(\omega)\| \\
&< (L+1).\frac{\varepsilon}{2(L+1)} + (L+1).\frac{\varepsilon}{2(L+1)} = \varepsilon
\end{aligned}$$

which implies that $T_l(\omega, q(\omega)) = q(\omega)$ for all $l \in I$ and for each $\omega \in \Omega$. Therefore $q \in F$. Thus $\{\xi_n\}$ converges strongly to a common random fixed point of $\{T_i : i \in I\}$. This completes the proof.

Lemma 3.1. Let X be a separable Banach space and C be a nonempty closed convex subset of X. Let $\{T_i : i \in I = \{1, 2, \ldots, N\}\}$ be N uniformly L-Lipschitzian asymptotically quasi-nonexpansive type random operators from $\Omega \times C$ to C. Assume that $F = \cap_{i=1}^{N} RF(T_i) \neq \emptyset$. Let $\{\xi_n(\omega)\}$ be the implicit random iterative sequence with errors defined by (2.1). Put

$$\begin{aligned}
A_{in}(\omega) &= \max\Big\{ \sup_{\xi(\omega)\in F,\ n\geq 1} \Big(\|T_i^n(\omega, \xi_n(\omega)) - \xi(\omega)\| - \|\xi_n(\omega) - \xi(\omega)\|\Big) \\
&\qquad \vee 0 : i \in I\Big\}
\end{aligned}$$

and

$$\begin{aligned}
B_{in}(\omega) &= \max\Big\{ \sup_{\xi(\omega)\in F,\ n\geq 1} \Big(\|T_i^n(\omega, \eta_n(\omega)) - \xi(\omega)\| - \|\eta_n(\omega) - \xi(\omega)\|\Big) \\
&\qquad \vee 0 : i \in I\Big\}
\end{aligned}$$

where $n = (k-1)N + i$ and $i = i(n) \in I$. Assume that $\sum_{n=1}^{\infty} A_{in}(\omega) < \infty$, $\sum_{n=1}^{\infty} B_{in}(\omega) < \infty$, $0 < \alpha \leq a_n, b_n \leq \beta < 1$ for some $\alpha, \beta \in (0,1)$, $\sum_{n=1}^{\infty} \gamma_n < \infty$ and $\sum_{n=1}^{\infty} c_n < \infty$. Then $\lim_{n\to\infty} \|\xi_n(\omega) - T_l(\omega, \xi_n(\omega))\| = 0$ for each $\omega \in \Omega$ and for all $l = 1, 2, \ldots, N$.

Proof. Let $\xi \in F$ be arbitrary. Since $\{f_n\}$, $\{g_n\}$ are bounded sequences of measurable functions from Ω to C, we can put for each $\omega \in \Omega$,

$$M(\omega) = \sup_{n\geq 1} \|f_n(\omega) - \xi(\omega)\| \vee \sup_{n\geq 1} \|g_n(\omega) - \xi(\omega)\|$$

obviously $M(\omega) < \infty$ for each $\omega \in \Omega$. From (3.5) we have that

$$\|\xi_n(\omega) - \xi(\omega)\| \leq \|\xi_{n-1}(\omega) - \xi(\omega)\| + \lambda_{ik(n)}(\omega) + \sigma_n(\omega),$$

where $\sum_{k(n)=1}^{\infty} \lambda_{ik(n)}(\omega) < \infty$ and $\sum_{n=1}^{\infty} \sigma_n(\omega) < \infty$ for all $i \in I$ and for each $\omega \in \Omega$. Hence by Lemma 2.2 we get that $\lim_{n\to\infty} \|\xi_n(\omega) - \xi(\omega)\|$ exists for all $\xi \in F$ and for each $\omega \in \Omega$. Thus $\{\xi_n(\omega)\}$ is a bounded sequence for each $\omega \in \Omega$. Let $\lim_{n\to\infty} \|\xi_n(\omega) - \xi(\omega)\| = R$ for some $R \geq 0$. From (3.3) we get that

$$\|\eta_n(\omega) - \xi(\omega)\| \leq \|\xi_n(\omega) - \xi(\omega)\| + A_{ik(n)}(\omega) + c_n M(\omega).$$

