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ON SET-INDEXERS OF GRAPHS

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Abstract This paper derives the set-indexing numbers of certain graphs subsequently identifying some set-semigraceful graphs.

1 Introduction

In the mathematical discipline of Graph Theory, labeling of graphs is a mapping that sends the edges or vertices or both of a graph to the set of numbers or subset of a set under certain conditions. Ever since the introduction of the concept labeling of graphs, it has been an active area of research in Graph Theory.

Most graph labelings trace their origin to a paper by Rosa [14]. In 1983, B. D. Acharya [1] initiated a general study of the labeling of the vertices and edges of a graph using the subset of a set known as set-valuation of graphs and indicated their potential applications in different areas of human inquiry. He introduced the concept of set-indexer of a graph and proved that every graph admits a set-indexer. Acharya also pioneered the notion of set-indexing number of a graph developing the classes of set-graceful as well as set-semigraceful graphs.

Meanwhile, Acharya and Hegde [5] undertook the study of yet another notion of set-valuation of graphs called set-sequential labeling as a set analogue of the well known sequential graphs and obtained some noteworthy results. Following this, Mollard and Payan [13] settled two conjectures about set-graceful graphs suggested by Acharya in [1]. Certain necessary conditions for a graph to have set-graceful and set-sequential labeling were obtained by Hegde in [9]. Acharya and Hegde [6] in 1999 published a lot of graph labeling problems including certain set-labeling problems. The survey of Acharya [3] on set-indexers of graphs focussing set-graceful graphs set a new momentum to this area of study in 2001. Later, many authors have investigated various aspects of set-valuation of graphs and derived new properties concerning them in [4], [10] and [15]. Recently, Vijayakumar [21] settled Hegde's [10] conjecture that every complete bipartite graph that has a set-graceful labeling is a star. Motivated by this, the authors studied set-indexers of graphs in [16], [17], [18] and [19].

In continuation of the study on set-indexers, this paper derives the set-indexing numbers of certain graphs. A relationship among the set-indexing numbers of the wheels, suns and cycles is established. The study also sheds some light on set-semigraceful graphs and obtains a characterization for set-semigraceful double stars.

2 Preliminaries

In this section we include certain definitions and known results needed for the subsequent development of the study. For a nonempty set X, the set of all subsets of X is denoted by 2^X. We always denote a graph under consideration by G and its vertex and edge sets by V and E respectively. By $G' \subseteq G$ we mean G' is a subgraph of G while $G' \subset G$ we mean G' is a proper subgraph of G. The empty graph of order n is denoted by N_n. The order and size of a graph G is denoted by $o(G)$ and $s(G)$ respectively. The basic notations and definitions in graph theory are assumed to be familiar to the reader.

Definition 2.1. [2] Let $G = (V, E)$ be a given graph and X be a nonempty set. Then a mapping $f : V \to 2^X$, or $f : E \to 2^X$, or $f : V \cup E \to 2^X$ is called a set-assignment or set-valuation of the vertices or edges or both.

Definition 2.2. [2] Let G be a given graph and X be a nonempty set. Then a set-valuation $f : V \cup E \to 2^X$ is a set-indexer of G if

(i) $f(u, v) = f(u) \oplus f(v), \forall (u, v) \in E$, where '$\oplus$' denotes the binary operation of taking the symmetric difference of the sets in 2^X

(ii) the restriction maps $f|_V$ and $f|_E$ are both injective.

In this case, X is called an indexing set of G. Clearly a graph can have many indexing sets and the minimum of the cardinalities of the indexing sets is said to be the set-indexing number of G, denoted by $\gamma(G)$. The set-indexing number of trivial graph K_1 is defined to be zero.

Theorem 2.3. [2] *Every graph has a set-indexer.*

Theorem 2.4. [2] *If X is an indexing set of $G = (V, E)$. Then*

(i) $|E| \leq 2^{|X|} - 1$ *and*

(ii) $\lceil \log_2(|E| + 1) \rceil \leq \gamma(G) \leq |V| - 1$, *where* $\lceil\ \rceil$ *is the ceiling function.*

Theorem 2.5. [12] *For any graph G,* $\lceil \log_2 |V| \rceil \leq \gamma(G)$.

Theorem 2.6. [2] *If G' is a subgraph of G, then* $\gamma(G') \leq \gamma(G)$.

