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A GENERALIZATION OF VOIGT FUNCTION INVOLVING GENERALIZED WHITTAKER AND BESSEL FUNCTIONS

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Abstract In the present paper we study a new generalization of Voigt functions involving the product of generalized Bessel and generalized Whittaker functions. We obtain its explicit representation and then derive partly bilateral and partly unilateral representation and two generating functions. Some special cases of main results have also been given.

1 Introduction

The familiar Voigt functions $K(x,y)$ and $L(x,y)$ play an important role in astrophysical spectroscopy, neutrons physics, plasma physics and several other diverse field of physics. Furthermore, the function $K(x,y)+iL(x,y)$ is, except for a numerical factor, identical to the so-called plasma dispersion function which is tabulated by Fried and Conte [2].

A number of authors namely Srivastava and Chen [15], Srivastava et al. [19], Pathan and Shahwan [10], Goyal and Mukherjee [3], Gupta et al [4], Pathan et al [8], Pathan et al [9], Garg and Jain [12] and Gupta and Gupta [5] studied various mathematical properties for the Voigt functions and their generalizations.

For the purposes of our present study, we begin by recalling here the following representations due to Srivastava and Miller [18, p.113, eq.(8)]:

$$V_{\mu,\nu}(x,y) \;=\; \sqrt{\frac{x}{2}}\int_0^{\infty} t^{\mu}\exp\left(-yt-\frac{1}{4}t^2\right) J_{\nu}(xt)\,dt \quad (x,y\in R^+;\ Re(\mu+\nu)>-1), \tag{1.1}$$

where $J_\nu(z)$ is well known Bessel function of order ν.

$$\text{So that}\ \ K(x,y)=V_{\frac{1}{2},-\frac{1}{2}}(x,y) \quad \text{and} \quad L(x,y)=V_{\frac{1}{2},\frac{1}{2}}(x,y). \tag{1.2}$$

Subsequently, following the work of Srivastava and Miller [18] closely, Klush [6] proposed a unification (and generalization) of the Voigt functions $K(x,y)$ and $L(x,y)$ in the form

$$\Omega_{\mu,\nu}(x,y,z) = \sqrt{\frac{x}{2}}\int_0^{\infty} t^{\mu}\exp\left(-yt-zt^2\right) J_{\nu}(xt)\,dt \quad (x,y,z\in R^+;\ Re(\mu+\nu)>-1). \tag{1.3}$$

In view of the above facts, we introduce and study a new generalization of Voigt functions involving the product of generalized Bessel and generalized Whittaker functions defined as follows

$$\Omega^{\mu,\rho,\sigma_1,\sigma_2\cdots,\sigma_n}_{\eta,\nu,\lambda}(x,y,z,u_1,u_2,\cdots,u_n)$$

$$= \sqrt{\frac{x}{2}}\int_0^{\infty} t^{\eta+2\rho}\exp\left(-yt-zt^2\right) J^{\mu}_{\nu,\lambda}(xt)\, M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1t^2,\cdots,2u_nt^2)\,dt \tag{1.4}$$

$(x,y,z,u_1,u_2,\cdots,u_n\in R^+,\ Re\{\eta+\nu+2(\rho+\lambda+\sigma_1+\sigma_2+\cdots+\sigma_n)\}>-2)$,

where $J^{\mu}_{\nu,\lambda}(z)$ is the generalization of Bessel function defined by Pathak [7] as follows

$$J^{\mu}_{\nu,\lambda}(z)=\sum_{m=0}^{\infty}\frac{(-1)^m\,(z/2)^{\nu+2\lambda+2m}}{\Gamma(\lambda+m+1)\,\Gamma(\nu+\lambda+\mu m+1)} \tag{1.5}$$

and $M_{\rho,\sigma_1,\cdots,\sigma_n}(z_1,\cdots,z_n)$ is the generalized Whittaker function [17, p.63(15)].

Taking $n=1$ in (1.4), we get the generalized Voigt function given by Gupta and Gupta [5]. In (1.4) for $n=1$, if we take $\sigma_1=-\frac{1}{2}-\rho$ and $u=\frac{z}{2}$ (Also See [5], [16]), we get the generalized Voigt function given by Srivastava et al [19]. Further, which gives the known generalization of Voigt function defined by Srivastava and Chen [15] on taking $z=\frac{1}{2}$.