Taking limsup on both sides of the above inequality we get that

$$\limsup_{n\to\infty} \|\eta_n(\omega) - \xi(\omega)\| \le R \text{ for each } \omega \in \Omega. \tag{3.11}$$

Now

$$\begin{aligned} R &= \lim_{n\to\infty} \|\xi_n(\omega) - \xi(\omega)\| \\ &= \lim_{n\to\infty} \|\alpha_n \xi_{n-1}(\omega) + \beta_n T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) + \gamma_n f_n(\omega) - \xi(\omega)\| \\ &= \lim_{n\to\infty} \|(1-\beta_n)(\xi_{n-1}(\omega) - \xi(\omega) + \gamma_n(f_n(\omega) - \xi_{n-1}(\omega))) \\ &\quad + \beta_n(T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi(\omega) + \gamma_n(f_n(\omega) - \xi_{n-1}(\omega)))\|. \end{aligned} \tag{3.12}$$

Now for each $\omega \in \Omega$,

$$\|\xi_{n-1}(\omega) - \xi(\omega) + \gamma_n(f_n(\omega) - \xi_{n-1}(\omega))\| \le \|\xi_{n-1}(\omega) - \xi(\omega)\| + \gamma_n \|f_n(\omega) - \xi_{n-1}(\omega)\|.$$

Taking limsup on both sides of the above inequality we get for each $\omega \in \Omega$

$$\begin{aligned} &\limsup_{n\to\infty} \|\xi_{n-1}(\omega) - \xi(\omega) + \gamma_n(f_n(\omega) - \xi_{n-1}(\omega))\| \\ \le\ &\limsup_{n\to\infty} (\|\xi_{n-1}(\omega) - \xi(\omega)\| + \gamma_n \|f_n(\omega) - \xi_{n-1}(\omega)\|) \\ =\ &R. \end{aligned} \tag{3.13}$$

Also,

$$\begin{aligned} &\left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi(\omega) + \gamma_n(f_n(\omega) - \xi_{n-1}(\omega))\right\| \\ \le\ &\left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi(\omega)\right\| + \gamma_n \|f_n(\omega) - \xi_{n-1}(\omega)\| \\ \le\ &\|\eta_n(\omega) - \xi(\omega)\| + B_{ik(n)}(\omega) + \gamma_n \|f_n(\omega) - \xi_{n-1}(\omega)\|. \end{aligned}$$

Taking limsup on both sides of the above inequality we get for each $\omega \in \Omega$

$$\limsup_{n\to\infty} \left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi(\omega) + \gamma_n(f_n(\omega) - \xi_{n-1}(\omega))\right\| \le R. \tag{3.14}$$

From (3.12), (3.13), (3.14) and Lemma 2.3 we get

$$\lim_{n\to\infty} \left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi_{n-1}(\omega)\right\| = 0 \text{ for each } \omega \in \Omega. \tag{3.15}$$

Again for each $\omega \in \Omega$, we have

$$\begin{aligned} \|\xi_n(\omega) - \xi_{n-1}(\omega)\| &= \left\|\alpha_n \xi_{n-1}(\omega) + \beta_n T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) + \gamma_n f_n(\omega) - \xi_{n-1}(\omega)\right\| \\ &\le \beta_n \left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi_{n-1}(\omega)\right\| + \gamma_n \|f_n(\omega) - \xi_{n-1}(\omega)\| \\ &\to 0 \text{ as } n \to \infty. \end{aligned} \tag{3.16}$$

Hence for each $\omega \in \Omega$,

$$\lim_{n\to\infty} \|\xi_n(\omega) - \xi_{n+l}(\omega)\| = 0 \text{ for each } \omega \in \Omega \text{ and for all } l \in I. \tag{3.17}$$

