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Quantum Codes Over $F_2 + uF_2 + vF_2$

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Abstract In this paper, it was constructed quantum codes from cyclic codes over finite ring $S = F_2 + uF_2 + vF_2, u^2 = u, v^2 = v, uv = vu = 0$ for arbitrary length n. It was given a new Gray map Ψ which is both an isometry and weight preserving map. It was shown that C is self orthogonal codes over S, so is $\Psi(C)$. It was given a necessary and sufficient condition for cyclic codes over S that contains its dual and it was determined the parameters of quantum codes which are obtained from cyclic codes over S.

1 Introduction

Although the theory quantum error correcting codes has striking differences from the theory classical error correcting codes, Calderbank et al. gave a way to construct quantum error correcting codes from classical error correcting codes in [3].

Many good quantum codes have been constructed by using classical cyclic codes over F_q with self orthogonal (or dual containing) properties.

Some authors constructed quantum codes by using linear codes over finite rings. For example, in [5], J. Qian et al. gave a new method to obtain self-orthogonal codes over F_2. They gave a construction for quantum error correcting codes starting from cyclic codes over finite ring, $F_2 + uF_2$, $u^2 = 0$. X. Kai, S. Zhu gave construction for quantum codes from linear and cyclic codes over $F_4 + uF_4$, $u^2 = 0$ in [6]. They derived Hermitian self-orthogonal codes over F_4 as Gray images of linear and cyclic codes over F_4+uF_4. In [7], X. Yin and W. Ma gave an existence condition of quantum codes which are derived from cyclic codes over finite ring $F_2+uF_2+u^2F_2$, $u^3 = 0$ with Lee metric. J. Qian gave a new method of constructing quantum error correcting codes from cyclic codes over finite ring $F_2 + vF_2$, $v^2 = v$, for arbitrary length n in [4]. A. Dertli et al. gave quantum codes over the finite ring in [1, 2].

This paper is organized as follows. In section 2, we give some basic knowledges about the finite ring S, cyclic code, dual code. In section 3, we define a new Gray map from S to F_2^3, Lee weights of elements of S. We show that if C is self orthogonal so is $\Psi(C)$. In section 4, a necessary and sufficient condition for cyclic code over S that contains its dual is given. The parameters of quantum error correcting codes are obtained from cyclic codes over S. In section 5, we give some examples.

2 Preliminaries

Let S be the ring $F_2 + uF_2 + vF_2$ where $u^2 = u$, $v^2 = v$, $uv = vu = 0$ and $F_2 = \{0, 1\}$ a finite commutative ring with 8 elements. S is semi local ring with three maximal ideals and a principal ideal ring. It is not finite chain ring. Let $w = 1 + u + v$. Addition and multiplication over S are given in the following tables:

$+$	0	1	u	v	$1+u$	$1+v$	$u+v$	w
0	0	1	u	v	$1+u$	$1+v$	$u+v$	w
1	1	0	$1+u$	$1+v$	u	v	w	$u+v$
u	u	$1+u$	0	$u+v$	1	w	v	$1+v$
v	v	$1+v$	$u+v$	0	w	1	u	$1+u$
$1+u$	$1+u$	u	1	w	0	$u+v$	$1+v$	v
$1+v$	$1+v$	v	w	1	$u+v$	0	$1+u$	u
$u+v$	$u+v$	w	v	u	$1+v$	$1+u$	0	1
w	w	$u+v$	$1+v$	$1+u$	v	u	1	0

$.$	0	1	u	v	$1+u$	$1+v$	$u+v$	$1+u+v$
0	0	0	0	0	0	0	0	0
1	0	1	u	v	$1+u$	$1+v$	$u+v$	w
u	0	u	u	0	0	u	u	0
v	0	v	0	v	v	0	v	0
$1+u$	0	$1+u$	0	v	$1+u$	w	v	w
$1+v$	0	$1+v$	u	0	w	$1+v$	u	w
$u+v$	0	$u+v$	u	v	v	u	$u+v$	0
$1+u+v$	0	w	0	0	w	w	0	w

The ideals are follows:

$$\begin{aligned}
I_0 &= \{0\},\ I_1 = S \\
I_u &= \{0,u\},\ I_v = \{0,v\},\ I_{1+u+v} = \{0,1+u+v\} \\
I_{u+v} &= \{0,u,v,u+v\},\ I_{1+u} = \{0,v,1+u,1+u+v\} \\
I_{1+v} &= \{0,u,1+v,1+u+v\}
\end{aligned}$$

A linear code C over S length n is a $S-$submodule of S^n. An element of C is called a codeword.