2 Explicit Representation

We make use of the series representation of the generalized Bessel function $J^{\mu}_{\nu,\lambda}(xt)$, the exponential function $\exp(-yt)$ and generalized Whittaker function $M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1t^2,2u_2t^2,\cdots,2u_nt^2)$ and interchanging the order of summation and integration, we get

$$\Omega^{\mu,\rho,\sigma_1,\sigma_2\cdots,\sigma_n}_{\eta,\nu,\lambda}(x,y,z,u_1,u_2,\cdots,u_n)=\left(\frac{x}{2}\right)^{\nu+2\lambda+1/2}(2u_1)^{\sigma_1+1/2}(2u_2)^{\sigma_2+1/2}\cdots(2u_n)^{\sigma_n+1/2}$$

$$\cdot\sum_{m,s,m_1,\cdots,m_n=0}^{\infty}\frac{(-1)^m(\sigma_1+\sigma_2+\cdots\sigma_n-\rho+\frac{n}{2})_{m_1+m_2+\cdots+m_n}(x/2)^{2m}}{\Gamma(\lambda+m+1)\Gamma(\nu+\lambda+\mu m+1)(2\sigma_1+1)_{m_1}(2\sigma_2+1)_{m_2}\cdots(2\sigma_n+1)_{m_n}}\frac{(-y)^s}{s!}$$

$$\cdot\frac{(2u_1)^{m_1}}{m_1!}\cdots\frac{(2u_n)^{m_n}}{m_n!}\int_0^{\infty}t^{\eta+2\rho+\nu+2\lambda+2(\sigma_1+\cdots+\sigma_n)+s+2m+n+2(m_1+\cdots+m_n)}\,e^{-(z+u_1+\cdots+u_n)t^2}\,dt. \tag{2.1}$$

Using the following result in (2.1)

$$\int_0^{\infty}t^{\lambda}e^{-zt^2}dt=\frac{1}{2}\Gamma\frac{(\lambda+1)}{2}\,(z)^{-(\lambda+1)/2}\qquad(Re(z)>0,\ Re(\lambda)>-1), \tag{2.2}$$

and adjusting the parameters, we get

$$\Omega^{\mu,\rho,\sigma_1,\sigma_2\cdots,\sigma_n}_{\eta,\nu,\lambda}(x,y,z,u_1,u_2,\cdots,u_n)=\frac{2^{\sigma_1+\sigma_2+\cdots+\sigma_n+\frac{n-3}{2}-\nu-2\lambda}\,x^{\nu+2\lambda+1/2}u_1^{\sigma_1+1/2}u_2^{\sigma_2+1/2}\cdots u_n^{\sigma_n+1/2}}{Z^{\alpha}}$$

$$\cdot\sum_{m,s,m_1,\cdots,m_n=0}^{\infty}\frac{(\sigma_1+\sigma_2+\cdots\sigma_n-\rho+\frac{n}{2})_{m_1+m_2+\cdots+m_n}\,\Gamma\{\alpha+m+(m_1+m_2+\cdots+m_n)+s/2\}}{\Gamma(\lambda+m+1)\Gamma(\nu+\lambda+\mu m+1)(2\sigma_1+1)_{m_1}(2\sigma_2+1)_{m_2}\cdots(2\sigma_n+1)_{m_n}}$$

$$\cdot\left(\frac{-x^2}{4Z}\right)^m\frac{(-y/\sqrt{Z})^s}{s!}\frac{(2u_1/Z)^{m_1}}{m_1!}\frac{(2u_2/Z)^{m_2}}{m_2!}\cdots\frac{(2u_n/Z)^{m_n}}{m_n!}, \tag{2.3}$$

where $Z=z+u_1+u_2+\cdots+u_n$ and $\alpha=\lambda+\frac{\eta+\nu+1}{2}+\rho+\frac{n}{2}+(\sigma_1+\sigma_2+\cdots+\sigma_n)$.

