Palestine Journal of Mathematics

Vol. 3(1) (2014) , 130–140

Anisotropic Bianchi Type-V Cosmology with Perfect Fluid and Heat Flow in S$\acute{a}$ez-Ballester Theory of Gravitation

Anirudh Pradhan, Ajay Kumar Singh and D. S. Chouhan

Communicated by Jose Luis Lopez-Bonilla

MSC 2010 Classifications: 83C15; 83F05.

Keywords and phrases: Bianchi type-V universe, exact solution, alternative gravitation theory, accelerating universe.

The authors would like to thank Prof. Jose Luis Lopez-Bonilla for his fruitful suggestions.

Abstract. In this paper, we obtain a spatially homogeneous and anisotropic Bianchi type-V cosmological model of the universe for perfect fluid distribution with heat flow within the framework of scalar-tensor theory of gravitation proposed by S$\acute{a}$ez and Ballester (Phys. Lett. 113:467, 1986). To prevail the deterministic solutions we consider time-dependent deceleration parameter (DP) which provides the value of scale factor as $a = [\sinh(\alpha t)]^{\frac{1}{n}}$, where α and n are arbitrary positive constants. This acclimates time-dependent DP representing models which generate a transition of the universe from the early decelerated phase to the recent accelerating phase. The modified Einstein's field equations are solved exactly and the derived model is found to be in good concordance with recent observations. Some physical and geometric properties of the models are also discussed.

1 Introduction

In the last few decades there has been much interest in alternative theories of gravitation, especially the scalar-tensor theories proposed by Brans and Dicke [1], Nordvedt [2], Wagoner [3], Rose [4], Dun [5], S$\acute{a}$ez and Ballester [6], Barber [7], Lau and Prokhovnik [8] are most important among them. The scalar-tensor theories are the generalizations of Einstein's of gravitation in which the metric is generated by a scalar gravitational field together with non-gravitational field (matter). The scalar gravitational field itself is generated by the non-gravitational fields via a wave equation in curved space-time. The strength of the coupling between gravity and scalar field is determined by an arbitrary coupling function ω. S$\acute{a}$ez- Ballester [6] developed a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field, an anti-gravity regime appears. This theory suggests a possible way to solve the missing-matter problem in non-flat FRW cosmologies. The Scalar-Tensor theories of gravitation play an important role to remove the graceful exit problem in the inflation era [9]. In earlier literature, cosmological models within the framework of S$\acute{a}$ez-Ballester scalar-tensor theory of gravitation, have been studied by Singh and Agrawal [10, 11], Reddy and Naidu [12], Rao et al. [13, 14], Adhav et al. [15], Singh [16], Pradhan and Singh [17]. Recently, Socorro and Sabido [18] and Naidu et al. [19, 20] (see the references therein) have studied cosmological models in S$\acute{a}$ez and Ballester scalar tensor theory of gravitation in different context.

Ram et al. [21] obtained Bianchi type-V cosmological models with perfect fluid and heat flow in S$\acute{a}$ez and Ballester theory by considering a variation law for Hubble's parameter with average scale factor which yields constant value of the deceleration parameter. In literature it is common to use a constant deceleration parameter as it duly gives a power law for metric function or corresponding quantity. But it is worth mentioned here that the universe is accelerated expansion at present as observed in recent observations of Type Ia supernova [22]$-$[26] and CMB anisotropies [27]$-$[29] and decelerated expansion in the past. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Now for a Universe which was decelerating in past and accelerating at the present time, the DP must show signature flipping [30]−[32]. So, in general, the DP is not a constant but time variable.

Recently, Pradhan et al. [33, 34] investigated some new exact Bianchi type-I cosmological models in scalar-tensor theory of gravitation with time dependent deceleration parameter. Motivated by these discussions and current observational facts, in this paper, we propose to study Bianchi type-V universe with perfect fluid and heat flow in Sáez-Ballester scalar-tensor theory of gravitation by considering a law of variation of scale factor as increasing function of time which yields a time dependent DP.