For any $x=(x_0,x_1,...,x_{n-1})$, $y=(y_0,y_1,...,y_{n-1})$ the inner product is defined as

$$x.y=\sum_{i=0}^{n-1} x_i y_i$$

If $x.y=0$ then x and y are said to be orthogonal. Let C be linear code of length n over S, the dual code of C

$$C^{\perp}=\{x:\forall y\in C, x.y=0\}$$

which is also a linear code over S of length n. A code C is self orthogonal if $C\subseteq C^{\perp}$ and self dual if $C=C^{\perp}$.

A cyclic code C over S is a linear code with the property that if $c=(c_0,c_1,...,c_{n-1})\in C$ then $\sigma(C)=(c_{n-1},c_0,...,c_{n-2})\in C$. A subset C of S^n is a linear cyclic code of length n iff it is polynomial representation is an ideal of $S[x]/\langle x^n-1\rangle$.

Let C be code over F_2 of length n and $\acute{c}=(\acute{c}_0,\acute{c}_1,...,\acute{c}_{n-1})$ be a codeword of C. The Hamming weight of $\acute{c}$ is defined as $w_H(\acute{c})=\sum_{i=0}^{n-1} w_H(\acute{c}_i)$ where $w_H(\acute{c}_i)=1$ if $\acute{c}_i=1$ and $w_H(\acute{c}_i)=0$ if $\acute{c}_i=0$. Hamming distance of C is defined as $d_H(C)=\min d_H(c,\acute{c})$, where for any $\acute{c}\in C$, $c\neq\acute{c}$ and $d_H(c,\acute{c})$ is Hamming distance between two codewords with $d_H(c,\acute{c})=w_H(c-\acute{c})$.

Let $a\in F_2^{3n}$ with $a=(a_0,a_1,...,a_{3n-1})=\left(a^{(0)}\left|a^{(1)}\right|a^{(2)}\right)$, $a^{(i)}\in F_2^n$ for $i=0,1,2$. Let φ be a map from F_2^{3n} to F_2^{3n} given by $\varphi(a)=\left(\sigma\left(a^{(0)}\right)\left|\sigma\left(a^{(1)}\right)\right|\sigma\left(a^{(2)}\right)\right)$ where σ is a cyclic shift from F_2^n to F_2^n given by $\sigma\left(a^{(i)}\right)=((a^{(i,n-1)}),(a^{(i,0)}),(a^{(i,1)}),...,(a^{(i,n-2)}))$ for every $a^{(i)}=(a^{(i,0)},...,a^{(i,n-1)})$ where $a^{(i,j)}\in F_2$, $0\leq j\leq n-1$. A code of length $3n$ over F_2 is said to be quasi cyclic code of index 3 if $\varphi(C)=C$.

3 Gray Map And Gray Images Of Cyclic Codes Over S

Let $x=a+ub+uc$ be an element of S where $a,b,c\in F_2$. We define Gray map Ψ from S to F_2^3 by

$$\begin{aligned}
\Psi &: S\to F_2^3 \\
\Psi(a+ub+vc) &= (a,a+b,a+c)
\end{aligned}$$

The Lee weight of elements of S are defined $w_L(a+ub+vc)=w_H(a,a+b,a+c)$ where w_H denotes the ordinary Hamming weight for binary codes. Hence, there is one element whose weight is 0, there are $u,v,1+u+v$ elements whose weights are 1, there are $1+u,1+v,u+v$ elements whose weights are 2, there is one element whose weight are 3.

Let C be a linear code over S of length n. For any codeword $c = (c_0, ..., c_{n-1})$ the Lee weight of c is defined as $w_L(c) = \sum_{i=0}^{n-1} w_L(c_i)$ and the Lee distance of C is defined as $d_L(C) = \min d_L(c, \acute{c})$, where for any $\acute{c} \in C$, $c \neq \acute{c}$ and $d_L(c, \acute{c})$ is Lee distance between two codewords with $d_L(c, \acute{c}) = w_L(c - \acute{c})$. Gray map Ψ can be extended to map from S^n to F_2^{3n}.