Theorem 2.7. [17] *If G is a star graph, then* $\gamma(G) = \lceil \log_2 o(G) \rceil$.

Theorem 2.8. [17] $\gamma(K_{1,n}) = \gamma(N_{n+1})$.

Theorem 2.9. [17] $\gamma(K_{2^m,n}) = m + l$; $l = \lfloor log_2 n \rfloor + 1$.

Definition 2.10. [2] A graph G is set-graceful if $\gamma(G) = \log_2(|E| + 1)$ and the corresponding set-indexer is called a set-graceful labeling of G.

Theorem 2.11. [13] *For any integer $n \geq 2$, the cycle C_{2^n-1} is set-graceful.*

Theorem 2.12. [18] $\gamma(C_4) = 3$.

Theorem 2.13. [3] $\gamma(C_5) = 4$.

Theorem 2.14. [19] *For* $k \geq 7$, $\gamma(C_k) = \begin{cases} n;\ k = 2^n - 1 \\ n+1;\ 2^n \leq k \leq 2^n + 2^{n-1} - 2. \end{cases}$

Definition 2.15. [7] An n-sun is a graph that consists of a cycle C_n and an edge terminating in a vertex of degree one attached to each vertex of C_n.

3 Set-Indexing Numbers

This section focusses mainly on the set-indexing numbers of certain suns, cycles and wheels. Upper bounds for the set-indexing numbers of complete k-partite graphs have been derived.

Theorem 3.1. *If m is not a power of 2, then* $\gamma(K_{m,n}) \leq \lceil \log_2 m \rceil + \lceil \log_2 n \rceil$.

Proof. Let $V = \{u_1, \ldots, u_m, v_1, \ldots, v_n\}$; $d(u_i) = n$ for $1 \leq i \leq m$ and $d(v_j) = m$ for $1 \leq j \leq n$. Consider the sets $X = \{x_1, \ldots, x_p\}$ and $Y = \{y_1, \ldots, y_q\}$ where $p = \lceil \log_2 m \rceil$ and $q = \lceil \log_2 n \rceil$. Assigning any m distinct nonempty subsets of X to the vertices $u_1, \ldots, u_m$ and any n distinct subsets of Y to the vertices $v_1, \ldots, v_n$ in any order we get a set-indexer of $K_{m,n}$ with indexing set $X \cup Y$. Consequently, $\gamma(K_{m,n}) \leq p + q$. □

Remark 3.2. If both m and n are powers of 2, the above upper bound need not be true. For example, take $K_{m,n} = K_{4,16}$. Then, 6 = $\lceil \log_2 m \rceil + \lceil \log_2 n \rceil < \gamma(K_{m,n})$ = 7, by Theorem 2.9.

Theorem 3.3. $\gamma(K_{n_1,\ldots,n_k}) \leq \sum_{i=1}^{k} \lceil \log_2 n_i \rceil + k - 1$.

Proof. Let $V = V_1 \cup \ldots \cup V_k$ be a partition of the vertex set of $K_{n_1,\ldots,n_k}$. Let $p_1 = \lceil \log_2 n_1 \rceil$ and $p_i = \lceil \log_2 n_i \rceil + 1$ for $2 \leq i \leq k$. Let $X_1, \ldots, X_k$ be any k disjoint sets with $|X_i| = p_i$ for $1 \leq i \leq k$. Now, assign the vertices in V_1 with distinct subsets of X_1 and the vertices in V_i; $2 \leq i \leq k$ with distinct nonempty subsets of X_i, in any order. This gives a set-indexer for $K_{n_1,\ldots,n_k}$ so that $\gamma(K_{n_1,\ldots,n_k}) \leq \sum_{i=1}^{k} \lceil \log_2 n_i \rceil + k - 1$. □

The upper bound for the set-indexing number of the cycles C_{2^n-2}; $n \geq 3$ given by the following

Theorem 3.4. [18] *For every integer* $n \geq 3$, $\gamma(C_{2^n-2}) \leq n+1$

is the best one possible. For, below we prove that all such cycles attain that upper bound.

Theorem 3.5. $\gamma(C_{2^n-2}) = n+1$*;* $n \geq 3$.

