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Generalized Sasakian-Space-Forms with Conharmonic curvature tensor

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Abstract The object of the present paper is to study conharmonically flat generalized Sasakian-space-forms and conharmonically locally $\phi-$symmetric generalized Sasakian-space-forms. Interesting relations between conharmonic curvature tensor, projective curvature tensor and conformal curvature tensor of a generalized Sasakian-space-form of dimension greater than three have been established. Obtained results are supported by illustrative examples.

1 Introduction

Recently, P. Alegre, D. Blair and A. Carriazo [2] introduced and studied generalized Sasakian-space-forms. These space-forms are defined as follows:
Given an almost contact metric manifold $M(\phi, \xi, \eta, g)$, we say that M is generalized Sasakian-space-form if there exist three functions f_1, f_2, f_3 on M such that the curvature tensor R of M is given by

$$\begin{aligned} R(X,Y)Z &= f_1\{g(Y,Z)X - g(X,Z)Y\} \\ &+ f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ &+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \end{aligned}$$

for any vector fields X, Y, Z on M. In such a case we denote the manifold as $M(f_1, f_2, f_3)$. These kind of manifolds appear as a generalization of the well known Sasakian-space-forms, which can be obtained as a particular case of generalized Sasakian-space-forms by taking $f_1 = \frac{c+3}{4}$, $f_2 = f_3 = \frac{c-1}{4}$. But, it is to be noted that generalized Sasakian-space-forms are not merely generalization of Sasakian-space-forms. It also contains a large class of almost contact manifolds. For example, it is known that [3] any three-dimensional $(\alpha, \beta)-$trans Sasakian manifold with α, β depending on ξ is a generalized Sasakian-space-form. However, we can find generalized Sasakian-space-forms with non-constant functions and arbitrary dimensions. In [2], the authors cited several examples of generalized Sasakian-space-forms in terms of warped product spaces. In [9], U. K. Kim studied conformally flat generalized Sasakian-space-forms and locally symmetric generalized Sasakian-space-forms. In Riemannian geometry, one of the basic interests is curvature property and to what extent this determines the manifold itself. Two important curvature properties are flatness and symmetry. In the paper [5], we have studied projectively flat generalized-Sasakian-space-forms. In [6], we also have studied locally $\phi-$symmetric generalized Sasakian-space-forms. In this connection, it should be mentioned that in [10], T. Takahashi introduced the notion of locally $\phi-$symmetric manifolds in the context of Sasakian geometry. In the present paper, we like to study conharmonically flat generalized Sasakian-space-forms and conharmonically locally $\phi-$symmetric generalized Sasakian-space-forms, because after conformal and quasi-conformal curvature tensor, conharmonic curvature tensor is an important one from the geometric point of view. Let M be a $(2n + 1)-$dimensional $(n > 1)$ Riemannian manifold of class C^∞. The conharmonic curvature tensor C is considered as an invariant of the conharmonic transformation defined by Y Ishii [8]. It satisfies all the symmetric properties of the Riemannian curvature tensor. Conharmonic curvature tensor are also important from the physical point of view. In [1], Abdussattar showed that sufficient condition for a space-time to be conharmonic to a flat space-time is that the tensor C vanishes identically. A conharmonically flat space-time is either empty, in which case it is flat, or, is filled with a distribution represented by energy momentum tensor T possessing the algebraic structure of an electromagnetic field, and is conformal to flat space-time [8]. Also, he described the gravitational field due to a distribution of pure radiation in presence of disordered radiation, by means of spherically symmetric

conharmonically flat space time. The present paper is organized as follows:
In Section 2 we review some preliminary results. In Section 3, we study conharmoically flat generalized Sasakian-space-forms and obtain that if a generalized Sasakian-space-form of dimension greater than three is conharmonically flat, then it is projectively flat, the converse holds when $f_1 = f_3$. Section 4 deals with conharmonically locally $\phi-$symmetric generalized Sasakian-space-forms. Here we prove that a conharmonically locally $\phi-$symmetric generalized Sasakian-space-form of dimension greater than three is conformally flat and the converse is also true if f_1 and f_3 are constants. In this section, we also show that if a generalized Sasakian space-form of dimension greater than three is conharmonically locally $\phi-$symmetric, then its scalar curvature is constant, the converse is valid if $f_2 = 0$. Both, Section 4 and Section 5 contains illustrative examples to show the validity of the obtained results.