Theorem 3.1. *The Gray map Ψ is a weight preserving map from (S^n, Lee weight) to (F_2^{3n}, Hamming weight). Moreover it is an isometry from S^n to F_2^{3n}.*

Theorem 3.2. *If C is an $[n, k, d_L]$ linear codes over S then $\Psi(C)$ is a $[3n, k, d_H]$ linear codes over F_2, where $d_H = d_L$.*

Proof. Let $x_1 = a_1 + ub_1 + vc_1$, $x_2 = a_2 + ub_2 + vc_2 \in S, \alpha \in F_2$ then

$$\begin{aligned}\Psi(x_1 + x_2) &= \Psi(a_1 + a_2 + u(b_1 + b_2) + v(c_1 + c_2)) \\ &= (a_1 + a_2, a_1 + a_2 + b_1 + b_2, a_1 + a_2 + c_1 + c_2) \\ &= (a_1, a_1 + b_1, a_1 + c_1) + (a_2, a_2 + b_2, a_2 + c_2) \\ &= \Psi(x_1) + \Psi(x_2) \\ \Psi(\alpha x_1) &= \Psi(\alpha a_1 + u\alpha b_1 + v\alpha c_1) \\ &= (\alpha a_1, \alpha a_1 + \alpha b_1, \alpha a_1 + \alpha c_1) \\ &= \alpha(a_1, a_1 + b_1, a_1 + c_1) \\ &= \alpha\Psi(x_1)\end{aligned}$$

so Ψ is linear. As Ψ is bijective then $|C| = |\Psi(C)|$. From Theorem 3.1 we have $d_H = d_L$. □

Theorem 3.3. *If C is self orthogonal, so is $\Psi(C)$.*

Proof. Let $x_1 = a_1 + ub_1 + vc_1$, $x_2 = a_2 + ub_2 + vc_2$ where $a_1, b_1, c_1, a_2, b_2, c_2 \in F_2$.

From $x_1.x_2 = a_1a_2 + u(a_1b_2 + b_1a_2 + b_1b_2) + v(a_1c_2 + c_1a_2 + c_1c_2)$, if C is self orthogonal, so we have $a_1a_2 = 0, a_1b_2 + b_1a_2 + b_1b_2 = 0, a_1c_2 + c_1a_2 + c_1c_2 = 0$. From

$$\begin{aligned}\Psi(x_1).\Psi(x_2) &= (a_1, a_1 + b_1, a_1 + c_1)(a_2, a_2 + b_2, a_2 + c_2) \\ &= a_1a_2 + a_1b_2 + b_1a_2 + b_1b_2 + a_1a_2 + a_1c_2 + c_1a_2 + c_1c_2 = 0\end{aligned}$$

Therefore, we have $\Psi(C)$ is self orthogonal. □

Proposition 3.4. *Let Ψ be Gray map from S^n to F_2^{3n}, let σ be cyclic shift and let φ be a map as in the preliminaries. Then $\Psi\sigma = \varphi\Psi$.*

Proposition 3.5. *Let σ and φ be as in the preliminaries. A code C of length n over S is cyclic code if and only if $\Psi(C)$ is quasi cyclic code of index 3 over F_2 with length $3n$.*

Proof. Similar to proof of in [8]. □

We denote that $A_1 \otimes A_2 \otimes A_3 = \{(a_1, a_2, a_3) : a_1 \in A_1, a_2 \in A_2, a_3 \in A_3\}$ and $A_1 \oplus A_2 \oplus A_3 = \{a_1 + a_2 + a_3 : a_1 \in A_1, a_2 \in A_2, a_3 \in A_3\}$

Let C be a linear code of length n over S. Define

$$\begin{aligned}C_1 &= \{a \in F_2^n : \exists b, c \in F_2^n, a + ub + vc \in C\} \\ C_2 &= \{a + b \in F_2^n : \exists c \in F_2^n, a + ub + vc \in C\} \\ C_3 &= \{a + c \in F_2^n : \exists b \in F_2^n, a + ub + vc \in C\}\end{aligned}$$

Then C_1, C_2 and C_3 are binary linear codes of length n. Moreover, the linear code C of length n over S can be uniquely expressed as $C = (1 + u + v)C_1 \oplus (u) C_2 \oplus (v) C_3$.

