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A New Similarity Measure of Generalized Fuzzy numbers Based on Left and Right Apex Angles (I)

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Abstract. In this paper, a new similarity measure of generalized fuzzy numbers based on left and right apex angles is presented. Left and Right Apex Angles combines the concepts of the center of gravity, the area, the perimeter and the height of generalized fuzzy numbers for calculating the degree of similarity between generalized fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

1 Introduction

The similarity measure of fuzzy numbers is very important in many research fields such as pattern recognition [[5],[6]] and risk analysis in fuzzy environment [[1],[3],[9]]. Some methods have been presented to calculate the degree of similarity between fuzzy numbers [[1]-[4],[8],[9]]. In [[10]], Wen presented A modified similarity measure of generalized fuzzy numbers. Also, Pandey et al.,[[7]] proposed a new aggregation operator for trapezoidal fuzzy numbers based on the arithmetic means of the left and right apex angles.
In this paper, a new similarity measure of generalized fuzzy numbers based on reft and right apex angles is presented. Left and Right Apex Angles combines the concepts of the center of gravity, the area, the perimeter and the height of generalized fuzzy numbers for calculating the degree of similarity between generalized fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

2 Preliminaries

Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions,

(i) μ_A is a continuous mapping from R to the closed interval [0,1],

(ii) $\mu_A(x) = 0, -\infty < x \leq a$,

(iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a, b]$,

(iv) $\mu_A(x) = w, b \leq x \leq c$,

(v) $\mu_A(x) = R(x)$ is strictly decreasing on $[c, d]$,

(vi) $\mu_A(x) = 0, d \leq x < \infty$

Where $0 < w \leq 1$ and a, b, c, and d are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by

$$A = (a, b, c, d; w) . \tag{2.1}$$

A $A = (a, b, c, d; w)$ is a fuzzy set of the real line R whose membership function $\mu_A(x)$ is defined as

$$\mu_A(x) = \begin{cases} w\frac{x-a}{b-a} & if \quad a \leq x \leq b \\ w & if \quad b \leq x \leq c \\ w\frac{d-x}{d-c} & if \quad c \leq x \leq d \\ 0 & \quad Otherwise \end{cases} \tag{2.2}$$

3 Proposed Approach

In this section some important results, that are useful for the proposed approach, are proved.

The concept of the method to calculate the degree of similarity between generalized fuzzy numbers, the horizontal center-of-gravity, the perimeter, the height and the area of the two fuzzy numbers are considered.

Suppose that $A_1 = (a_1, b_1, c_1, d_1; w_1)$ and $A_2 = (a_2, b_2, c_2, d_2; w_2)$ be the generalized trapezoidal fuzzy numbers. where $0 \leq a_1 \leq b_1 \leq c_1 \leq d_1 \leq 1$ and $0 \leq a_2 \leq b_2 \leq c_2 \leq d_2 \leq 1$. Then the degree of similarity $S(A_1, A_2)$ between the generalized trapezoidal fuzzy numbers A_1 and A_2 is calculated as follows:

$$S(A_1, A_2) = [1- \mid x^*_{A_1} - x^*_{A_2} \mid] \times [1- \mid w_{A_1} - w_{A_2} \mid] \times \frac{min(P(A_1), P(A_2)) + min(A(A_1), A(A_2))}{max(P(A_1), P(A_2)) + max(A(A_1), A(A_2))} \tag{3.1}$$

Where $x^*_{A_1}$ and $x^*_{A_2}$ are the horizontal center-of-gravity of the generalized trapezoidal fuzzy numbers A_1 and A_2 is calculated as follows:

$$x^*_{A_1} = \frac{y^*_{A_1}(c_1 + b_1) + (d_1 + a_1)(w_{A_1} - y^*_{A_1})}{2w_{A_1}} \tag{3.2}$$

$$y^*_{A_1} = \begin{cases} \frac{w_{A_1}(\frac{c_1 - b_1}{d_1 - a_1} + 2)}{6} & if \quad a_1 \neq d_1 \; and \, 0 < w_{A_1} \leq 1 \\ \frac{w_{A_1}}{2} & if \quad a_1 = d_1 \; and \, 0 < w_{A_1} \leq 1 \end{cases} \tag{3.3}$$