2 The Metric and Basic Equations

We consider anisotropic Bianchi type-V line element, given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}\left[B^2(t)dy^2 + C^2(t)dz^2\right], \tag{2.1}$$

where A, B and C are metric functions and m is a constant.

We define the following parameters to be used in solving Einstein's modified field equations for the metric (2.1).

The average scale factor a of Bianchi type-V model (2.1) is defined as

$$a = (ABC)^{\frac{1}{3}}. \tag{2.2}$$

A volume scale factor V is given by

$$V = a^3 = ABC. \tag{2.3}$$

In analogy with FRW universe, we also define the generalized Hubble parameter H as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3), \tag{2.4}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble factors in the directions of x-, y- and z-axes respectively. Here, and also in what follows, a dot indicates ordinary differentiation with respect to t.

Further, the deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{2.5}$$

We introduce the kinematical quantities such as expansion scalar (θ), shear scalar (σ^2) and anisotropy parameter (A_m), defined as follows:

$$\theta = u^i_{;i}, \tag{2.6}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{2.7}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{2.8}$$

where $u^i = (0, 0, 0, 1)$ is the matter 4-velocity vector and

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;\alpha}P^{\alpha}_j + u_{j;\alpha}P^{\alpha}_i\right) - \frac{1}{3}\theta P_{ij}. \tag{2.9}$$

Here the projection tensor P_{ij} has the form

$$P_{ij} = g_{ij} - u_i u_j. \tag{2.10}$$

These dynamical scalars, in Bianchi type-V, have the forms

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{2.11}$$

$$2\sigma^2 = \left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{3}. \tag{2.12}$$

3 Field Equations and their Quadrature Solutions

The field equations in the scalar-tensor theory, proposed by Seáz and Ballester [6], are given by

$$G_{ij} - \omega\phi^r \left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij}, \tag{3.1}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ and $8\pi G = c = 1$. The scalar field ϕ satisfies the equation

$$2\phi^r \phi^{;i}_{;i} + r\phi^{r-1}\phi_{,k}\phi^{,k} = 0. \tag{3.2}$$

Here r is an arbitrary constant and ω is a dimensionless coupling constant. Comma and semicolon respectively denote ordinary and covariant derivative with respect to cosmic time t. T_{ij} is the energy-momentum tensor of the matter.

The energy-momentum tensor is the source of gravitational field through which the effect of the perfect fluid with heat flow in the evolution of the universe is performed. The energy-momentum tensor of a perfect fluid with heat flow has the form given by [16, 21].

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} + h_i u_j + h_j u_i, \tag{3.3}$$

where ρ is the energy density, p is the thermodynamic pressure, u_i is the four-velocity of the fluid and h_i is the heat flow vector satisfying

$$h^i u_i = 0. \tag{3.4}$$

We assume that the heat flow is in x direction only so that $h_i = (h_1, 0, 0, 0)$, h_1 being a function of time. Considering the form of the energy-momentum tensor (3.3), the Einstein's modified field equations (3.1), for the Bianchi type-V space-time (2.1) in Sáez-Ballester theory, are given explicitly as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{3.5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{3.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{3.7}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3m^2}{A^2} = \rho - \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{3.8}$$

$$m\left(2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = h_1, \tag{3.9}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{r}{2\phi}\dot{\phi}^2 = 0. \tag{3.10}$$

From the energy conservation equation $T^{j}_{i;j} = 0$, we obtain

$$\dot{\rho} + (p+\rho)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{2m}{A^2}h_1. \tag{3.11}$$

Equations (3.5)-(3.8) can be written in terms of H, q, σ^2 and ϕ as

$$p = H^2(2q-1) - \sigma^2 + \frac{m^2}{A^2} + \frac{1}{2}\omega\phi^r\dot{\phi}^2. \tag{3.12}$$

$$\rho = 3H^2 - \sigma^2 - \frac{3m^2}{A^2} + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{3.13}$$