Theorem 3.6. *Let C be a linear code of length n over S. Then $\Psi(C) = C_1 \otimes C_2 \otimes C_3$ and $|C| = |C_1| |C_2| |C_3|$.*

Proof. For any $(a_0, a_1, ..., a_{n-1}, b_0, b_1, ..., b_{n-1}, c_0, c_1, ..., c_{n-1}) \in \Psi(C)$. Let $r_i = a_i + u(a_i + b_i) + v(a_i + c_i)$, $i = 0, 1, ..., n-1$. Since Ψ is a bijection $r = (r_0, r_1, ..., r_{n-1}) \in C$. By definitions of C_1, C_2 and C_3 we have $(a_0, a_1, ..., a_{n-1}) \in C_1$, $(b_0, b_1, ..., b_{n-1}) \in C_2$, $(c_0, c_1, ..., c_{n-1}) \in C_3$. So, $(a_0, a_1, ..., a_{n-1}, b_0, b_1, ..., b_{n-1}, c_0, c_1, ..., c_{n-1}) \in C_1 \otimes C_2 \otimes C_3$. That is $\Psi(C) \subseteq C_1 \otimes C_2 \otimes C_3$.

On the other hand, for any $(a_0, a_1, ..., a_{n-1}, b_0, b_1, ..., b_{n-1}, c_0, c_1, ..., c_{n-1}) \in C_1 \otimes C_2 \otimes C_3$ where $(a_0, a_1, ..., a_{n-1}) \in C_1$, $(b_0, b_1, ..., b_{n-1}) \in C_2$, $(c_0, c_1, ..., c_{n-1}) \in C_3$. There are $x = (a_0, a_1, ..., a_{n-1})$, $y = (b_0, b_1, ..., b_{n-1})$, $z = (c_0, c_1, ..., c_{n-1}) \in C$ such that $x_i = a_i + (u+v)p_i$, $y_i = b_i + (1 + u) q_i$, $z_i = c_i + (1 + v) s_i$ where $p_i, q_i, s_i \in F_2$ and $0 \leq i \leq n-1$. Since C is linear we have $r = (1 + u + v)x + (u)y + (v)z = a + u(a + b) + v(a + c) \in C$. It follows then $\Psi(r) = (a_0, a_1, ..., a_{n-1}, b_0, b_1, ..., b_{n-1}, c_0, c_1, ..., c_{n-1})$, which gives $C_1 \otimes C_2 \otimes C_3 \subseteq \Psi(C)$. Therefore, $\Psi(C) = C_1 \otimes C_2 \otimes C_3$. The second result is easy to verify. □

Corollary 3.7. *If* $\Psi(C) = C_1 \otimes C_2 \otimes C_3$, *then* $C = (1+u+v)C_1 \oplus (u)\, C_2 \oplus (v)\, C_3$.

It is easy to see that,

$$\begin{aligned} |C| &= |C_1|\,|C_2|\,|C_3| = 2^{n-\deg(f_1)} 2^{n-\deg(f_2)} 2^{n-\deg(f_3)} \\ &= 2^{3n-(\deg(f_1)+\deg(f_2)+\deg(f_3))} \end{aligned}$$

where f_1, f_2 and f_3 are the generator polynomials of C_1, C_2 and C_3, respectively.

Corollary 3.8. *If* G_1, G_2, G_3 *and* G_4 *are generator matrices of binary linear codes* C_1, C_2 *and* C_3 *respectively, then the generator matrix of* C *is*

$$G = \begin{bmatrix} (1+u+v)G_1 \\ (u)\, G_2 \\ (v)\, G_3 \end{bmatrix}$$

We have

$$\Psi(G) = \begin{bmatrix} \Psi((1+u+v)G_1) \\ \Psi((u)\, G_2) \\ \Psi((v)\, G_3) \end{bmatrix} = \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{bmatrix}.$$

Let d_L minimum Lee weight of linear code C over S. Then,

$$d_L = d_H(\Psi(C)) = \min\{d_H(C_1), d_H(C_2), d_H(C_3)\}$$

where $d_H(C_i)$ denotes the minimum Hamming weights of binary codes C_1, C_2 and C_3, respectively.