Since

$$\left\|\xi_n(\omega) - T_{i(n)}^{k(n)}(\omega, \eta_n(\omega))\right\| \leq \|\xi_n(\omega) - \xi_{n-1}(\omega)\| + \left\|\xi_{n-1}(\omega) - T_{i(n)}^{k(n)}(\omega, \eta_n(\omega))\right\|,$$

by using (3.15) and (3.16) we get

$$\lim_{n\to\infty} \left\|\xi_n(\omega) - T_{i(n)}^{k(n)}(\omega, \eta_n(\omega))\right\| = 0 \text{ for each } \omega \in \Omega. \tag{3.18}$$

Now

$$\begin{aligned}
\|\eta_n(\omega) - \xi_n(\omega)\| &= \left\|a_n\xi_n(\omega) + b_nT_{i(n)}^{k(n)}(\omega, \xi_n(\omega)) + c_ng_n(\omega) - \xi_n(\omega)\right\| \\
&\leq b_n \left\|T_{i(n)}^{k(n)}(\omega, \xi_n(\omega)) - \xi_n(\omega)\right\| + c_n \|g_n(\omega) - \xi_n(\omega)\| \\
&\leq b_n\Big[\left\|T_{i(n)}^{k(n)}(\omega, \xi_n(\omega)) - T_{i(n)}^{k(n)}(\omega, \eta_n(\omega))\right\| + \left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi_n(\omega)\right\| \Big] \\
&\quad + c_n \|g_n(\omega) - \xi_n(\omega)\| \\
&\leq b_n\Big[L\|\xi_n(\omega) - \eta_n(\omega)\| + \left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi_n(\omega)\right\|\Big] \\
&\quad + c_n \|g_n(\omega) - \xi_n(\omega)\| \\
&\leq (1-a_n)L\|\xi_n(\omega) - \eta_n(\omega)\| + (1-a_n)\left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi_n(\omega)\right\| \\
&\quad + c_n \|g_n(\omega) - \xi_n(\omega)\| \\
&\leq (1-a_nL)\|\xi_n(\omega) - \eta_n(\omega)\| + (1-a_n)\left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi_n(\omega)\right\| \\
&\quad + c_n \|g_n(\omega) - \xi_n(\omega)\| \qquad (3.19)
\end{aligned}$$

which implies that

$$a_nL\|\eta_n(\omega) - \xi_n(\omega)\| \leq a_n\big(\frac{1}{a_n} - 1\big)\left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi_n(\omega)\right\| + c_n\|g_n(\omega) - \xi_n(\omega)\|$$

which implies that

$$\|\xi_n(\omega) - \eta_n(\omega)\| \leq \frac{1}{L}\big(\frac{1}{\alpha} - 1\big)\left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - \xi_n(\omega)\right\| + \frac{c_n}{\alpha L}\|g_n(\omega) - \xi_n(\omega)\|. \tag{3.20}$$

Since $\sum_{n=1}^{\infty} c_n < \infty$, we have $\lim_{n\to\infty} c_n = 0$ and using (3.18) in (3.20), we get that for each $\omega \in \Omega$

$$\|\xi_n(\omega) - \eta_n(\omega)\| \to 0 \text{ as } n \to \infty. \tag{3.21}$$

Now

$$\begin{aligned}
\|\xi_{n-1}(\omega) - T_n(\omega, \xi_n(\omega))\| &\leq \left\|\xi_{n-1}(\omega) - T_{i(n)}^{k(n)}(\omega, \eta_n(\omega))\right\| + \left\|T_{i(n)}^{k(n)}(\omega, \eta_n(\omega)) - T_n(\omega, \xi_n(\omega))\right\| \\
&\leq \left\|\xi_{n-1}(\omega) - T_{i(n)}^{k(n)}(\omega, \eta_n(\omega))\right\| + L\left\|T_{i(n)}^{k(n)-1}(\omega, \eta_n(\omega)) - \xi_n(\omega)\right\| \\
&= \rho_n(\omega) + L\left\|T_{i(n)}^{k(n)-1}(\omega, \eta_n(\omega)) - \xi_n(\omega)\right\|, \qquad (3.22)
\end{aligned}$$