2 Preliminaries

In an almost contact metric manifold we have [4]

$$\phi^2 X = -X + \eta(X)\xi, \quad \phi\xi = 0, \tag{2.1}$$

$$\eta(\xi) = 1, \quad g(X,\xi) = \eta(X), \quad \eta(\phi X) = 0, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.3}$$

$$g(\phi X, Y) = -g(X, \phi Y), \quad g(\phi X, X) = 0, \tag{2.4}$$

$$(\nabla_X \eta)(Y) = g(\nabla_X \xi, Y), \tag{2.5}$$

where ϕ is a $(1,1)$ tensor, ξ is a vector field, η is an $1-$form and g is a Riemannian metric. The metric g induces an inner product on the tangent space of the manifold. Again, we know that [2] in a generalized Sasakian-space-form

$$\begin{aligned} R(X,Y)Z = {} & f_1\{g(Y,Z)X - g(X,Z)Y\} \\ & + f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ & + f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ & + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \end{aligned} \tag{2.6}$$

for any vector fields X, Y, Z on M, where R denotes the curvature tensor of M and f_1, f_2, f_3 are smooth functions on the manifold. The Ricci operator Q, Ricci tensor S and the scalar curvature r of the manifold of dimension $(2n+1)$ are respectively given by [9]

$$QX = (2nf_1 + 3f_2 - f_3)X - (3f_2 + (2n-1)f_3)\eta(X)\xi, \tag{2.7}$$

$$S(X,Y) = (2nf_1 + 3f_2 - f_3)g(X,Y) - (3f_2 + (2n-1)f_3)\eta(X)\eta(Y), \tag{2.8}$$

$$r = 2n(2n+1)f_1 + 6nf_2 - 4nf_3. \tag{2.9}$$

For a $(2n+1)-$dimensional $(n > 1)$ almost contact metric manifold the conharmonic curvature tensor C is given by

$$\begin{aligned} C(X,Y)Z = {} & R(X,Y)Z - \frac{1}{2n-1}[S(Y,Z)X - S(X,Z)Y \\ & + g(Y,Z)QX - g(X,Z)QY]. \end{aligned} \tag{2.10}$$

3 Conharmonically flat generalized Sasakian-space-forms

Definition 3.1. A $(2n+1)-$dimensional $(n > 1)$ generalized Sasakian-space-form M is called conharmonically flat if it satisfies

$$C(X,Y)Z = 0$$

for any vector fields X, Y, Z on the manifold.

Let us consider that M is conharmonically flat. Then, by Definition 3.1 and using (2.6), (2.8) and (2.10), we get

$$\begin{aligned}
&f_1\{g(Y,Z)X - g(X,Z)Y\} \\
&+ f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\
&+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\
&+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \\
&= \frac{2}{2n-1}(2nf_1 + 3f_2 - f_3)(g(Y,Z)X - g(X,Z)Y) \\
&+ \frac{1}{2n-1}(3f_2 + (2n-1)f_3)(\eta(Y)\eta(Z)X \\
&- \eta(X)\eta(Z)Y + \eta(X)\xi - \eta(Y)\xi). \qquad (3.1)
\end{aligned}$$

Replacing Z by ϕZ, we obtain from above

$$\begin{aligned}
&f_1\{g(Y,\phi Z)X - g(X,\phi Z)Y\} \\
&+ f_2\{g(X,\phi^2 Z)\phi Y - g(Y,\phi^2 Z)\phi X + 2g(X,\phi Y)\phi^2 Z\} \\
&+ f_3\{g(X,\phi Z)\eta(Y)\xi - g(Y,\phi Z)\eta(X)\xi\} \\
&= \frac{2}{2n-1}(2nf_1 + 3f_2 - f_3)(g(Y,\phi Z)X - g(X,\phi Z)Y) \\
&+ \frac{1}{2n-1}(3f_2 + (2n-1)f_3)(\eta(X)\xi - \eta(Y)\xi). \qquad (3.2)
\end{aligned}$$

In (3.2), putting $X = Z = \xi$, we get

$$\frac{1}{2n-1}(3f_2 + (2n-1)f_3)(\xi - \eta(Y)\xi) = 0. \qquad (3.3)$$

The above equation is true for any Y. If we choose Y other than ξ, then the above equation yields

$$f_3 = \frac{3f_2}{1-2n}.$$

It is known that [5] a $(2n+1)-$dimensional $(n > 1)$ generalized Sasakian-space-form is projectively flat if and only $f_3 = \frac{3f_2}{1-2n}$. Hence, we see that the manifold under consideration is projectively flat.