On separating the s-series into its even and odd terms, we get

$$\Omega^{\mu,\rho,\sigma_1,\sigma_2\cdots,\sigma_n}_{\eta,\nu,\lambda}(x,y,z,u_1,u_2,\cdots,u_n)=\frac{2^{\sigma_1+\sigma_2+\cdots+\sigma_n+\frac{n-3}{2}-\nu-2\lambda}\,x^{\nu+2\lambda+1/2}u_1^{\sigma_1+1/2}u_2^{\sigma_2+1/2}\cdots u_n^{\sigma_n+1/2}}{Z^{\alpha}}$$

$$\cdot\sum_{m,s=0}^{\infty}\frac{(-x^2/4Z)^m(y^2/4Z)^s}{\Gamma(\lambda+m+1)\Gamma(\nu+\lambda+\mu m+1)s!}\frac{\Gamma(\alpha+m+s)}{(1/2)_s}$$

$$\cdot\Big\{F_c^{(n)}[\alpha+m+s,\sigma_1+\sigma_2+\cdots+\sigma_n-\rho+\frac{n}{2};2\sigma_1+1,2\sigma_2+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z},\cdots,\frac{2u_n}{Z}]$$

$$-\tfrac{y\ (\alpha+m+s)_{1/2}}{\sqrt{Z}\ (2s+1)}$$

$$F_c^{(n)}[\alpha+1/2+m+s,\sigma_1+\sigma_2+\cdots+\sigma_n-\rho+\frac{n}{2};2\sigma_1+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z},\cdots,\frac{2u_n}{Z}]\Big\} \tag{2.4}$$

$$\big(x,y,z,u_1,u_2,\cdots,u_n\in R^+,\ \ Re\{\eta+\nu+2(\rho+\lambda+\sigma_1+\sigma_2+\cdots+\sigma_n)\}>-2\big),$$

where $F_c^{(n)}$ denotes one of the Lauricella function of n variables defined in Srivastava and Manocha [17, p.60(3)]

$$F_c^{(n)}[a,b;c_1,c_2,\cdots,c_n;x_1,x_2\cdots,x_n] =$$

$$\sum_{m_1,m_2,\cdots,m_n=0}^{\infty} \frac{(a)_{m_1+m_2+\cdots+m_n}(b)_{m_1+m_2+\cdots+m_n}}{(c_1)_{m_1}(c_2)_{m_2}\cdots(c_n)_{m_n}} \frac{{x_1}^{m_1}}{m_1!}\frac{{x_2}^{m_2}}{m_2!}\cdots\frac{{x_n}^{m_n}}{m_n!} \quad \left(\sqrt{|x_1|}+\cdots+\sqrt{|x_n|}<1\right). \tag{2.5}$$

Taking $n=1$ in equations (2.3) and (2.4), we get the explicit form of generalized Voigt function given by Gupta and Gupta [5].

3 Partly Bilateral and Partly Unilateral Representation

We have known result given by Srivastava, Bin-saad and Pathan [14, p.8(1.3)]

$$\exp\left[s+t-\frac{xt}{s}\right] = \sum_{m=-\infty}^{\infty}\sum_{p=0}^{\infty}\frac{s^m\ t^p}{m!\ p!}\ {}_1F_1[-p\,;m+1;\ x], \tag{3.1}$$

where ${}_1F_1[a\,;b\,;\ x]$ is the confluent hypergeometric function [13].

On replacing s,t and x by $s\xi^2$, $t\xi^2$ and $x\xi^2$ respectively, multiplying both sides by $\xi^{\eta+2\rho}\ \exp(-w\xi-z\xi^2)\ J^{\mu}_{\nu,\lambda}(q\xi)\ M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1\xi^2,\cdots,2u_n\xi^2)$ in (3.1) and integrating with respect to ξ from 0 to ∞, we get

$$\int_0^{\infty}\xi^{\eta+2\rho}\exp[-w\xi-(z-s-t+\frac{xt}{s})\xi^2]\ J^{\mu}_{\nu,\lambda}(q\xi)\ M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1\xi^2,\cdots,2u_n\xi^2)\,d\xi = \sum_{m=-\infty}^{\infty}\sum_{p=0}^{\infty}\frac{s^m\ t^p}{m!\ p!}$$

$$\cdot\int_0^{\infty}\xi^{\eta+2\rho+2m+2p}\exp[-w\xi-z\xi^2]\ J^{\mu}_{\nu,\lambda}(q\xi)\ M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1\xi^2,\cdots,2u_n\xi^2){}_1F_1[-p\,;m+1;\ x\xi^2]\,d\xi. \tag{3.2}$$