Now, we follow the approaches of Ram et al. [21] and Pradhan et al. [33, 34] to solve the field equations (3.5)-(3.8). Subtracting Eq. (3.5) from (3.6), Eq. (3.5) from (3.7) and Eq. (3.6) from (3.7) and taking second integral of each expression, we get the following relations respectively:

$$\frac{A}{B} = d_1 \exp\left(k_1 \int \frac{dt}{a^3}\right), \tag{3.14}$$

$$\frac{A}{C} = d_2 \exp\left(k_2 \int \frac{dt}{a^3}\right), \tag{3.15}$$

$$\frac{B}{C} = d_3 \exp\left(k_3 \int \frac{dt}{a^3}\right), \tag{3.16}$$

where d_1, d_2, d_3 and k_1, k_2, k_3 are constants of integration. From eqs.(3.14)-(3.16), the metric functions can be obtained explicitly as

$$A(t) = l_1 a \exp\left(\frac{X_1}{3}\int \frac{dt}{a^3}\right), \tag{3.17}$$

$$B(t) = l_2 a \exp\left(\frac{X_2}{3}\int \frac{dt}{a^3}\right), \tag{3.18}$$

$$C(t) = l_3 a \exp\left(\frac{X_3}{3}\int \frac{dt}{a^3}\right), \tag{3.19}$$

where

$$l_1 = \sqrt[3]{d_1 d_2}, \quad l_2 = \sqrt[3]{d_1^{-1} d_3}, \quad l_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$

$$X_1 = k_1 + k_2, \quad X_2 = k_3 - k_1, \quad X_3 = -(k_2 + k_3),$$

where the constants X_1, X_2, X_3 and l_1, l_2,l_3 satisfy the relations

$$X_1 + X_2 + X_3 = 0, \quad l_1 l_2 l_3 = 1. \tag{3.20}$$

The quadrature expression for the dimensionless scalar field function ϕ, from eq. (3.10), is found as

$$\phi = \left[\frac{\phi_0(r+2)}{2}\int \frac{dt}{a^3}\right]^{2/(r+2)}, \tag{3.21}$$

where ϕ_0 is a constant.

It is clear from Eqs. (3.17)-(3.21) that once we get the value of the average scale factor a, we can easily calculate the metric functions A, B, C and the scalar function ϕ.

We define the deceleration parameter q as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = b(t) \text{ say}. \tag{3.22}$$

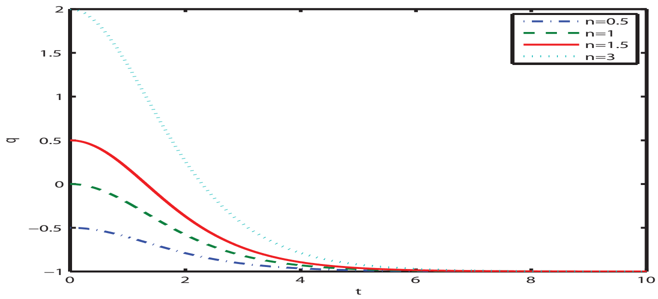

Figure 1. The plot of deceleration parameter q versus t.

The equation (3.22) may be rewritten as

$$\frac{\ddot{a}}{a} + b\frac{\dot{a}^2}{a^2} = 0. \tag{3.23}$$

In order to solve the Eq. (3.23), we assume $b = b(a)$. It is important to note here that one can assume $b = b(t) = b(a(t))$, as a is also a time dependent function. It can be done only if there is a one to one correspondences between t and a. But this is only possible when one avoid singularity like big bang or big rip because both t and a are increasing function.

Following Pradhan et al. [35], the general solution of Eq. (3.23) with assumption $b = b(a)$, is given by

$$a = (\sinh(\alpha t))^{\frac{1}{n}}, \tag{3.24}$$

where α is an arbitrary constant and n is a positive constant.