$P(A_1)$ and $P(A_2)$ are the perimeters of two generalized trapezoidal fuzzy numbers which are calculated as follows:

$$P(A_1) = \sqrt{(a_1 - b_1)^2 + w^2_{A_1}} + \sqrt{(c_1 - d_1)^2 + w^2_{A_1}} + (c_1 - b_1) + (d_1 - a_1) \tag{3.4}$$

$$P(A_2) = \sqrt{(a_2 - b_2)^2 + w^2_{A_2}} + \sqrt{(c_2 - d_2)^2 + w^2_{A_2}} + (c_2 - b_2) + (d_2 - a_2) \tag{3.5}$$

$A(A_1)$ and $A(A_2)$ are the areas of two generalized trapezoidal fuzzy numbers which are calculated as follows:

$$A(A_1) = \frac{1}{2} w_{A_1}(c_1 - b_1 + d_1 - a_1) \tag{3.6}$$

$$A(A_2) = \frac{1}{2} w_{A_2}(c_2 - b_2 + d_2 - a_2) \tag{3.7}$$

The larger the value of $S(A_1, A_2)$, the more the similarity measure between two generalized trapezoidal fuzzy numbers A_1 and A_2.

Aggregation operations on fuzzy numbers are operations by which several fuzzy numbers are combined to produce a single fuzzy number.

i) Arithmetic Mean

The arithmetic mean aggregation operator defined on n trapezoidal fuzzy numbers (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2),..., (a_n, b_n, c_n, d_n) produces the result (a, b, c, d) where

$a = \frac{1}{n}\sum_1^n a_i$, $b = \frac{1}{n}\sum_1^n b_i$, $c = \frac{1}{n}\sum_1^n c_i$ and $d = \frac{1}{n}\sum_1^n d_i$.

ii) Geometric Mean

The arithmetic mean aggregation operator defined on n trapezoidal fuzzy numbers (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2),..., (a_n, b_n, c_n, d_n) produces the result (a, b, c, d) where

$a = (\prod_1^n a_i)^{\frac{1}{n}}$, $b = (\prod_1^n b_i)^{\frac{1}{n}}$, $c = (\prod_1^n c_i)^{\frac{1}{n}}$ and $d = (\prod_1^n d_i)^{\frac{1}{n}}$.

An Aggregation Operators trapezoidal fuzzy numbers are given in [7].

Consider the trapezoidal fuzzy number shown in Figure 1. If the value of this trapezoidal fuzzy number is $v \in [b, c]$ the corresponding possibility $\mu = 1$. The left side apex angle of this trapezoidal fuzzy number is $\mathcal{L}apb$. The right side apex angle of this trapezoidal fuzzy number is $\mathcal{L}drc$. The left and right side apex angles of the trapezoid refer to the apex angles subtended to the left and the right of the interval [b,c] respectively. But

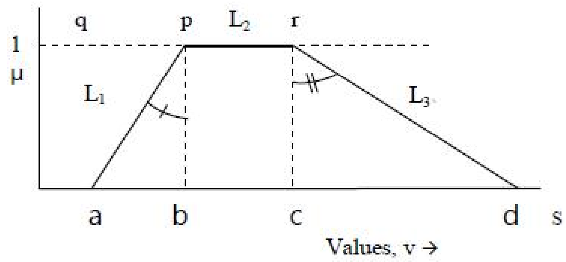

Figure 1. Trapezoidal Fuzzy Number

$$\mathcal{L}apb = \frac{\pi}{2} - \mathcal{L}bap \tag{3.8}$$

and

$$\mathcal{L}drc = \mathcal{L}sdr - \frac{\pi}{2} \tag{3.9}$$

Considering the left side and averaging over n trapezoidal fuzzy numbers we have

$$\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i = \frac{1}{n}\sum_{1}^{n}(\mathcal{L}(\frac{\pi}{2} - bap)_i) \tag{3.10}$$

$$\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i = \frac{\pi}{2} - \frac{1}{n}\sum_{1}^{n}(\mathcal{L}bap)_i \tag{3.11}$$

The left side of the above equation represents the contributions of the left lines(aggregate apex angle). It can be seen that

$$\tan(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i) = \frac{1}{\tan(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}bap)_i)} \tag{3.12}$$

It can be shown that

$$\tan(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i) = [tan(\frac{1}{n}\sum_{1}^{n}tan^{-1}(b_i - a_i))]^{-1} \tag{3.13}$$