where $\rho_n(\omega) = \left\|\xi_{n-1}(\omega) - T_{i(n)}^{k(n)}(\omega, \eta_n(\omega))\right\|$ for each $\omega \in \Omega$. From (3.15) we have for each $\omega \in \Omega$, $\rho_n(\omega) \to 0$ as $n \to \infty$. Again

$$\begin{aligned}
\left\|T_{i(n)}^{k(n)-1}(\omega, \eta_n(\omega)) - \xi_n(\omega)\right\| \leq\ & \left\|T_{i(n)}^{k(n)-1}(\omega, \eta_n(\omega)) - T_{i(n-N)}^{k(n)-1}(\omega, \xi_{n-N}(\omega))\right\| \\
& + \left\|T_{i(n-N)}^{k(n)-1}(\omega, \xi_{n-N}(\omega)) - T_{i(n-N)}^{k(n)-1}(\omega, \eta_{n-N}(\omega))\right\| \\
& + \left\|T_{i(n-N)}^{k(n)-1}(\omega, \eta_{n-N}(\omega)) - \xi_{(n-N)-1}(\omega)\right\| \\
& + \left\|\xi_{(n-N)-1}(\omega) - \xi_n(\omega)\right\|. \qquad (3.23)
\end{aligned}$$

Now for each $n > N$, $n = (n-N)(mod\ N)$. Again since $n = (k(n)-1)N + i(n)$, we have $k(n-N) = k(n)-1$ and $i(n-N) = i(n)$. So from (3.23) we have

$$\begin{aligned}
\left\|T_{i(n)}^{k(n)-1}(\omega, \eta_n(\omega)) - \xi_n(\omega)\right\| \leq\ & \left\|T_{i(n)}^{k(n)-1}(\omega, \eta_n(\omega)) - T_{i(n-N)}^{k(n)-1}(\omega, \xi_{n-N}(\omega))\right\| \\
& + \left\|T_{i(n-N)}^{k(n)-1}(\omega, \xi_{n-N}(\omega)) - T_{i(n-N)}^{k(n)-1}(\omega, \eta_{n-N}(\omega))\right\| \\
& + \left\|T_{i(n-N)}^{k(n)-1}(\omega, \eta_{n-N}(\omega)) - \xi_{(n-N)-1}(\omega)\right\| \\
& + \left\|\xi_{(n-N)-1}(\omega) - \xi_n(\omega)\right\| \\
\leq\ & L\left\|\eta_n(\omega) - \xi_{n-N}(\omega)\right\| + L\left\|\xi_{n-N}(\omega) - \eta_{n-N}(\omega)\right\| \\
& + \rho_{n-N}(\omega) + \left\|\xi_{(n-N)-1}(\omega) - \xi_n(\omega)\right\|. \qquad (3.24)
\end{aligned}$$

So from (3.22) and (3.24) we have for each $\omega \in \Omega$

$$\begin{aligned}
\left\|\xi_{n-1}(\omega) - T_n(\omega, \xi_n(\omega))\right\| \leq\ & \rho_n(\omega) + L^2\left\|\eta_n(\omega) - \xi_{n-N}(\omega)\right\| \\
& + L^2\left\|\xi_{n-N}(\omega) - \eta_{n-N}(\omega)\right\| + L\rho_{n-N}(\omega) \\
& + L\left\|\xi_{(n-N)-1}(\omega) - \xi_n(\omega)\right\| \\
\leq\ & \rho_n(\omega) + L^2\big(\left\|\eta_n(\omega) - \xi_n(\omega)\right\| + \left\|\xi_n(\omega) - \xi_{n-N}(\omega)\right\|\big) \\
& + L^2\left\|\xi_{n-N}(\omega) - \eta_{n-N}(\omega)\right\| + L\rho_{n-N}(\omega) \\
& + L\left\|\xi_{(n-N)-1}(\omega) - \xi_n(\omega)\right\|. \qquad (3.25)
\end{aligned}$$

Now for each $\omega \in \Omega$, it follows that

$$\left\|\xi_{n-1}(\omega) - T_n(\omega, \xi_n(\omega))\right\| \to 0 \text{ as } n \to \infty. \qquad (3.26)$$