On comparing (3.2) with (1.4), we get following expression

$$\Omega^{\mu,\rho,\sigma_1,\sigma_2\cdots,\sigma_n}_{\eta,\nu,\lambda}(q,w,z-s-t+\frac{xt}{s},u_1,u_2,\cdots,u_n) = \sqrt{\frac{q}{2}}\sum_{m=-\infty}^{\infty}\sum_{p=0}^{\infty}\frac{s^m\ t^p}{m!\ p!}$$

$$\cdot\int_0^{\infty}\xi^{\eta+2\rho+2m+2p}\exp[-w\xi-z\xi^2]\ J^{\mu}_{\nu,\lambda}(q\xi)\ M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1\xi^2,\cdots,2u_n\xi^2){}_1F_1[-p\,;m+1;\ x\xi^2]\,d\xi \tag{3.3}$$

$$\left(q,w,z,z-s-t+\frac{xt}{s},u_1,u_2,\cdots,u_n\in R^+, Re[\eta+\nu+2\{\rho+\lambda+(\sigma_1+\cdots+\sigma_n)\}]>-2\right).$$

Now expanding the exponential function $\exp(-w\xi)$, generalized Bessel function $J^{\mu}_{\nu,\lambda}(q\xi)$ and generalized Whittaker function $M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1\xi^2,\cdots,2u_n\xi^2)$ in series form and using the following known result [1, p.337(9)]

$$\int_0^{\infty}x^{s-1}e^{-\alpha x^2}\ {}_1F_1(a\,;b\,;\beta x^2)\,dx = \frac{1}{2}\alpha^{-s/2}\ \Gamma(s/2)\ {}_2F_1[a\,,s/2\,;b\,;\beta/\alpha] \tag{3.4}$$

$$(Re(s)>0;\ Re(\alpha)>\ max\{0,Re(\beta)\}),$$

we arrive at

$$\Omega^{\mu,\rho,\sigma_1,\sigma_2\cdots,\sigma_n}_{\eta,\nu,\lambda}(q,w,z-s-t+\frac{xt}{s},u_1,\cdots,u_n) = \frac{q^{\nu+2\lambda+1/2}\ 2^{\sigma_1+\cdots+\sigma_n+n/2-3/2-\nu-2\lambda}\ u_1^{\sigma_1+1/2}\cdots u_n^{\sigma_n+1/2}}{Z^{\alpha}}$$

$$\cdot\sum_{m=-\infty}^{\infty}\sum_{p=0}^{\infty}\frac{(s/Z)^m\ (t/Z)^p}{m!\ p!}\sum_{r,j,l_1,l_2,\cdots,l_n=0}^{\infty}\frac{(\sigma_1+\cdots+\sigma_n-\rho+\frac{n}{2})_{l_1+l_2+\cdots+l_n}}{\Gamma(\lambda+j+1)\ \Gamma(\nu+\lambda+\mu j+1)}$$

$$\cdot\frac{\Gamma(m+p+j+l_1+\cdots+l_n+\frac{r}{2}+\alpha)}{(2\sigma_1+1)_{l_1}(2\sigma_2+1)_{l_2}\cdots(2\sigma_n+1)_{l_n}}\frac{(\frac{-w}{\sqrt{Z}})^r(\frac{-q^2}{4Z})^j}{r!}\ {}_2F_1[-p,m+p+j+l_1+\cdots+l_n+r/2+\alpha;\ m+1;\ \frac{x}{Z}]$$

$$\cdot\frac{(\frac{2u_1}{Z})^{l_1}}{l_1!}\frac{(\frac{2u_2}{Z})^{l_2}}{l_2!}\cdots\frac{(\frac{2u_n}{Z})^{l_n}}{l_n!}. \tag{3.5}$$

Now expanding ${}_2F_1$ in series form and separating r-series into its even and odd term, we get

$$\Omega^{\mu,\rho,\sigma_1,\sigma_2\cdots,\sigma_n}_{\eta,\nu,\lambda}(q,w,z-s-t+\frac{xt}{s},u_1,\cdots,u_n)=\frac{q^{\nu+2\lambda+1/2}\ 2^{\sigma_1+\cdots+\sigma_n+n/2-3/2-\nu-2\lambda}\ u_1^{\sigma_1+1/2}\cdots u_n^{\sigma_n+1/2}}{Z^{\alpha}}$$

$$.\sum_{m=-\infty}^{\infty}\sum_{p=0}^{\infty}\frac{(s/Z)^m\ (t/Z)^p}{m!\ p!}\sum_{j,r,k=0}^{\infty}\frac{\Gamma(m+p+j+r+k+\alpha)\ (-p)_k\ (-\frac{q^2}{4Z})^j\ (\frac{w^2}{4Z})^r\ (\frac{x}{Z})^k}{(1/2)_r(m+1)_k\ \Gamma(\lambda+j+1)\ \Gamma(\nu+\lambda+\mu j+1)\ r!\ k!}$$