Under identical treatment, it can be shown that

$$b = \frac{1}{n}\sum_{1}^{n}b_i, \quad c = \frac{1}{n}\sum_{1}^{n}c_i \ . \tag{3.14}$$

Subsequently it is possible to show that

$$a = \frac{1}{n}\sum_{1}^{n}b_i - \tan[\frac{1}{n}\sum_{1}^{n}\tan^{-1}(b_i - a_i)] \tag{3.15}$$

and

$$d = \frac{1}{n}\sum_{1}^{n}c_i + \tan[\frac{1}{n}\sum_{1}^{n}\tan^{-1}(d_i - c_i)] \tag{3.16}$$

So, Left and Right Apex Angles combines the concepts of the center of gravity, the area, the perimeter and the height of generalized fuzzy numbers for new approach. Means, Put the values (16-18) in (4-9). The proposed method is now presented as follows:

$*$ Step 1

Find y_A^* and y_B^*

$$y_A^* = \begin{cases} w_A([\frac{1}{n}\sum_1^n c_i - \frac{1}{n}\sum_1^n b_i/\frac{1}{n}\sum_1^n c_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(d_i-c_i)] - \frac{1}{n}\sum_1^n b_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i-a_i)]] + 2)/6 \\ if\ \frac{1}{n}\sum_1^n b_i - \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i-a_i)] \neq \frac{1}{n}\sum_1^n c_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(d_i-c_i)]\ and\ 0 < w_A \leq 1 \\ w_A/2 \\ if\ \frac{1}{n}\sum_1^n b_i - \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i-a_i)] = \frac{1}{n}\sum_1^n c_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(d_i-c_i)]\ and\ 0 < w_A \leq 1 \end{cases} \tag{3.17}$$

and

$$y_B^* = \begin{cases} w_B([\frac{1}{n}\sum_1^n c_i - \frac{1}{n}\sum_1^n b_i/\frac{1}{n}\sum_1^n c_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(d_i-c_i)] - \frac{1}{n}\sum_1^n b_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i-a_i)]] + 2)/6 \\ if\ \frac{1}{n}\sum_1^n b_i - \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i-a_i)] \neq \frac{1}{n}\sum_1^n c_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(d_i-c_i)]\ and\ 0 < w_B \leq 1 \\ w_B/2 \\ if\ \frac{1}{n}\sum_1^n b_i - \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i-a_i)] = \frac{1}{n}\sum_1^n c_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(d_i-c_i)]\ and\ 0 < w_B \leq 1 \end{cases} \tag{3.18}$$

$*$ Step 2

Find x_A^* and x_B^*

$$x_A^* = [y_A^*(\frac{1}{n}\sum_1^n c_i + \frac{1}{n}\sum_1^n b_i)$$

$$+(\frac{1}{n}\sum_1^n c_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(d_i-c_i)] + \frac{1}{n}\sum_1^n b_i - \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i-a_i)])(w_A - y_A^*)]/2w_A \tag{3.19}$$

and

$$x_B^* = [y_B^*(\frac{1}{n}\sum_1^n c_i + \frac{1}{n}\sum_1^n b_i)$$

$$+(\frac{1}{n}\sum_1^n c_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(d_i-c_i)] + \frac{1}{n}\sum_1^n b_i - \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i-a_i)])(w_B - y_B^*)]/2w_B \tag{3.20}$$

$*$ Step 3

Find $P(A)$ and $P(B)$

$$P(A) = \sqrt{(\frac{1}{n}\sum_1^n b_i - \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i-a_i)] - \frac{1}{n}\sum_1^n b_i)^2 + w_A^2}$$

$$+\sqrt{(\frac{1}{n}\sum_1^n c_i - \frac{1}{n}\sum_1^n c_i - \tan[\frac{1}{n}\sum_1^n \tan^{-1}(d_i-c_i)])^2 + w_A^2} + (\frac{1}{n}\sum_1^n c_i - \frac{1}{n}\sum_1^n b_i)$$

$$+(\frac{1}{n}\sum_1^n c_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(d_i-c_i)] - \frac{1}{n}\sum_1^n b_i + \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i-a_i)]) \tag{3.21}$$