From (2.5) and (3.24), we obtain the time varying deceleration parameter as

$$q = n\left[1 - (\tanh(\alpha t))^2\right] - 1. \tag{3.25}$$

From Eq. (3.25), we observe that $q > 0$ for $t < \frac{1}{\alpha}\tanh^{-1}(1 - \frac{1}{n})^{\frac{1}{2}}$ and $q < 0$ for $t > \frac{1}{\alpha}\tanh^{-1}(1 - \frac{1}{n})^{\frac{1}{2}}$. It is also observed that for $0 < n \leq 1$, our model is in accelerating phase but for $n > 1$, our model is evolving from decelerating phase to accelerating phase. Also, recent observations of SNe Ia, expose that the present universe is accelerating and the value of DP lies to some place in the range $-1 \leq q < 0$. It follows that in our derived model, one can choose the value of DP consistent with the observations. Figure 1 depicts the variation of the deceleration parameter (q) versus time (t) which gives the behavior of q for different values of n. It is also clear from the figure that for $n \leq 1$, the model is evolving only in accelerating phase whereas for

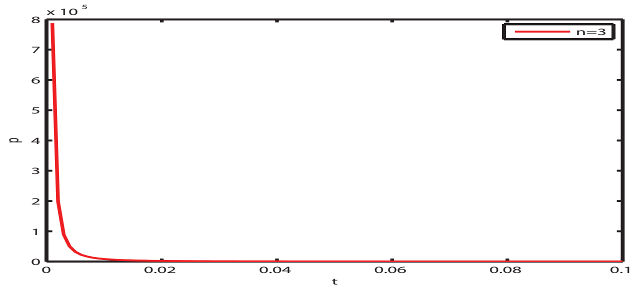

Figure 2. The plot of isotropic pressure p versus t. Here $\omega = \alpha = \beta_2 = X_1 = m = \phi_0 = 1$.

$n > 1$ the model is evolving from the early decelerated phase to the present accelerating phase.

Using (3.24) in Eqs. (3.17)-(3.19), we obtain the following expressions for scale factors:

$$A(t) = l_1(\sinh(\alpha t))^{1/n} \exp\left(\frac{X_1}{3}\int \frac{dt}{(\sinh(\alpha t))^{3/n}}\right), \tag{3.26}$$

$$B(t) = l_2(\sinh(\alpha t))^{1/n} \exp\left(\frac{X_2}{3}\int \frac{dt}{(\sinh(\alpha t))^{3/n}}\right), \tag{3.27}$$

$$C(t) = l_3(\sinh(\alpha t))^{1/n} \exp\left(\frac{X_3}{3}\int \frac{dt}{(\sinh(\alpha t))^{3/n}}\right). \tag{3.28}$$

Hence the geometry of the universe (2.1) is reduced to

$$\begin{aligned} ds^2 = dt^2 - l_1^2(\sinh(\alpha t))^{2/n} \exp\left(\frac{2X_1}{3}\int \frac{dt}{(\sinh(\alpha t))^{3/n}}\right) dx^2 - \\ e^{2mx}\Bigg[l_2^2(\sinh(\alpha t))^{2/n} \exp\left(\frac{2X_2}{3}\int \frac{dt}{(\sinh(\alpha t))^{3/n}}\right) dy^2 + \\ l_3^2(\sinh(\alpha t))^{2/n} \exp\left(\frac{2X_3}{3}\int \frac{dt}{(\sinh(\alpha t))^{3/n}}\right) dz^2 \Bigg]. \end{aligned} \tag{3.29}$$

4 Some Physical and Geometric Properties

The solution for scalar function ϕ, from (3.21), is obtained as

$$\phi = \left[\frac{\phi_0(r+2)}{2}\int \frac{dt}{(\sinh(\alpha t))^{3/n}}\right]^{2/(r+2)}. \tag{4.1}$$

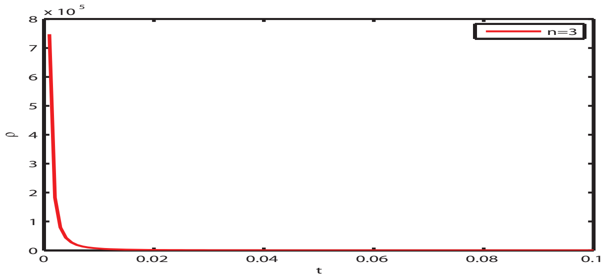

Figure 3. The plot of energy density ρ versus t. Here $\omega = \alpha = \beta_2 = X_1 = m = \phi_0 = 1$.