Now by (3.26) and (3.16) we get that for each $\omega \in \Omega$

$$\begin{aligned}
\left\|\xi_{n-1}(\omega) - T_n(\omega, \xi_n(\omega))\right\| \leq\ & \left\|\xi_n(\omega) - \xi_{n-1}(\omega)\right\| \\
& + \left\|\xi_{n-1}(\omega) - T_n(\omega, \xi_n(\omega))\right\| \\
& \to 0 \text{ as } n \to \infty. \qquad (3.27)
\end{aligned}$$

Now for each $l \in \{1, 2, \ldots, N\} = I$, by using (3.27) and (3.17) we get that

$$\begin{aligned}
\|\xi_n(\omega) - T_{n+l}(\omega, \xi_n(\omega))\| &\leq \|\xi_n(\omega) - \xi_{n+l}(\omega)\| + \|\xi_{n+l}(\omega) - T_{n+l}(\omega, \xi_{n+l}(\omega))\| \\
&\quad + \|T_{n+l}(\omega, \xi_{n+l}(\omega)) - T_{n+l}(\omega, \xi_n(\omega))\| \\
&\leq \|\xi_n(\omega) - \xi_{n+l}(\omega)\| + \|\xi_{n+l}(\omega) - T_{n+l}(\omega, \xi_{n+l}(\omega))\| \\
&\quad + L\,\|\xi_{n+l}(\omega) - \xi_n(\omega)\| \\
&= (1+L)\,\|\xi_n(\omega) - \xi_{n+l}(\omega)\| + \|\xi_{n+l}(\omega) - T_{n+l}(\omega, \xi_{n+l}(\omega))\| \\
&\quad \to 0 \text{ as } n \to \infty \text{ and for each } \omega \in \Omega. \qquad (3.28)
\end{aligned}$$

Consequently, we have

$$\|\xi_n(\omega) - T_l(\omega, \xi_n(\omega))\| \to 0 \text{ as } n \to \infty \text{ for each } \omega \in \Omega \text{ and for each } l \in I.$$

This completes the proof.

Theorem 3.2. Let X be a separable Banach space and C be a nonempty closed convex subset of X. Let $\{T_i : i \in I = \{1, 2, \ldots, N\}\}$ be N uniformly L-Lipschitzian asymptotically quasi-nonexpansive type random operators from $\Omega \times C$ to C. Assume that $F = \cap_{i=1}^{N} RF(T_i) \neq \emptyset$. Let $\{\xi_n(\omega)\}$ be the implicit random iterative sequence with errors defined by (2.1). Put

$$\begin{aligned}
A_{in}(\omega) &= \max\Big\{ \sup_{\xi(\omega)\in F,\, n\geq 1} \Big(\|T_i^n(\omega, \xi_n(\omega)) - \xi(\omega)\| - \|\xi_n(\omega) - \xi(\omega)\| \Big) \\
&\qquad \vee 0 : i \in I \Big\}
\end{aligned}$$

and

$$\begin{aligned}
B_{in}(\omega) &= \max\Big\{ \sup_{\xi(\omega)\in F,\, n\geq 1} \Big(\|T_i^n(\omega, \eta_n(\omega)) - \xi(\omega)\| - \|\eta_n(\omega) - \xi(\omega)\| \Big) \\
&\qquad \vee 0 : i \in I \Big\}
\end{aligned}$$

where $n = (k-1)N + i$ and $i = i(n) \in I$. Assume that $\sum_{n=1}^{\infty} A_{in}(\omega) < \infty$, $\sum_{n=1}^{\infty} B_{in}(\omega) < \infty$, $0 < \alpha \leq \alpha_n, \beta_n \leq \beta < 1$ for some $\alpha, \beta \in (0, 1)$, $\sum_{n=1}^{\infty} \gamma_n < \infty$ and $\sum_{n=1}^{\infty} c_n < \infty$. If the family $\{T_i : i \in I = \{1, 2, \ldots, N\}\}$ satisfies condition (B) for each $\omega \in \Omega$, then $\{\xi_n\}$ converges strongly to a common random fixed point of the random operators $\{T_i : i \in I\}$.