and

$$P(B)=\sqrt{(\frac{1}{n}\sum_{1}^{n}b_i-\text{tan}[\frac{1}{n}\sum_{1}^{n}\text{tan}^{-1}(b_i-a_i)]-\frac{1}{n}\sum_{1}^{n}b_i)^2+w_B^2}$$

$$+\sqrt{(\frac{1}{n}\sum_{1}^{n}c_i-\frac{1}{n}\sum_{1}^{n}c_i-\text{tan}[\frac{1}{n}\sum_{1}^{n}\text{tan}^{-1}(d_i-c_i)])^2+w_B^2}+(\frac{1}{n}\sum_{1}^{n}c_i-\frac{1}{n}\sum_{1}^{n}b_i)$$

$$+(\frac{1}{n}\sum_{1}^{n}c_i+\text{tan}[\frac{1}{n}\sum_{1}^{n}\text{tan}^{-1}(d_i-c_i)]-\frac{1}{n}\sum_{1}^{n}b_i+\text{tan}[\frac{1}{n}\sum_{1}^{n}\text{tan}^{-1}(b_i-a_i)]) \quad (3.22)$$

$*$ Step 4

Find $A(A)$ and $A(B)$

$$A(A)=\frac{1}{2}w_A(\frac{1}{n}\sum_{1}^{n}c_i-\frac{1}{n}\sum_{1}^{n}b_i+\frac{1}{n}\sum_{1}^{n}c_i+\text{tan}[\frac{1}{n}\sum_{1}^{n}\text{tan}^{-1}(d_i-c_i)]$$

$$-\frac{1}{n}\sum_{1}^{n}b_i+\text{tan}[\frac{1}{n}\sum_{1}^{n}\text{tan}^{-1}(b_i-a_i)]) \quad (3.23)$$

and

$$A(B)=\frac{1}{2}w_B(\frac{1}{n}\sum_{1}^{n}c_i-\frac{1}{n}\sum_{1}^{n}b_i+\frac{1}{n}\sum_{1}^{n}c_i+\text{tan}[\frac{1}{n}\sum_{1}^{n}\text{tan}^{-1}(d_i-c_i)]$$

$$-\frac{1}{n}\sum_{1}^{n}b_i+\text{tan}[\frac{1}{n}\sum_{1}^{n}\text{tan}^{-1}(b_i-a_i)]) \quad (3.24)$$

$*$ Step 5

Calculating $S(A,B)$ with eq.(3) .

4 A comparison of the similarity measures

In this section, we extend 15 sets of fuzzy numbers presented in Wei and Chen [[9]] into 18 sets of fuzzy numbers, shown in Fig. 2, and compare the calculation results of the proposed method with the results of the existing similarity measures, shown in Table 1.

Example 1. Let $A=(0.1,0.2,0.3,0.4;1)$ and $B=(0.1,0.25,0.25,0.4;1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B)=[1-\mid 0.25-0.25\mid]\times[1-\mid 1-1\mid]\times\frac{min(2.36,2.36)+min(0.34,0.34)}{max(2.36,2.36)+max(0.34,0.34)}=1$$

Example 2. Let $A=(0.1,0.2,0.3,0.4;1)$ and $B=(0.1,0.2,0.3,0.4;1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B)=[1-\mid 0.275-0.275\mid]\times[1-\mid 1-1\mid]\times\frac{min(2.41,2.41)+min(0.2,0.2)}{max(2.41,2.41)+max(0.2,0.2)}=1$$

Example 3. Let $A=(0.1,0.2,0.3,0.4;1)$ and $B=(0.4,0.55,0.55,0.7;1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B)=[1-\mid 0.4-0.4\mid]\times[1-\mid 1-1\mid]\times\frac{min(2.36,2.36)+min(0.17,0.17)}{max(2.36,2.36)+max(0.17,0.17)}=1$$

Example 4. Let $A = (0.1, 0.2, 0.3, 0.4; 1)$ and $B = (0.4, 0.5, 0.6, 0.7; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.4 - 0.4 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.41, 2.41) + min(0.2, 0.2)}{max(2.41, 2.41) + max(0.2, 0.2)} = 1$$

Example 5. Let $A = (0.1, 0.2, 0.3, 0.4; 1)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.8)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.275 - 0.25 \mid] \times [1- \mid 1 - 0.8 \mid] \times \frac{min(2.41, 2.02) + min(0.2, 0.16)}{max(2.41, 2.02) + max(0.2, 0.16)}$$

$$= [1 - 0.025] \times [1 - 0.2] \times \frac{2.02 + 0.16}{2.41 + 0.2} = 0.975 \times 0.8 \times 0.83 = 0.6474$$