By using the values of the metric functions from Eqs. (3.26)-(3.28) into Eq. (3.9), the expression for the heat flow function h_1 is given by

$$h_1 = \frac{m\beta_1}{3(\sinh(\alpha t))^{3/n}}, \tag{4.2}$$

where $\beta_1 = 2X_1 - X_2 - X_3$.

From Eqs. (3.12) and (3.13), the energy density and pressure for the model (3.29) are given by

$$p = \frac{(2n-3)\alpha^2}{n^2}(\coth(\alpha t))^2 + \left[\left(\frac{1}{2}\omega\phi_0^2 - \frac{\beta_2}{18}\right)\frac{1}{(\sinh(\alpha t))^{6/n}}\right]$$
$$-\frac{2\alpha^2}{n} + \left[\frac{m^2}{l_1^2(\sinh(\alpha t))^{2/n}}\exp\left(\frac{-2X_1}{3}\int\frac{dt}{(\sinh(\alpha t))^{3/n}}\right)\right], \tag{4.3}$$

$$\rho = \frac{3\alpha^2}{n^2}(\coth(\alpha t))^2 + \left[\left(\frac{1}{2}\omega\phi_0^2 - \frac{\beta_2}{18}\right)\frac{1}{(\sinh(\alpha t))^{6/n}}\right] -$$
$$\left[\frac{3m^2}{l_1^2(\sinh(\alpha t))^{2/n}}\exp\left(\frac{-2X_1}{3}\int\frac{dt}{(\sinh(\alpha t))^{3/n}}\right)\right]. \tag{4.4}$$

In view of (3.20), it is observed that the above set of solutions satisfy the energy conservation equation (3.11) identically and hence represent exact solutions of the Einstein's modified field equations (3.5)-(3.10). From Eqs. (4.3) and (4.4), we observe that isotropic pressure p and the energy density ρ are always positive and decreasing function of time and both approach to zero as $t \to \infty$. Figures 2 and 3 depict p and ρ, respectively, versus time t showing the positive decreasing function of t and approaching to zero at $t \to \infty$.

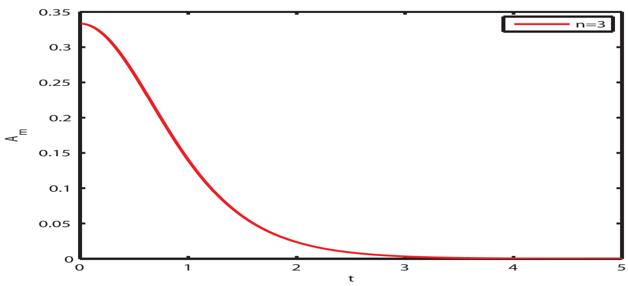

Figure 4. The plot of anisotropic parameter A_m versus t. Here $\alpha = \beta_2 = 1$.

The physical parameters such as spatial volume (V), directional Hubble factors (H_i), Hubble parameter (H), expansion scalar (θ), shear scalar (σ) and anisotropy parameter (A_m) for the model (3.29) are given by

$$V = (\sinh(\alpha t))^{\frac{3}{n}}, \tag{4.5}$$

$$H_i = \frac{\alpha}{n}\coth(\alpha t) + \frac{X_i}{3(\sinh(\alpha t))^{3/n}}, \tag{4.6}$$

$$\theta = 3H = \frac{3\alpha}{n}\coth(\alpha t), \tag{4.7}$$

$$\sigma^2 = \frac{\beta_2}{18(\sinh(\alpha t))^{6/n}}, \tag{4.8}$$

$$A_m = \frac{\beta_2 n^2}{27\alpha^2}\frac{(\tanh(\alpha t))^2}{(\sinh(\alpha t))^{6/n}}, \tag{4.9}$$

where $\beta_2 = X_1^2 + X_2^2 + X_3^2$.