Proof. By Theorem 2.4 and Theorem 3.4, $n \leq \gamma(C_{2^n-2}) \leq n+1$ where $C_{2^n-2} = (v_1, \ldots, v_{2^n-2}, v_1)$. Suppose $\gamma(C_{2^n-2}) = n$ and let f be the corresponding set-indexer with indexing set X. Let $f(v_i) = A_i$; $1 \leq i \leq 2^n - 2$. Then the edge labels given by $A_1 \oplus A_2$, $A_2 \oplus A_3$, $\ldots$, $A_{2^n-2} \oplus A_1$ are $2^n - 2$ distinct nonempty subsets of X. Hence, there exists exactly one nonempty subset say, B of X such that $B \notin f(E)$. Now,
$(A_1 \oplus A_2) \oplus (A_2 \oplus A_3) \oplus \ldots \oplus (A_{2^n-2} \oplus A_1) \oplus B = \emptyset \Rightarrow B = \emptyset$ – a contradiction.
Consequently, $\gamma(C_{2^n-2}) = n+1$, as required. □

Now for the sake of completeness, we combine certain known results on the set-indexing numbers of cycles below:

Theorem 3.6. *For* $k \geq 7$, $\gamma(C_k)= \begin{cases} n & ;\ k = 2^n - 1 \\ n+1; & k = 2^n - 2 \text{ or } 2^n \leq k \leq 2^n + 2^{n-1} - 2 \end{cases}$.

Theorem 3.7. [18] *If G is the* $(2^n - 1)$*-sun;* $n \geq 2$*, then* $\gamma(G) = n+1$.

Theorem 3.8. [18] *For* $n \geq 2$, $\gamma(2^n{-}sun) = n+2$.

Theorem 3.9. $\gamma(5{-}sun) = 4$.

Proof. The set-valuation given in Figure 1 together with Theorem 2.5 shows that $\gamma(5{-}sun) = 4$. □

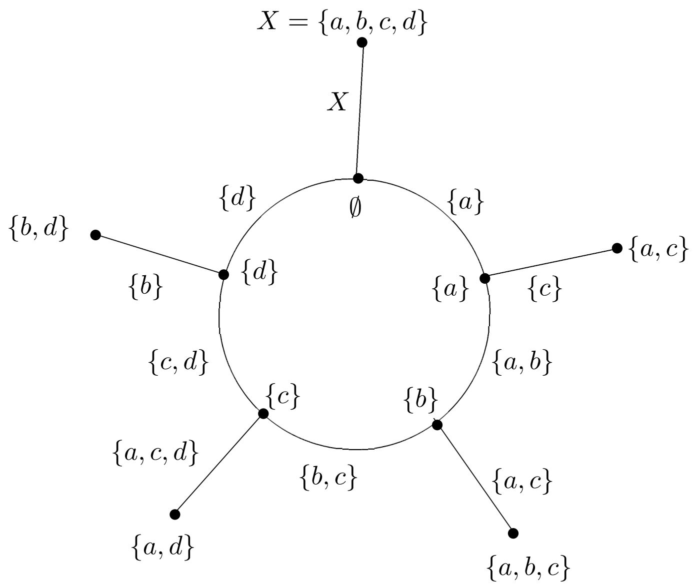

Figure 1: An optimal set-indexer of the 5-sun

Theorem 3.10. [18] *If G is the* $(2^n + 1)$*-sun;* $n \geq 3$*, then* $\gamma(G) = n+2$.

Theorem 3.11. $\gamma(6{-}sun) = 4$.