$$.\Big\{F_c^{(n)}[m+p+j+r+k+\alpha,\ \sigma_1+\cdots+\sigma_n-\rho+\frac{n}{2};\ 2\sigma_1+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z},\cdots,\frac{2u_n}{Z}]$$

$$-\frac{w\ (m+p+j+r+k+\alpha)_{1/2}}{\sqrt{Z}\ (2r+1)}$$

$$F_c^{(n)}[m+p+j+r+k+\alpha+1/2,\sigma_1+\cdots+\sigma_n-\rho+\frac{n}{2};\ 2\sigma_1+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z},\cdots,\frac{2u_n}{Z}]\Big\} \tag{3.6}$$

$$\Big(q,w,z,z-s-t+\frac{xt}{s},u_1,u_2,\cdots,u_n\in R^+,\ \ Re\{\eta+\nu+2(\rho+\lambda+\sigma_1+\sigma_2+\cdots+\sigma_n)\}>-2\Big).$$

4 Generating Functions

On expanding left hand side of (3.6) by using (2.4), we get

$$\left(\frac{Z}{Z'}\right)^{\alpha}\sum_{m,j=0}^{\infty}\frac{\{-\frac{q^2}{4Z'}\}^m\ \{-\frac{w^2}{4Z'}\}^j\ \Gamma(\alpha+m+j)}{\Gamma(\lambda+m+1)\ \Gamma(\nu+\lambda+\mu m+1)(1/2)_j\ j!}$$

$$.\Big\{F_c^{(n)}[\alpha+m+j,\sigma_1+\cdots+\sigma_n-\rho+\frac{n}{2};2\sigma_1+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z'},\cdots,\frac{2u_n}{Z'}]$$

$$-\frac{w\ (\alpha+m+j)_{1/2}}{\sqrt{Z'}\ (2j+1)}$$

$$F_c^{(n)}[\alpha+1/2+m+j,\sigma_1+\cdots+\sigma_n-\rho+\frac{n}{2};\ 2\sigma_1+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z'},\cdots,\frac{2u_n}{Z'}]\Big\}$$

$$=\sum_{l=-\infty}^{\infty}\sum_{p=0}^{\infty}\frac{(\frac{s}{Z})^l(\frac{t}{Z})^p}{l!\ p!}\sum_{i,r,k=0}^{\infty}\frac{(-p)_k\ (-\frac{q^2}{4Z})^i\ (\frac{w^2}{4Z})^r\ (\frac{x}{Z})^k\ \Gamma(l+p+i+r+k+\alpha)}{(l+1)_k\ (1/2)_r\ \Gamma(\lambda+i+1)\ \Gamma(\nu+\lambda+\mu i+1)\ r!\ k!}$$

$$.\Big\{F_c^{(n)}[l+p+i+r+k+\alpha,\ \sigma_1+\cdots+\sigma_n-\rho+\frac{n}{2};\ 2\sigma_1+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z},\cdots,\frac{2u_n}{Z}]$$

$$-\frac{w\ (l+p+i+r+k+\alpha)_{1/2}}{\sqrt{Z}\ (2r+1)}$$

$$F_c^{(n)}[l+p+i+r+k+\alpha+1/2,\sigma_1+\cdots+\sigma_n-\rho+\frac{n}{2};\ 2\sigma_1+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z},\cdots,\frac{2u_n}{Z}]\Big\}, \tag{4.1}$$

where $Z^{'}=(Z-s-t+\frac{xt}{s})$.

On setting $n=1$ for σ_n and u_n, the equation (4.1) reduce to generating function of Gupta and Gupta [5].

Next, we derive another generating function by using the Kummer's first formula.

We have Kummer's first formula [13, p.125(1)]

$$e^{-v}\ {}_1F_1(a;\ b;\ v)=\sum_{k=0}^{\infty}\frac{(b-a)_k}{(b)_k}\ \frac{(-v)^k}{k!}. \tag{4.2}$$

On replacing v by vt, multiplying both sides by $t^{\eta+2\rho}\ e^{-yt-zt^2}\ J^{\mu}_{\nu,\lambda}(xt)\ M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1t^2,$ $\cdots, 2u_nt^2)$ in (4.2) and integrating with respect to t from 0 to ∞, we get

$$\int_0^\infty t^{\eta+2\rho}\ e^{-yt-zt^2}\ J^{\mu}_{\nu,\lambda}(xt)\ M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1t^2,\cdots,2u_nt^2)\ e^{-vt}\,{}_1F_1(a;\ b;\ vt)\ dt$$

$$= \sum_{k=0}^{\infty} \frac{(b-a)_k(-v)^k}{(b)_k\ k!} \int_0^\infty t^{\eta+2\rho+k}\ e^{-yt-zt^2}\ J^{\mu}_{\nu,\lambda}(xt)\ M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1t^2,\cdots,2u_nt^2)\ dt. \quad (4.3)$$