From Eqs. (4.5) and (4.7), we observe that the spatial volume is zero at $t = 0$ and the expansion scalar is infinite, which show that the universe starts evolving with zero volume at $t = 0$ which is big bang scenario. From Eqs. (3.26)-(3.28), we observe that the spatial scale factors are zero at the initial epoch $t = 0$ and hence the model has a point type singularity [36]). We observe that proper volume increases with time.

The dynamics of the mean anisotropy parameter depends on the constant $\beta_2 = X_1^2+X_2^2+X_3^2$. From Eq. (4.9), we observe that at late time when $t \to \infty$, $A_m \to 0$. Thus, our model has transition from initial anisotropy to isotropy at present epoch which is in good harmony with current observations. Figure 4 depicts the variation of anisotropic parameter (A_m) versus cosmic time t. From the figure, we observe that A_m decreases with time and tends to zero as $t \to \infty$. Thus, the

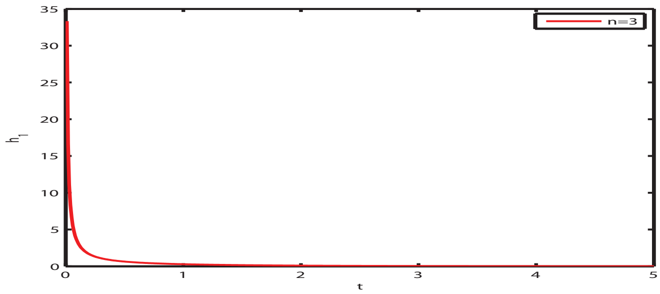

Figure 5. The plot of heat flow function h_1 versus t. Here $\beta_1 = m = \alpha = 1$.

observed isotropy of the universe can be achieved in our model at present epoch.

It is important to note here that $\lim_{t\to 0}\left(\frac{\rho}{\theta^2}\right)$ spread out to be constant. Therefore the model of the universe goes up homogeneity and matter is dynamically negligible near the origin. This is in good agreement with the result already given by Collins [37].

The flow of heat along the x-direction was maximum in early universe, and it diminishes as $t \to \infty$. Figure 5 describe the variation of heat flow versus cosmic time t which shows the nature of h_1. From Eqs. (4.8) and (4.2), we also observe that $\frac{\sigma^2}{h_1^2}$ = constant which shows that shear scalar is proportional to heat conduction.

5 Concluding Remarks

In this paper we have studied a spatially homogeneous and anisotropic Bianchi type-V space-time within the framework of the scalar-tensor theory of gravitation proposed by Sáez and Ballester [6]. The field equations have been solved exactly with suitable physical assumptions. The solutions satisfy the energy conservation Eq. (3.11) identically. Therefore, new, exact and physically viable Bianchi type-V model has been obtained. To find the deterministic solution, we have considered scale factor which yields time dependent deceleration parameters. As we have already discussed in Introduction that for a Universe which was deceleration in past and accelerating at present time, the DP must show signature flipping [30-32] and so there is no scope for a constant DP. The main features of the model are as follows:

• The model is based on exact and new solutions of Einstein's modified field equations for the anisotropic Bianchi type-V space time filled with perfect fluid and heat flow.

• Our special choice of scale factor yields a time dependent deceleration parameter which represents a model of the Universe which evolves from decelerating phase to an accelerating phase. This scenario is consistent with recent observations [22-26].

• It has been observed that $\lim_{t\to 0}\left(\frac{\rho}{\theta^2}\right)$ turn out to be constant. Thus the model approaches homogeneity and matter is dynamically negligible near the origin.

• We also observe that $\frac{\sigma^2}{h_1^2}$ = constant which shows that shear scalar is proportional to heat conduction (i.e. $\sigma \propto h_1$).

Finally, the solutions presented here can be one of the potential candidates to describe the observed universe.

References

[1] C. H. Brans, R. H. Dicke, *Mach's principle and a relativistic theory of gravitation*, Phys. Rev. A, **124**, No. 3 (1961), 925-935.