4 Quantum Codes From Cyclic Codes Over S

Theorem 4.1. (**CSS Construction**) *Let* C *and* $\acute{C}$ *be two binary codes with parameters* $[n, k_1, d_1]$ *and* $[n, k_2, d_2]$, *respectively. If* $C^\perp \subseteq \acute{C}$, *then an* $[[n, k_1 + k_2 - n, \min\{d_1, d_2\}]]$ *quantum code can be constructed. Especially, if* $C^\perp \subseteq C$, *then there exists an* $[[n, 2k_1 - n, d_1]]$ *quantum code.*

Proposition 4.2. *Let* $C = (1+u+v)C_1 \oplus (u)\, C_2 \oplus (v)\, C_3$ *be a linear code over* S. *Then* C *is a cyclic code over* S *iff* C_1, C_2 *and* C_3 *are binary cyclic codes.*

Proof. Let $(a_0, a_1, ..., a_{n-1}) \in C_1$, $(b_0, b_1, ..., b_{n-1}) \in C_2$ and $(c_0, c_1, ..., c_{n-1}) \in C_3$. Assume that $m_i = (1+u+v)a_i + (u)\, b_i + (v)\, c_i$ for $i = 0, 1, ..., n-1$. Then $(m_0, m_1, ..., m_{n-1}) \in C$. Since C is a cyclic code, it follows that $(m_{n-1}, m_0, ..., m_{n-2}) \in C$. Note that $(m_{n-1}, m_0, ..., m_{n-2}) = (1+u+v)(a_{n-1}, a_0, ..., a_{n-2}) + (u)\, (b_{n-1}, b_0, ..., b_{n-2}) + (v)\, (c_{n-1}, c_0, ..., c_{n-2})$.Hence $(a_{n-1}, a_0, ..., a_{n-2}) \in C_1$, $(b_{n-1}, b_0, ..., b_{n-2}) \in C_2$ and $(c_{n-1}, c_0, ..., c_{n-2}) \in C_3$. Therefore, C_1, C_2 and C_3 cyclic codes over F_2.

Conversely, suppose that C_1, C_2 and C_3 cyclic codes over F_2. Let $(m_0, m_1, ..., m_{n-1}) \in C$ where $m_i = (1+u+v)a_i + (u)\, b_i + (v)\, c_i$ for $i = 0, 1, ..., n-1$. Then $(a_0, a_1, ..., a_{n-1}) \in C_1$, $(b_0, b_1, ..., b_{n-1}) \in C_2$ and $(c_0, c_1, ..., c_{n-1}) \in C_3$. Note that $(m_{n-1}, m_0, ..., m_{n-2}) = (1+u+v)\, (a_{n-1}, a_0, ..., a_{n-2}) + (u)\, (b_{n-1}, b_0, ..., b_{n-2}) + (v)\, (c_{n-1}, c_0, ..., c_{n-2}) \in C = (1+u+v)C_1 \oplus (u)\, C_2 \oplus (v)\, C_3$. So, C is cyclic code over S. □

Proposition 4.3. *Suppose* $C = (1+u+v)C_1 \oplus (u)\, C_2 \oplus (v)\, C_3$ *is a cyclic code of length* n *over* S. *Then*

$$C = <(1+u+v)f_1, (u)\, f_2, (v)\, f_3>$$

and $|C| = 2^{3n-(\deg f_1 + \deg f_2 + \deg f_3)}$ *where* f_1, f_2 *and* f_3 *generator polynomials of* C_1, C_2 *and* C_3 *respectively.*

Proposition 4.4. *Suppose* C *is a cyclic code of length* n *over* S, *then there is a unique polynomial* $f(x)$ *such that* $C = \langle f(x) \rangle$ *and* $f(x) \mid x^n - 1$ *where* $f(x) = (1+u+v)f_1(x) + (u)\, f_2(x) + (v)\, f_3(x)$.

Proposition 4.5. *If* $C = (1+u+v)C_1 \oplus (u)\, C_2 \oplus (v)\, C_3$ *is a cyclic code of length* n *over* S. *Then*

$$C^\perp = \langle (1+u+v)h_1^* + (u)\, h_2^* + (v)\, h_3^* \rangle$$

and $\left|C^\perp\right| = 2^{\deg f_1 + \deg f_2 + \deg f_3}$ *where for* $i = 1, 2, 3$, h_i^* *are the reciprocal polynomials of* h_i *i.e.,* $h_i(x) = (x^n - 1)/f_i(x)$, $h_i^*(x) = x^{\deg h_i} h_i\left(x^{-1}\right)$ *for* $i = 1, 2, 3$.