Conversely, suppose that the manifold M is projectively flat. It is well known that a projectively flat Riemannian manifold is a manifold of constant curvature. Hence, M is of constant curvature λ, (say). Therefore, we have

$$R(X,Y)Z = \lambda(g(Y,Z)X - g(X,Z)Y). \qquad (3.4)$$

The above equation yields

$$S(X,Y) = 2n\lambda g(X,Y). \qquad (3.5)$$

From (2.10), (3.4) and (3.5), we get

$$C(X,Y)Z = \frac{\lambda(1+2n)}{1-2n}(g(Y,Z)X - g(X,Z)Y). \qquad (3.6)$$

From (3.6), it follows that M is not conharmonically flat if λ is non-zero. It is known that [5] a projectively flat generalized Sasakian-space-form is flat if $f_1 = f_3$. Now, if we consider $f_1 = f_3$, then the manifold is flat and hence by (3.4), $\lambda = 0$. In such case (3.6) yields $C(X,Y)Z = 0$. Thus, we are in a position to state the following:

Theorem 3.2. *If a* $(2n+1)-$*dimensional* $(n > 1)$ *generalized Sasakian-space-form is conharmonically flat, then it is projectively flat, the converse holds when* $f_1 = f_3$.

It is known that [5] a $(2n+1)-$dimensional $(n > 1)$ generalized Sasakian-space-form is projectively flat if and only if it is Ricci semisymmetric. So, we can state the following:

Corollary 3.3. *If a* $(2n+1)-$*dimensional* $(n > 1)$ *generalized Sasakian-space-form is conharmonically flat, then it is Ricci semisymmetric, the converse holds when* $f_1 = f_3$.

Example 3.4. Let $N(a,b)$ be a generalized complex space-form of dimension 4, then by [2], $M = \mathbb{R} \times_f N$, endowed with the almost contact metric structure (ϕ, ξ, η, g_f) is a generalized Sasakian-space-form $M(f_1, f_2, f_3)$ of dimension 5 with

$$f_1 = \frac{a - f'^2}{f^2}, \quad f_2 = \frac{b}{f^2}, \quad f_3 = \frac{a - f'^2}{f^2} + \frac{f''}{f}$$

where f is a function of $t \in \mathbb{R}$ and f' denotes differentiation of f with respect to t. Let us choose f as a constant. and $a = -b$. Then $f_3 = \frac{3f_2}{1-2.2}$ and $f_1 = f_3$. Therefore, by Theorem 3.1 M is conharmonically flat.

4 Conharmonically locally $\phi-$symmetric generalized Sasakian-space-forms

Definition 4.1. A $(2n+1)-$dimensional $(n > 1)$ generalized Sasakian-space-form will be called conharmonically locally $\phi-$symmetric if it satisfies $\phi^2(\nabla_W C)(X,Y)Z = 0$, for all X, Y, Z orthogonal to ξ.

In this connection it should be mentioned that the notion of locally $\phi-$symmetric manifolds was introduced by T. Takahashi [10] in the context of Sasakian geometry.