[2] K. Nordverdt, *Post newtonian metric for a general class of scalar-tensor gravitational theories and observational consequences*, The Astrophys. Journ. **161** (1970), 1059-1067.

[3] R. V. Wagoner, *Scalar-tensor theory and gravitational waves*, Phys. Rev. D **1**, No. 2 (1970), 3209-3216.

[4] D. K. Ross, *Scalar-tensor theory of gravitation*, Phys. Rev. D, **5**, No. 2 (1972), 284-292.

[5] K. A. Dunn, *A scalar-tensor theory of gravitation*, Jour. Math. Phys. **15**, No. 12 (1974), 2229-2231.

[6] D. Sáez, V. J. Ballester, *A simple coupling with cosmological implications*, Phys. Lett. A **113**, No. 9 (1985), 467-470.

[7] G. A. Barber, *On two self creation cosmologies*, Gen. Rel. Gravit., **14**, No. 2 (1985), 117-136.

[8] D. La, P. J. Steinhardt, *Extended inflationary cosmology, Physics Review Letter*, **62** (1989), 376-378.

[9] L. O. Piemental, *New exact vacuum solutions in Brans-Deke theory*, Mod. Phys. Lett. A **12**, No. 25 (1997), 1865-1870.

[10] T. Singh, A. K. Agrawal, *Some Bianchi-type cosmological models in a new scalar-tensor theory*, Astrophys. Space Sci. **182**, No. 2, (1991), 289-312.

[11] T. Singh, A. K. Agrawal, *Bianchi type-II, VIIIm and IX in certain new theories of gravitation*, Astrophys. Space Sci. **191**, No. 1 (1992), 61-68.

[12] D. R. K. Reddy, R. L. Naidu, *Bianchi type-IX cosmic string in a scalar tensor theory of gravitation*, Astrophys. Space Sci. **312**, No. 1-2 (2007), 99-102.

[13] V. U. M. Rao, T. Vinutha, M. V. Shanthi, G. S. D. Kumari, *Bianchi type-V anisotropic cosmological models in certain theories of gravitation with perfect fluid distribution*, Astrophys. Space Sci. **317**, No. 1-2 (2008), 89-94.

[14] V. M. U. Rao, G. S. D. Kumari, K. V. S. Sireesha, *Anisotropic universe with cosmic strings and bulk viscosity in a scalarâĂŞtensor theory of gravitation*, Astrophys. Space Sci. **335**, No. 2 (2011), 635-638.

[15] K. S. Adhav, M. R. Ugale, C. B. Kale, M. P. Bhende, *Bianchi type VI string cosmological model in SaezâĂŞBallesterâĂŹs scalar-tensor theory of gravitation*, Int. Jour, Theor. Phys. **46**, No. 12 (2007), 3122-3127.

[16] C. P. Singh, *LRS Bianchi type-V cosmology with heat flow in scalar-tensor theory*, Braz. Jour. Phys. **39**, No. 4 (2009), 669-628.

[17] A. Pradhan, S. K. Singh, *Some exact Bianchi type-V cosmological models in scalar-tensor theory: kinematic test*, Elect. Jour. Theor. Phys. **7**, No. 24 (2010), 407-424.

[18] J. Socorro, M. Sabido, *Anisotropic cosmology in Sáez-Ballester theory: classical and quantum solutions*, Revista Mexicana De $\acute{F}$isica, **56**, No. 2 (2010), 166-171.

[19] R. L. Naidu, B. Satyanarayana, D. R. K. Reddy, *LRS Bianchi type-II dark energy model in a scalarâĂŞtensor theory of gravitation*, Astrophys. Space Sci. **338**, (2012), 333-336.

[20] R. L. Naidu, B. Satyanarayana, D. R. K. Reddy, *LRS Bianchi type-II universe with cosmic strings and bulk viscosity in a scalar tensor theory of gravitation*, Astrophys. Space Sci. **338** (2012), 351-354.

[21] S. Ram, M. Zeyauddin, C. P. Singh, *Bianchi type-V cosmological models with perfect fluid and heat flow in Saez-Ballester theory*, Pramana - Journal of Physics, **72**, No. 2 (2009), 415-427.