Lemma 4.6. *A binary linear cyclic code C with generator polynomial $f(x)$ contains its dual code iff*

$$x^n - 1 \equiv 0\,(mod f f^*)$$

where f^* is the reciprocal polynomial of f.

Theorem 4.7. *Let $C = \langle (1+u+v)f_1, (u)\,f_2, (v)\,f_3 \rangle$ be a cyclic code of length n over S. Then $C^\perp \subseteq C$ iff $x^n - 1 \equiv 0\,(mod f_i f_i^*)$ for $i = 1, 2, 3$.*

Proof. Let $x^n - 1 \equiv 0\,(mod f_i f_i^*)$ for $i = 1, 2, 3$. Then $C_1^\perp \subseteq C_1, C_2^\perp \subseteq C_2, C_3^\perp \subseteq C_3$. By using $(1+u+v)C_1^\perp \subseteq (1+u+v)C_1$, $(u)\,C_2^\perp \subseteq (u)\,C_2$, $(v)\,C_3^\perp \subseteq (v)\,C_3$. We have $(1+u+v)C_1^\perp \oplus (u)\,C_2^\perp \oplus (v)\,C_3^\perp \subseteq (1+u+v)C_1 \oplus (u)\,C_2 \oplus (v)\,C_3$. So, $< (1+u+v)h_1^* + (u)\,h_2^* + (v)\,h_3^* > \subseteq < (1+u+v)f_1, (u)\,f_2, (v)\,f_3 >$. That is $C^\perp \subseteq C$.

Conversely, if $C^\perp \subseteq C$, then $(1+u+v)C_1^\perp \oplus (u)\,C_2^\perp \oplus (v)\,C_3^\perp \subseteq (1+u+v)C_1 \oplus (u)\,C_2 \oplus (v)\,C_3$. By thinking $mod\,(1+u+v)$, $mod\,(u)$ and $mod\,(v)$ respectively we have $C_i^\perp \subseteq C_i$ for $i = 1, 2, 3$. Therefore, $x^n - 1 \equiv 0\,(mod f_i f_i^*)$ for $i = 1, 2, 3$. □

Corollary 4.8. *$C = (1+u+v)C_1 \oplus (u)\,C_2 \oplus (v)\,C_3$ is a cyclic code of length n over S. Then $C^\perp \subseteq C$ iff $C_i^\perp \subseteq C_i$ for $i = 1, 2, 3$.*

Example 4.9. Let $n = 7, S = F_2 + uF_2 + vF_2$
$x^7 - 1 = (x+1)\left(x^3+x+1\right)\left(x^3+x^2+1\right) = f_1 f_2 f_3$ in $F_2[x]$. Hence,

$$\begin{aligned} f_1^* &= x + 1 = f_1 \\ f_2^* &= x^3 + x^2 + 1 = f_3 \\ f_3^* &= x^3 + x + 1 = f_2 \end{aligned}$$

Let $C = \langle (1+u+v)f_3, (u)\,f_2, (v)\,f_3 \rangle$. Obviously $x^n - 1$ is divisibly by $f_i f_i^*$ for $i = 2, 3$. Thus we have $C^\perp \subseteq C$.

Using Theorem 4.1 and Theorem 4.7 we can construct quantum codes.

Theorem 4.10. *Let $C = C = (1+u+v)C_1 \oplus (u)\,C_2 \oplus (v)\,C_3$ be a cyclic code of arbitrary length n over S with type $8^{k_1}4^{k_2}2^{k_3}$. If $C_i^\perp \subseteq C_i$ where $i = 1, 2, 3$ then $C^\perp \subseteq C$ and there exists a quantum error-correcting code with parameters $[[3n, 3k_1 + 2k_2 + k_3 - 3n, d_L]]$ where d_L is the minimum Lee weights of C.*

5 Examples

n	C_1	C_2	C_3	$\Psi(C)$	$[[N, K, D]]$
7	$[7,4,3]$	$[7,4,3]$	$[7,4,3]$	$[21,12,3]$	$[[21,3,3]]$
8	$[8,6,2]$	$[8,4,2]$	$[8,6,2]$	$[24,16,2]$	$[[24,8,2]]$
15	$[15,11,3]$	$[15,8,4]$	$[15,11,3]$	$[45,30,3]$	$[[45,15,3]]$
30	[30,18,5]	$[30,21,4]$	$[30,17,6]$	$[90,56,4]$	$[[90,22,4]]$

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