Proof. The Figure 2 and Theorem 2.4 account for the proof. □

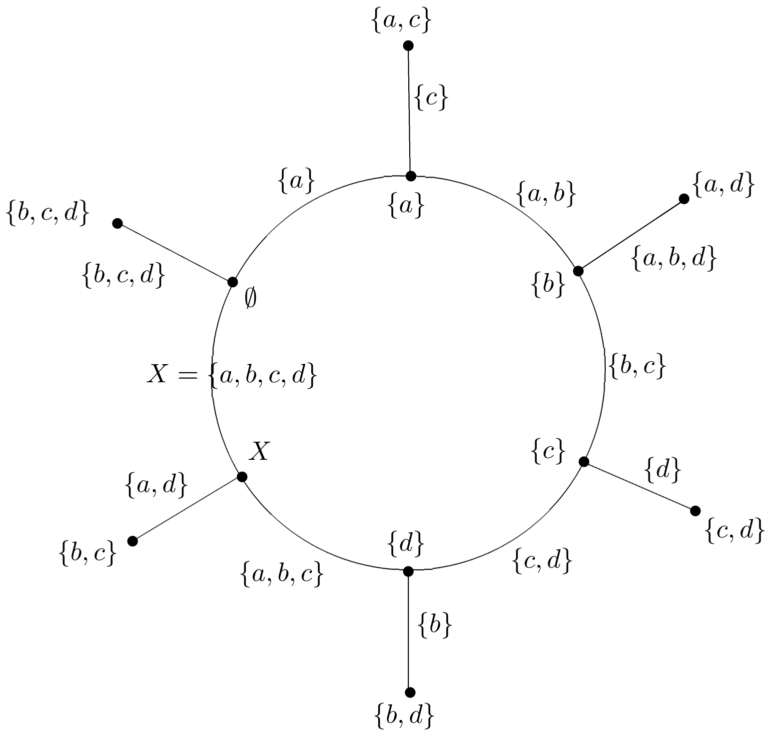

Figure 2: An optimal set-indexer of the 6-sun

Theorem 3.12. [18] *If G is the* (2^n+2)*-sun;* $n \geq 3$*, then* $\gamma(G) = n+2$.

For the convenience of future references, the results of Theorem 3.7, Theorem 3.8, Theorem 3.9, Theorem 3.10, Theorem 3.11 and Theorem 3.12 are summarized as follows:

Theorem 3.13. $\gamma(k{-}sun) = \begin{cases} n+1 \; if \; k = 2^n-1; \; n \geq 2 \\ n+2 \; if \; k = 2^n, 2^n+1, 2^n+2; \; n \geq 2 \end{cases}$

.

Definition 3.14. [11] The wheel graph with n spokes, W_n, is the graph that consists of an n-cycle and one additional vertex, say u, that is adjacent to all the vertices of the cycle.

Theorem 3.15. [18] *The set-indexing number of the wheel* W_{2^n-1}*;* $n \geq 2$ *is* $n+1$.

Theorem 3.16. [18] *For every integer* $n \geq 2$*, the set-indexing number of the wheel* W_{2^n} *is* $n+2$.

Theorem 3.17. $\gamma(W_5) = 4$.

Proof. The set-valuation of W_5 given in Figure 3 together with Theorem 2.4 shows that $\gamma(W_5)$ = 4. □

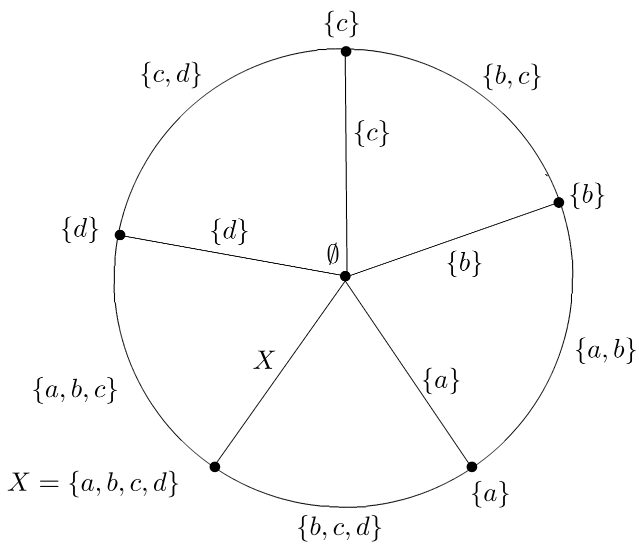

Figure 3: An optimal set-indexer of the wheel W_5

Theorem 3.18. [18] *The set-indexing number of the wheel* W_{2^n+1}; $n \geq 3$ *is* $n+2$.

Theorem 3.19. *The set-indexing number of the wheel* W_6 *is* 4.

Proof. In the light of Theorem 2.4, the proof is evident from the set-indexer given in Figure 4. □