From (2.6), (2.7), (2.8) and (2.10), we get by covariant differentiation

$$\begin{aligned}
(\nabla_W C)(X,Y)Z = {} & df_1(W)\{g(Y,Z)X - g(X,Z)Y\} \\
& + df_2(W)\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\
& + f_2\{g(X,\phi Z)(\nabla_W \phi)Y + g(X,(\nabla_W \phi)Z)\phi Y \\
& - g(Y,\phi Z)(\nabla_W \phi)X - g(Y,(\nabla_W \phi)Z)\phi X \\
& + 2g(X,\phi Y)(\nabla_W \phi)Z + 2g(X,(\nabla_W \phi)Y)\phi Z\} \\
& + df_3(W)\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\
& + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \\
& + f_3\{(\nabla_W \eta)(X)\eta(Z)Y + \eta(X)(\nabla_W \eta)(Z)Y \\
& - (\nabla_W \eta)(Y)\eta(Z)X - \eta(Y)(\nabla_W \eta)(Z)X \\
& + g(X,Z)(\nabla_W \eta)(Y)\xi + g(X,Z)\eta(Y)(\nabla_W \xi) \\
& - g(Y,Z)(\nabla_W \eta)(X)\xi - g(Y,Z)\eta(X)(\nabla_W \xi)\} \\
& - \frac{1}{2n-1}[d(2nf_1 + 3f_2 - f_3)(W)g(Y,Z)X \\
& - (3f_2 + (2n-1)f_3)((\nabla_W \eta)(Y)\eta(Z) + \eta(Y)(\nabla_W \eta)(Z))X \\
& - d(3f_2 + (2n-1)f_3)\eta(Y)\eta(Z)X \\
& - d(2nf_1 + 3f_2 - f_3)(W)g(X,Z)Y \\
& + (3f_2 + (2n-1)f_3)((\nabla_W \eta)(X)\eta(Z) + \eta(X)(\nabla_W \eta)(Z)) \\
& + d(3f_2 + (2n-1)f_3)(W)\eta(X)\eta(Z)Y] \\
& - \frac{1}{2n-1}[d(2nf_1 + 3f_2 - f_3)(W)(g(Y,Z)X - g(X,Z)Y) \\
& - d(3f_2 + (2n-1)f_3)(W)(\eta(X)\xi - \eta(Y)\xi \\
& + (\nabla_W \eta)(X)\xi - (\nabla_W \eta)(Y)\xi \\
& + \eta(X)\nabla_W \xi - \eta(Y)\nabla_W \xi)],
\end{aligned} \tag{4.1}$$

where ∇ denotes covariant differentiation on M with respect to Levi-Civita connection. Taking X, Y, Z orthogonal to ξ, we get from (4.1)

$$
\begin{aligned}
(\nabla_W C)(X,Y)Z = {} & df_1(W)\{g(Y,Z)X - g(X,Z)Y\} \\
& + df_2(W)\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\
& + f_2\{g(X,\phi Z)(\nabla_W \phi)Y + g(X,(\nabla_W \phi)Z)\phi Y \\
& - g(Y,\phi Z)(\nabla_W \phi)X - g(Y,(\nabla_W \phi)Z)\phi X \\
& + 2g(X,\phi Y)(\nabla_W \phi)Z + 2g(X,(\nabla_W \phi)Y)\phi Z\} \\
& - \frac{2}{2n-1} d(2nf_1 + 3f_2 - f_3)(W)(g(Y,Z)X - g(X,Z)Y). \qquad (4.2)
\end{aligned}
$$

From (4.2), using (2.1) and considering X, Y, Z orthogonal to ξ, we get

$$
\begin{aligned}
\phi^2(\nabla_W C)(X,Y)Z = {} & - df_1(W)\{g(Y,Z)X - g(X,Z)Y\} \\
& - df_2(W)\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\
& - f_2\{g(X,\phi Z)(\nabla_W \phi)Y + g(X,(\nabla_W \phi)Z)\phi Y \\
& - g(Y,\phi Z)(\nabla_W \phi)X - g(Y,(\nabla_W \phi)Z)\phi X \\
& + 2g(X,\phi Y)(\nabla_W \phi)Z + 2g(X,(\nabla_W \phi)Y)\phi Z\} \\
& + \frac{2}{2n-1} d(2nf_1 + 3f_2 - f_3)(W)(g(Y,Z)X - g(X,Z)Y). \qquad (4.3)
\end{aligned}
$$