On expanding ${}_1F_1$ into series form, we arrive at

$$\sum_{l=0}^{\infty} \frac{(a)_l v^l}{(b)_l\ l!} \int_0^\infty t^{\eta+2\rho+l}\ e^{-(y+v)t-zt^2}\ J^{\mu}_{\nu,\lambda}(xt)\ M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1t^2,\cdots,2u_nt^2)\ dt$$

$$= \sum_{k=0}^{\infty} \frac{(b-a)_k(-v)^k}{(b)_k\ k!} \int_0^\infty t^{\eta+2\rho+k}\ e^{-yt-zt^2}\ J^{\mu}_{\nu,\lambda}(xt)\ M_{\rho,\sigma_1,\cdots,\sigma_n}(2u_1t^2,\cdots,2u_nt^2)\ dt. \quad (4.4)$$

Now using (2.4) on both sides of (4.4), we get

$$\sum_{l=0}^{\infty} \frac{(a)_l(\frac{v}{\sqrt{Z}})^l}{(b)_l\ l!} \sum_{m,s=0}^{\infty} \frac{(-\frac{x^2}{4Z})^m\{\frac{(y+v)^2}{4Z}\}^s\ \Gamma(\alpha+l/2+m+s)}{\Gamma(\lambda+m+1)\ \Gamma(\nu+\lambda+\mu m+1)\ (1/2)_s\ s!}$$

$$.\Big\{F_c^{(n)}[\alpha+l/2+m+s,\ \sigma_1+\cdots+\sigma_n-\rho+\frac{n}{2};\ 2\sigma_1+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z},\cdots,\frac{2u_n}{Z}]$$

$$-\tfrac{(y+v)\ (\alpha+l/2+m+s)_{1/2}}{\sqrt{Z}\ (2s+1)}$$

$$F_c^{(n)}[\alpha+l/2+1/2+m+s,\sigma_1+\cdots+\sigma_n-\rho+\frac{n}{2};\ 2\sigma_1+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z},\cdots,\frac{2u_n}{Z}]\Big\}$$

$$= \sum_{k=0}^{\infty} \frac{(b-a)_k(-\frac{v}{\sqrt{Z}})^k}{(b)_k\ k!} \sum_{m,s=0}^{\infty} \frac{(-\frac{x^2}{4Z})^m(\frac{y^2}{4Z})^s\ \Gamma(\alpha+k/2+m+s)}{\Gamma(\lambda+m+1)\ \Gamma(\nu+\lambda+\mu m+1)\ (1/2)_s\ s!}$$

$$.\Big\{F_c^{(n)}[\alpha+k/2+m+s,\ \sigma_1+\cdots+\sigma_n-\rho+\frac{n}{2};\ 2\sigma_1+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z},\cdots,\frac{2u_n}{Z}]$$

$$-\tfrac{y\ (\alpha+k/2+m+s)_{1/2}}{\sqrt{Z}\ (2s+1)}$$

$$F_c^{(n)}[\alpha+k/2+1/2+m+s,\sigma_1+\cdots+\sigma_n-\rho+\frac{n}{2};\ 2\sigma_1+1,\cdots,2\sigma_n+1;\frac{2u_1}{Z},\cdots,\frac{2u_n}{Z}]\Big\} \quad (4.5)$$

$$(x,y,y+v,z,u_1,\cdots,u_n\in R^+,\ Re\{\eta+\nu+l+2(\rho+\lambda+\sigma_1+\cdots+\sigma_n)\}>-2,$$
$$Re\{\eta+\nu+k+2(\rho+\lambda+\sigma_1+\cdots+\sigma_n)\}>-2).$$

5 Conclusion

We investigate a new generalization of Voigt functions involving the product of generalized Whittaker and generalized Bessel functions and express it in terms of Lauricella function of n variables. We further obtain a partly bilateral and partly unilateral representation and two generating functions. The results established in this paper may be useful in some areas of mathematical physics and engineering such as in the study of generalized heat equations and wireless communications.

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