[22] S. Perlmutter, *et al.*, *Measurements of Ω and Λ from 42 High-Redshift Supernovae*, Astrophys. J. **517**, No. 2 (1999), 565-586.

[23] A.G. Riess, *et al.*, *Observational evidence from Supernovae for an accelerating universe and a cosmological constant*, Astron. J. **116**, Issue 3 (1998), 1009-1038.

[24] A. G. Riess, *et al.*, *Type Ia Supernova discoveries at $z > 1$ from the Hubble Space Telescope: evidence for post deceleration and constraints on dark energy evolution*, Astrophys. J. **607**, Issue 2 (2004), 665-687.

[25] J. L. Tonry, *et al.*, *Cosmological results from high-z Supernovae*, Astrophys. J. **594**, No. 1 (2003), 1-24.

[26] A. Clocchiatti, *et al.*, *Hubble Space Telescope and ground-based observations of type Ia Supernovae at redshift 0.5: cosmological implications*, Astrophys. J. **642**, No. 1 (2006), 1-21.

[27] C. L. Bennett, *et al.*, *First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: preliminary maps and basic results*, Astrophys. J. Suppl. **148**, No. 1 (2003), 1-43.

[28] P. de Bernardis *et al.*, *A Flat universe from high-resolution maps of the cosmic microwave background radiation*, Nature, **404**, (2000), 955-959.

[29] S. Hanany, *et al.*, *MAXIMA-1: A Measurement of the cosmic microwave background anisotropy on angular scales of 10 arcminutes to 5 degrees*, Astrophys. J. **545** (2000) L5.

[30] T. Padmanabhan and T. Roychowdhury, *A theoretician's analysis of the Supernova data and the limitations in determining the nature of dark energy*, Mon. Not. R. Astron. Soc. **344** (2003), 823-834.

[31] L. Amendola, *Acceleration at $z > 1$*, Mon. Not. R. Astron. Soc. **342**, Issue 1 (2003), 221-226.

[32] A. G. Riess *et al.*, *The farthest known Supernova: support for an accelerating universe and a glimpse of the epoch of deceleration*, Astrophys. J. **560**, Issue 1 (2001), 49-71.

[33] A. Pradhan, A.S. Dubey, R.K. Khare, *Some exact Bianchi type-I cosmological models in scalar-tensor theory of gravitation with time dependent deceleration parameter*, Rom. Jour. Phys. **57**, No. 3-4, (2012), 1222-1235.

[34] A. Pradhan, A.K. Singh, H. Amirhashchi, *A new class of Bianchi type-I cosmological models in scalar-tensor theory of gravitation and late time acceleration*, Int. Jour. Theor. Phys. 51, No. 2, (2012), 3769-3786. arXiv:1204.5173[physics.gen-ph].

[35] A. Pradhan, R. Jaiswal, K. Jotania, R. K. Khare, *Dark energy models with anisotropic fluid in Bianchi type-VI_0 space-time with time dependent deceleration parameter*, Astrophys. Space Sci. **337**, No. 1 (2012), 401-413.

[36] M. A. H. MacCallum, *A class of homogeneous cosmological models III: asymptotic behaviour*, Commun. Math. Phys. **20**, No. 1 (1971), 57-84.

[37] C. B. Collins, *Global structure of the "Kantowski-Sachs" cosmological models*, J. Math. Phys. **18**, Issue 11 (1977), 2116-2124.

Author information

Anirudh Pradhan, Ajay Kumar Singh, Department of Mathematics, Hindu Post-graduate College, Zamania-232 331, Ghazipur, India.
E-mail: `pradhan@iucaa.ernet.in` `pradhan.anirudh@gmail.com`

D. S. Chouhan, Department of Mathematics, School of Engineering Sir Padampat Singhania University, Bhatewar, Udaipur - 313 601, India.
E-mail: `ds.chouhan@spsu.ac.in`

Received: October 24, 2012.

Accepted: January 7, 2013