Suppose that the manifold is conharmonically locally $\phi-$symmetric. Then from (4.3) we obtain

$$
\begin{aligned}
& df_1(W)\{g(Y,Z)g(X,W) - g(X,Z)g(Y,W)\} \\
& + df_2(W)\{g(X,\phi Z)g(\phi Y,W) - g(Y,\phi Z)g(\phi X,W) + 2g(X,\phi Y)g(\phi Z,W)\} \\
& + f_2\{g(X,\phi Z)g((\nabla_W \phi)Y,W) + g(X,(\nabla_W \phi)Z)g(\phi Y,W) \\
& - g(Y,\phi Z)g((\nabla_W \phi)X,W) - g(Y,(\nabla_W \phi)Z)g(\phi X,W) \\
& + 2g(X,\phi Y)g((\nabla_W \phi)Z,W) + 2g(X,(\nabla_W \phi)Y)g(\phi Z,W)\} \\
& - \frac{2}{2n-1} d(2nf_1 + 3f_2 - f_3)(W)(g(Y,Z)g(X,W) - g(X,Z)g(Y,W)) \\
& = 0. \qquad (4.4)
\end{aligned}
$$

Putting $X = W = e_i$, where $\{e_i\}$ is an orthonormal basis of the tangent space at each point of the manifold, and taking summation over i, $i = 1, 2, ..., 2n+1$, we get

$$
\begin{aligned}
& 2ndf_1(W)g(Y,Z) \\
& + 3df_2(W)g(Y,Z) \\
& + f_2\{g(\phi Z,(\nabla_{e_i}\phi)Y) + g((\nabla_{e_i}\phi)Z,\phi Y) \\
& - \sum_i g(Y,\phi Z)g((\nabla_{e_i}\phi)e_i,e_i) \\
& + 2g(\phi Y,(\nabla_{e_i}\phi)Z) + 2g((\nabla_{e_i}\phi)Y,\phi Z) \\
& - \frac{4n}{2n-1} d(2nf_1 + 3f_2 - f_3)(W)g(Y,Z). \qquad (4.5)
\end{aligned}
$$

Putting $Z = \phi Y$, we have from the above equation

$$
\begin{aligned}
& f_2\{g(\phi^2 Y,(\nabla_{e_i}\phi Y)) + g((\nabla_{e_i}\phi)\phi Y,\phi Y) \\
& - \sum_i g(Y,\phi^2 Y)g((\nabla_{e_i}\phi)e_i,e_i) \\
& + 2g(\phi Y,(\nabla_{e_i}\phi)\phi Y) + 2g((\nabla_{e_i}\phi)Y,\phi^2 Y) \\
& = 0. \qquad (4.6)
\end{aligned}
$$

The above equation is true for any arbitrary Y orthogonal to ξ. Considering Y other than ξ, we get from (4.6)

$$f_2 = 0. \qquad (4.7)$$

It is known that [9] a generalized Sasakian-space-form is conformally flat if and only if $f_2 = 0$. Thus, the manifold under consideration is conformally flat.

Conversely, let the manifold is conformally flat. Therefore, $f_2 = 0$. Then (4.3) yields

$$\phi^2(\nabla_W C)(X,Y)Z = -df_1(W) + \frac{2}{2n-1}d(2nf_1 - f_3)(W)(g(Y,Z)X - g(X,Z)Y). \quad (4.8)$$

From the above equation it follows that if f_1 and f_3 are constants, then it is conharmonically locally $\phi-$symmetric. The above discussion helps us to state the following:

Theorem 4.2. *If a $(2n+1)-$dimensional $(n > 1)$ generalized Sasakian-space-form is conharmonically locally $\phi-$symmetric, then it is conformally flat. The converse is true when f_1 and f_3 are constants.*

Again suppose that the manifold is conharmonically locally $\phi-$symmetric. Then by (4.7) and (4.4), it follows that

$$\begin{aligned} &df_1(W)(g(Y,Z)g(X,W) - g(X,Z)g(Y,W)) \\ &- \frac{2}{2n-1}d(2nf_1 - f_3)(W)(g(Y,Z)g(X,W) - g(X,Z)g(Y,W)) = 0. \end{aligned} \quad (4.9)$$

From the above equation

$$df_1(W) - \frac{2}{2n-1}d(2nf_1 - f_3)(W) = 0.$$

The above equation gives

$$f_1 - \frac{2}{2n-1}(2nf_1 - f_3) = \text{constant}.$$

The above equation yields

$$(2n+1)f_1 - 2f_3 = k, \quad (4.10)$$

where k is constant. From (2.9), and (4.7) we have

$$\begin{aligned} r &= 2n(2n+1)f_1 - 4nf_3 \\ &= 2n((2n+1)f_1 - 2f_3). \end{aligned} \quad (4.11)$$

In view of (4.10) and (4.11), it follows that $r =$ a constant. Hence, we see that if M is conharmonically locally $\phi-$symmetric, then r is a constant.