Example 6. Let $A = (0.3, 0.3, 0.3, 0.3; 1)$ and $B = (0.3, 0.3, 0.3, 0.3; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.3 - 0.3 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2, 2) + min(0, 0)}{max(2, 2) + max(0, 0)} = 1$$

Example 7. Let $A = (0.2, 0.2, 0.2, 0.2; 1)$ and $B = (0.3, 0.3, 0.3, 0.3; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.25 - 0.25 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2, 2) + min(0, 0)}{max(2, 2) + max(0, 0)} = 1$$

Example 8. Let $A = (0.1, 0.2, 0.2, 0.3; 1)$ and $B = (0.3, 0.3, 0.3, 0.3; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.25 - 0.25 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(0.2, 0.2) + min(0.05, 0.05)}{max(0.2, 0.2) + max(0.05, 0.05)} = 1$$

Example 9. Let $A = (0.1, 0.2, 0.2, 0.3; 1)$ and $B = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.25 - 0.25 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.21, 2.21) + min(0.1, 0.1)}{max(2.21, 2.21) + max(0.1, 0.1)} = 1$$

Example 10. Let $A = (0.1, 0.4, 0.4, 0.7; 1)$ and $B = (0.3, 0.4, 0.4, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.4 - 0.4 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(0.84, 0.84) + min(0.2, 0.2)}{max(0.84, 0.84) + max(0.2, 0.2)} = 1$$

Example 11. Let $A = (0.2, 0.3, 0.5, 0.6; 1)$ and $B = (0.3, 0.4, 0.4, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.4 - 0.4 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.41, 2.41) + min(0.2, 0.2)}{max(2.41, 2.41) + max(0.2, 0.2)} = 1$$

Example 12. Let $A = (0.4, 0.4, 0.4, 0.8; 1)$ and $B = (0.3, 0.4, 0.4, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.5 - 0.5 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.23, 2.23) + min(0.1, 0.1)}{max(2.23, 2.23) + max(0.1, 0.1)} = 1$$

Example 13. Let $A = (0.2, 0.3, 0.4, 0.5; 1)$ and $B = (0.3, 0.4, 0.5, 0.6; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.4 - 0.4 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.41, 2.41) + min(0.2, 0.2)}{max(2.41, 2.41) + max(0.2, 0.2)} = 1$$

Example 14. Let $A = (0.1, 0.2, 0.2, 0.3; 1)$ and $B = (0.1, 0.2, 0.2, 0.3; 0.7)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.2 - 0.14 \mid] \times [1- \mid 1 - 0.7 \mid] \times \frac{min(1.205, 1.62) + min(0.1, 0.07)}{max(1.205, 1.62) + max(0.1, 0.07)}$$

$$= \mid 1 - 0.06 \mid \times \mid 1 - 0.3 \mid \times \frac{1.205 + 0.07}{1.62 + 0.1} = 0.94 \times 0.7 \times 0.74 = 0.467$$

Example 15. Let $A = (0.1, 0.2, 0.2, 0.3; 1)$ and $B = (0.2, 0.2, 0.2, 0.2; 0.7)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.2 - 0.14 \mid] \times [1- \mid 1 - 0.7 \mid] \times \frac{min(2.102, 1.5) + min(0.05, 0.035)}{max(2.102, 1.5) + max(0.05, 0.035)}$$

$$= \mid 1 - 0.06 \mid \times \mid 1 - 0.3 \mid \times \frac{1.5 + 0.035}{2.102 + 0.05} = 0.94 \times 0.7 \times 0.713 = 0.47$$

Example 16. Let $A = (0.1, 0.4, 0.4, 0.7; 0.825)$ and $B = (0.3, 0.4, 0.4, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.4 - 0.33 \mid] \times [1- \mid 0.825 - 1 \mid] \times \frac{min(2.1, 2.44) + min(0.165, 0.2)}{max(2.1, 2.44) + max(0.165, 0.2)}$$

$$= \mid 1 - 0.07 \mid \times \mid 1 - 0.175 \mid \times \frac{2.1 + 0.165}{2.44 + 0.2} = 0.93 \times 0.825 \times 0.86 = 0.66$$