Proof. By Theorem 3.1 we know that $\lim_{n\to\infty} d(\xi_n(\omega), F)$ exists for each $\omega \in \Omega$. Again by Lemma 3.1 and condition (B), we have that $\lim_{n\to\infty} f(d(\xi_n(\omega), F)) = 0$. Since $f\colon [0, \infty) \to [0, \infty)$ is a nondecreasing function with $f(0) = 0$ so we have $\lim_{n\to\infty} d(\xi_n(\omega), F) = 0$. Hence the proof of Theorem 3.2 follows from Theorem 3.1. This completes the proof.

Theorem 3.3. Let X be a separable Banach space and C be a nonempty closed convex subset of X. Let $\{T_i : i \in I = \{1, 2, \ldots, N\}\}$ be N uniformly L-Lipschitzian asymptotically quasi-nonexpansive type random operators from $\Omega \times C$ to C. Assume that $F = \cap_{i=1}^{N} RF(T_i) \neq \emptyset$ and let one member of the family $\{T_i : i \in I\}$ to be semi-compact random operator. Let $\{\xi_n(\omega)\}$ be the implicit random iterative sequence with errors defined by (2.1). Put

$$\begin{aligned}
A_{in}(\omega) &= \max\Big\{ \sup_{\xi(\omega)\in F,\, n\geq 1} \Big(\|T_i^n(\omega, \xi_n(\omega)) - \xi(\omega)\| - \|\xi_n(\omega) - \xi(\omega)\| \Big) \\
&\qquad \vee 0 : i \in I \Big\}
\end{aligned}$$

and

$$B_{in}(\omega) = \max\Big\{ \sup_{\xi(\omega)\in F,\ n\geq 1} \Big(\|T_i^n(\omega, \eta_n(\omega)) - \xi(\omega)\| - \|\eta_n(\omega) - \xi(\omega)\| \Big) \vee 0 : i \in I \Big\}$$

where $n = (k-1)N + i$ and $i = i(n) \in I$. Assume that $\sum_{n=1}^{\infty} A_{in}(\omega) < \infty$, $\sum_{n=1}^{\infty} B_{in}(\omega) < \infty$, $0 < \alpha \leq \alpha_n, \beta_n \leq \beta < 1$ for some $\alpha, \beta \in (0,1)$, $\sum_{n=1}^{\infty} \gamma_n < \infty$ and $\sum_{n=1}^{\infty} c_n < \infty$. Then $\{\xi_n\}$ converges strongly to a common random fixed point of the random operators $\{T_i : i \in I\}$.

Proof. Let us suppose that T_1 is semi-compact random operator. By Lemma 3.1, we have $\lim_{n\to\infty} \|\xi_n(\omega) - T_1(\omega, \xi_n(\omega))\| = 0$ for each $\omega \in \Omega$. So there exists a subsequence $\{\xi_{n_k}(\omega)\}$ of $\{\xi_n(\omega)\}$ such that $\xi_{n_k}(\omega) \to \xi(\omega)$ for each $\omega \in \Omega$, where ξ is a measurable mapping from Ω to C. Now again by Lemma 3.1, we have

$$\|\xi(\omega) - T_l(\omega, \xi(\omega))\| = \lim_{n\to\infty} \|\xi_{n_k}(\omega) - T_l(\omega, \xi_{n_k}(\omega))\| = 0,$$

for each $\omega \in \Omega$ and for each $l \in I$. This implies that $\xi \in F$. Since $\{\xi_n(\omega)\}$ has a subsequence $\{\xi_{n_k}(\omega)\}$ such that $\xi_{n_k}(\omega) \to \xi(\omega)$ for each $\omega \in \Omega$, we have that $\lim_{n\to\infty} d(\xi_n(\omega), F) = 0$. By Theorem 3.1, we obtain $\{\xi_n\}$ converges strongly to a common random fixed point of the random operators $\{T_i : i \in I\}$. This completes the proof.

Remark 3.1. Our results extend and improve the corresponding results of Plubtieng et al. [12] to the case of more general class of asymptotically quasi-nonexpansive random operators and composite implicit random iteration process considered in this paper.