Conversely, if r is a constant, then by (2.9), $((2n+1)f_1 + 3f_2 - 2f_3)$ is a constant. Which implies

$$d((2n+1)f_1 + 3f_2 - 2f_3)(W) = 0. \quad (4.12)$$

If we consider $f_2 = 0$, then the above equation yields

$$d(f_1)(W) = \frac{2}{2n-1}(2nf_1 - f_3)(W). \quad (4.13)$$

In view of (4.13), (4.3) takes the form

$$\phi^2(\nabla_W C)(X,Y)Z = 0. \quad (4.14)$$

From (4.14), it follows that the manifold is conharmonically locally $\phi-$symmetric. Now, we are in a position to state the following:

Theorem 4.3. *If a $(2n+1)-$dimensional $(n > 1)$ generalized Sasakian-space-form is conharmonically locally $\phi-$symmetric then its scalar curvature is constant, the converse holds when $f_2 = 0$.*

Example 4.4. Let us now give an example of a generalized Sasakian-space-form which is conharmonically locally $\phi-$symmetric.

In [2], it is shown that $\mathbb{R} \times_f \mathbb{C}^m$ is a generalized Sasakian-space-form with

$$f_1 = -\frac{(f'^2}{f^2}, \quad f_2 = 0, \quad f_3 = -\frac{(f'^2}{f^2} + \frac{f''}{f},$$

where $f = f(t)$, $t \in \mathbb{R}$ and f' denotes derivative of f with respect to t. If we choose $m = 4$, and $f(t) = e^t$, then M is a $5-$dimensional conformally flat generalized Sasakian-space-form, because $f_2 = 0$. We also see that f_1 and f_3 are constants. Therefore, by Theorem 4.1 M is conharmonically locally $\phi-$symmetric. Again from (2.9), and the values of f_1, f_2, f_3, it follows that the scalar curvature of the manifold is constant.

Example 4.5. For a Sasakian-space-form of dimension greater than three and of constant $\phi-$sectional curvature 1, $f_1 = 1, f_2 = f_3 = 0$. So, it is conharmonically locally $\phi-$symmetric and its scalar curvature is constant.

Remark 4.6. The notion of quarter-symmetric metric connection was introduced by S. Golab [7]. The torsion tensor of the quarter-symmetric metric connection is given by

$$T(X,Y) = \eta(Y)X - \eta(X)Y.$$

If X, Y are orthogonal to ξ, then the torsion tensor vanishes and the quarter-symmetric metric connection reduces to Levi-Civita connection. Therefore, all the results of Section 4 are of the same form with respect to quarter-symmetric metric connection and Levi-Civita connection.

References

[1] Abdussattar, D. B., On conharmonic transformations in general relativity, Bull. Cal. Math. Soc., 6(1996), 465-700.

[2] Alegre, P., Blair, D. E. and Carriazo, A., Generalized Sasakian-space-forms, Israel J. Math., 14(2004), 157-183.

[3] Alegre P. and Cariazo, A., Structures on generalized Sasakian-space-forms, Differential Geom. and its application, 26(2008), 656-666.

[4] Blair, D. E., Contact Manifolds in Riemannian geometry, Lecture notes in Mathematics, 509, Springer-Verlag Berlin (1976).

[5] De, U. C. and Sarkar, A., On the projective curvature tensor of generalized Sasakian space-forms, Quaestiones Mathematicae, 33(2010), 245-252.

[6] De, U. C. and Sarkar, A., Some results on generalized Sasakian-space-forms, Thai. J. Math., 8(2010), 1-10.

[7] Golab, S., On semi-symmetric and quarter-symmetric linear connections, Tensor (N. S.), 29(1975), 249-254.

[8] Ishii, Y., On conharmonic transformations, Tensor(N. S.), 7(1957), 73-80.

[9] Kim, U. K., Conformally flat generalized Sasakian-space-forms and locally symmetric generalized Sasakian-space-forms, Note di matemetica, 26(2006), 55-67.

[10] Takahashi, T., Sasakian $\phi-$symmetric spaces, Tohoku Math. J., 29(1977), 91-113.

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