Example 17. Let $A = (0.2, 0.3, 0.5, 0.6; 0.79205)$ and $B = (0.3, 0.4, 0.4, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.4 - 0.317 \mid] \times [1- \mid 0.79205 - 1 \mid] \times \frac{min(2, 2.41) + min(0.158, 0.2)}{max(2, 2.41) + max(0.158, 0.2)}$$

$$= \mid 1 - 0.083 \mid \times \mid 1 - 0.20795 \mid \times \frac{2 + 0.158}{2.41 + 0.2} = 0.917 \times 0.79205 \times 0.83 = 0.603$$

Example 18. Let $A = (0.2, 0.3, 0.5, 0.6; 1)$ and $B = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.35 - 0.35 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.41, 2.41) + min(0.2, 0.2)}{max(2.41, 2.41) + max(0.2, 0.2)} = 1$$

Table (1): A comparison of the ranking results for different approaches

sets	Chen[2]	Lee[4]	Chen[1]	Wei[9]	Hejazi[3]	Wen[10]	proposed method
set 1	0.975	0.9617	0.8357	0.95	0.9004	0.9473	1
set 2	1	1	1	1	1	1	1
set 3	0.7	0.5	0.42	0.682	0.6465	0.6631	1
set 4	0.7	0.5	0.49	0.7	0.7	0.7	1
set 5	1	1	0.8	0.8248	0.6681	0.6659	0.6474
set 6	1	$*$	1	1	1	1	1
set 7	0.9	0	0.81	0.9	0.9	0.9	1
set 8	0.9	0.5	0.54	0.8411	0.37	0.3896	1
set 9	0.9	0.6667	0.81	0.9	0.9	0.9	1
set 10	0.9	0.8333	0.9	0.7833	0.6261	0.7731	1
set 11	0.9	0.75	0.72	0.8003	0.6448	0.7938	1
set 12	0.9	0.8	0.8325	0.8289	0.7361	0.7478	1
set 13	0.9	0.75	0.81	0.9	0.9	0.9	1
set 14	1	1	0.7	0.7209	0.5113	0.5104	0.467
set 15	0.95	0.75	0.9048	0.6215	0.383	0.4242	0.47
set 16	0.9	0.8333	0.7425	0.814	0.6261	0.7321	0.66
set 17	0.9	0.75	0.8911	0.838	0.6448	0.7432	0.603
set 18	0.9	0.75	0.6976	0.8003	0.6448	0.7144	1