Remark 3.2. Our results also extend and improve the corresponding results of Beg and Thakur [4] to the case of more general class of asymptotically quasi-nonexpansive random operators and composite implicit random iteration process with errors considered in this paper.

Remark 3.3. Our results also extend the corresponding results of Benerjee and Choudhury [1] to the case of more general class of asymptotically quasi-nonexpansive random operators considered in this paper.

Remark 3.4. Our results also extend and improve the corresponding results of [2, 14, 15] to the case of composite implicit random iteration process with errors for a finite family of asymptotically quasi-nonexpansive type random operators considered in this paper

References

[1] S. Banerjee and B. S. Choudhury, *Composite implicit random iterations for approximating common random fixed point for a finite family of asymptotically nonexpansive random operators*, Commun. Korean Math. Soc. 20(1) (2011), 23-35.

[2] I. Beg and M. Abbas, *Iterative procedures for solutions of random operator equations in Banach spaces*, Journal of Mathematical Analysis and Applications 315(2006), no. 1, 181-201.

[3] I. Beg and M. Abbas, *Random fixed point theorems for a random operator on an unbounded subset of a Banach space*, Appl. Math. Lett. (2007), doi:10.1016/j.aml.2007.10.015.

[4] I. Beg and B. Thakur, *Solution of random operator equations using general composite implicit iteration process*, Int. J. Modern Math. 4(1)(2009), 19-34.

[5] A. T. Bharucha-Reid, *Fixed point theorems in Probabilistic analysis*, Bulletin of the American Mathematical Society 82(1976), no. 5, 641-657.

[6] B. S. Choudhury, *Convergence of a random iteration scheme to a random fixed point*, J. Appl. Math. Stochastic Anal. 8(2) (1995), 139-142.

[7] B. S. Choudhury, *Random Mann iteration scheme*, Appl. Math. Lett. 16(2003), 93-96.

[8] B. S. Choudhury, *An iteration for finding a common random fixed point*, J. Appl. Math. Stochastic Anal. 4(2004), 385-394.

[9] C. J. Himmelberg, *Measurable relations*, Fundamenta Mathematicae 87(1975), 53-72.

[10] S. Itoh, *Random fixed point theorems with an application to random differential equations in Banach spaces*, Journal of Mathematical Analysis and Applications 67(1979), no. 2, 261-273.

[11] T. C. Lin, *Random approximations and random fixed point theorems for continuous* 1*-set contractive random maps*, Proc. Amer. math. Soc. 123(1995), 1167-1176.

[12] S. Plubtieng, P. Kumam and R. Wangkeeree, *Approximation of a common random fixed point for a finite family of random operators*, Int. J. Math. Math. Sci. Vol. 2007, Article ID 69626, 12 pages, doi:10.1155/2007/69626.

[13] B. E. Rhoades, *Iteration to obtain random solutions and fixed points of operators in uniformly convex Banach space*, Soochow J. Math. 27(4) (2001), 401-404.

[14] G. S. Saluja, *Random fixed point of three-step random iterative process with errors for asymptotically quasi-nonexpansive random operator*, The Math. Stud. 77(1-4) (2008), 161-176.

[15] G. S. Saluja, *Convergence of implicit random iteration process with errors for a finite family of symptotically quasi-nonexpansive random operators*, The J. Nonlinear Sci. Appl. 4(4) (2011), 292-307.

[16] J. Schu, *Weak and strong convergence theorems to fixed points of asymptotically nonexpansive mappings*, Bull. Austral. Math. Soc. 43(1991), 153-159.

[17] N. Shahzad, *Random fixed points of pseudo-contractive random operators*, J. Math. Anal. Appl. 296(2004), 302-308.

[18] K. K. Tan and H. K. Xu, *Approximating fixed points of nonexpansive mappings by the Ishikawa iteration process*, J. Math. Anal. Appl. 178(1993), 301-308.

[19] H. K. Xu, *Some random fixed point theorems for condensing and nonexpansive operators*, Proc. Amer. Math. Soc. 100(1990), 103-123.

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