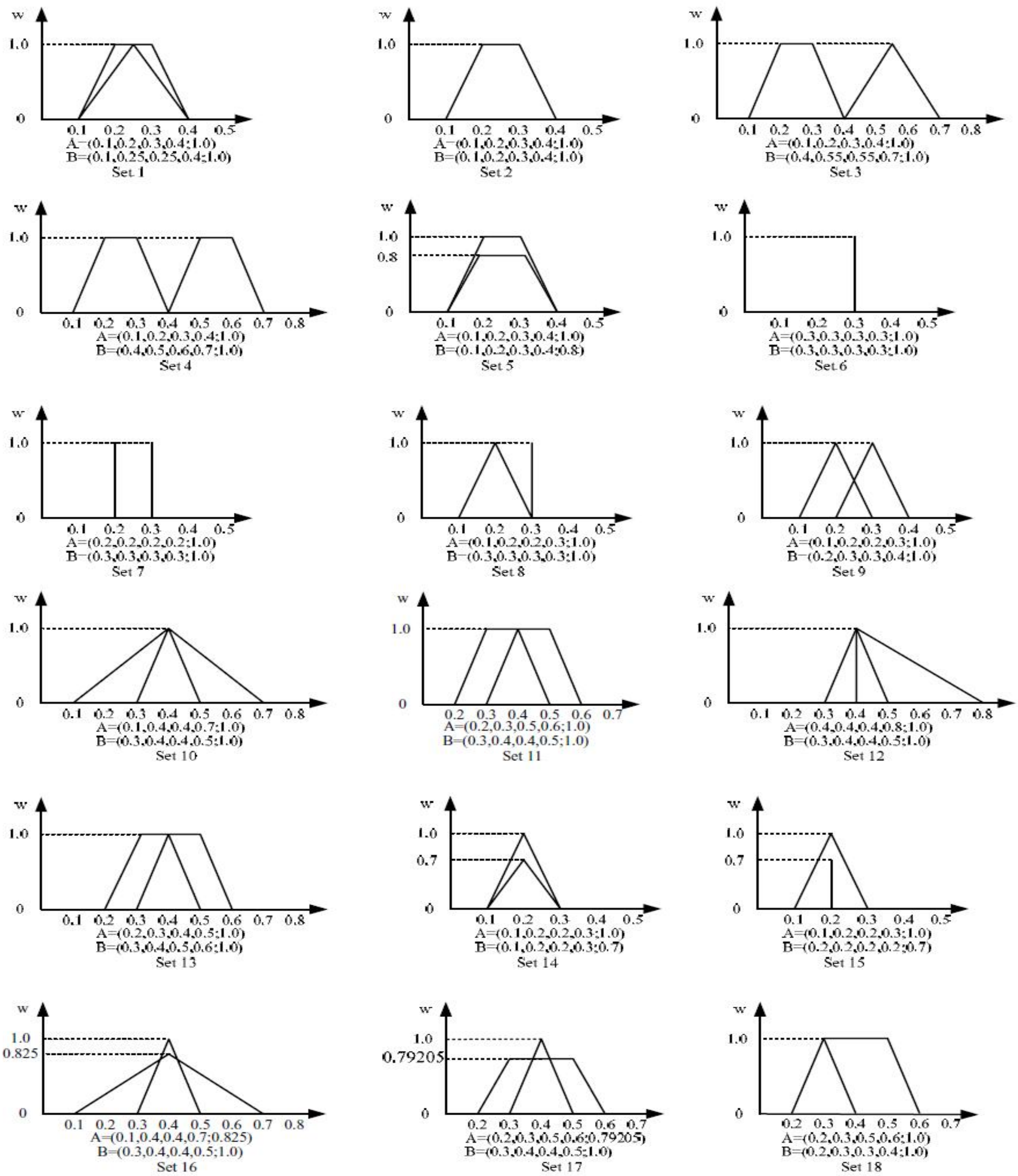

Figure 2. Eighteen sets of generalized fuzzy numbers

5 Conclusions

In this paper, left and right apex angles combines the concepts of the center of gravity, the area, the perimeter and the height of generalized fuzzy numbers for calculating the degree of similarity between generalized fuzzy numbers. The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

References

[1] Chen S. J., Chen S. M., Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers, IEEE Trans. *Fuzzy Syst.*, vol. 11, no. 1, p. 45-56 (2003).

[2] Chen S. M., New methods for subjective mental workload assessment and fuzzy risk analysis, Cybern. Syst.: *Int. J*, vol. 27, no. 5, p. 449-472 (1996).

[3] Hejazi S.R., Doostparast A., Hosseini S.M., An improved fuzzy risk analysis based on a new similarity measures of generalized fuzzy numbers, *Expert Systems with Applications* Volume 38, Issue 8, P. 9179-9185 (August 2011).

[4] Lee H. S., Optimal consensus of fuzzy opinions under group decision making environment, *Fuzzy sets Syst.*, vol. 132, no. 3, p. 303-315 (2002).

[5] Li, D.F., Cheng, C.T., New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions. *Pattern Recognition Lett.* 23, p. 221Ű225 (2002).

[6] Liang, Z.Z., Shi, P.F., Similarity measures on intuitionistic fuzzy numbers. *Pattern Recognition Lett.* 24, p. 2687Ű2693 (2003).

[7] Pandey et al., New Aggregation Operator for Trapezoidal Fuzzy Numbers based on the Arithmetic Means of the Left and Right Apex Angles, *International Journal of Advanced Research in Computer Science and Software Engineering*, Volume 2, Issue 3, p. 430-433 (March 2012).

[8] S. Rezvani, Three-tier FMCDM Problems with Trapezoidal Fuzzy Number, *World Applied Sciences Journal* 10 (9), p. 1106-1113 (2010).

[9] Wei, S. H., Chen, S. M. A new approach for fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. *Expert Systems with Applications*, 36, p. 589Ű598 (2009).

[10] Jiang Wen et al., A modified similarity measure of generalized fuzzy numbers, *Procedia Engineering* 15, p. 2773-2777 (2011).

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