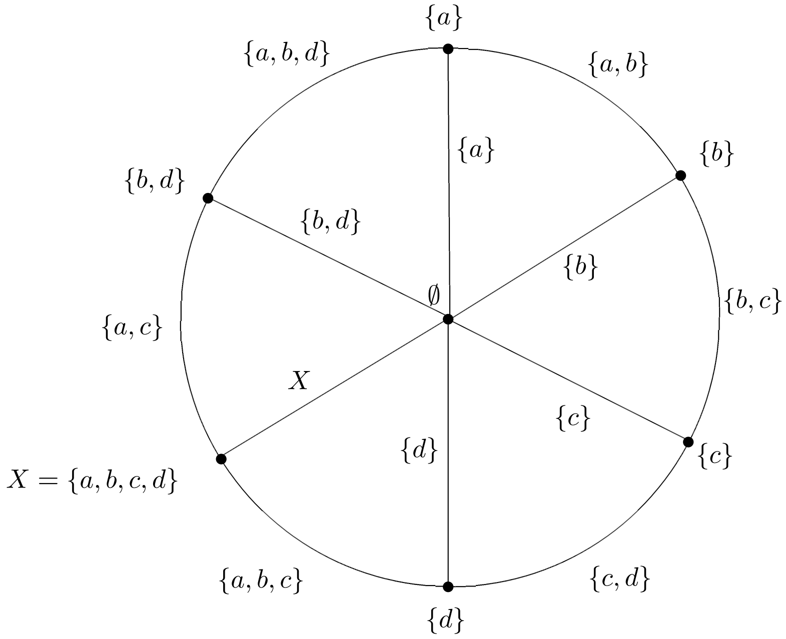

Figure 4: An optimal set-indexer of the wheel W_6

Theorem 3.20. [18] *The set-indexing number of the wheel* W_{2^n+2}; $n \geq 3$ *is* $n+2$.

Now we summarize the results of Theorem 3.15, Theorem 3.16, Theorem 3.17, Theorem 3.18, Theorem 3.19 and Theorem 3.20 for the convenience of future references.

Theorem 3.21.

$$\gamma(W_k) = \begin{cases} n+1 \; if \; k = 2^n - 1; \; n \geq 2 \\ n+2 \; if \; k = 2^n, 2^n+1, 2^n+2; \; n \geq 2 \end{cases}$$

The following Theorem shows that the set-indexing number of W_n is either $\gamma(C_n)$ or $\gamma(C_n)+1$.

Theorem 3.22. $\gamma(C_n) \leq \gamma(W_n) \leq \gamma(C_n) + 1$.

Proof. Let f be any set-indexer of C_n with indexing set X. Then f can be extended to a set-indexer g of W_n with indexing set $X \cup \{a\}$; $a \notin X$ defined as follows:
$g(v) = f(v)$ for all $v \in V(C_n)$ and $g(w) = \{a\}$; $V(K_1) = \{w\}$.
Hence, $\gamma(W_n) \leq \gamma(C_n) + 1$. Since $C_n \subset W_n$, the first inequality follows from Theorem 2.6. □

Similarly, the set-indexing number of n-sun is also either $\gamma(C_n)$ or $\gamma(C_n) + 1$.

Theorem 3.23. $\gamma(C_n) \leq \gamma(n{-}sun) \leq \gamma(C_n) + 1$.

Proof. Let the n-sun G contain the cycle $C_n = (v_1, \ldots, v_n, v_1)$ and $\{(u_i, v_i);\ i = 1, \ldots, n\}$ be the pendant edges of G. Let f be any set-indexer of C_n with indexing set X. Now we can define a set-indexer g of G with indexing set $X \cup \{a\}$; $a \notin X$ as follows:

$g(v_i) = f(v_i)$; $i \in \{1, \ldots, n\}$ and
$g(u_i) = f(v_{i-1}) \cup \{a\}$; $i \in \{1, \ldots, n\}$, $v_0 = v_n$.

Now by Theorem 2.6, $\gamma(C_n) \leq \gamma(n - sun) \leq \gamma(C_n) + 1$. □

Thus, $\gamma(C_n) \leq \gamma(W_n)$, $\gamma(\text{n}{-}sun) \leq \gamma(C_n) + 1$ and it has been already proved that, $\gamma(W_k) = \gamma(\text{k}{-}sun) = \gamma(C_k) + 1$ where $k = 2^n - 1$, 2^n, $2^n + 1$, $2^n + 2$ and $n \geq 3$. Also, $\gamma(W_k) = \gamma(\text{k}{-}sun) = \gamma(C_k) + 1$ where $k = 3, 4$ and $\gamma(W_k) = \gamma(\text{k}{-}sun) = \gamma(C_k)$ where $k = 5, 6$.

In the light of the above observations we put forward the following:

Conjecture 3.24. $\gamma(W_n) = \gamma(\text{n}{-}sun)$; $n \geq 3$.

4 Set-semigraceful Graphs

In 1986, B. D. Acharya introduced the concept of set-semigraceful graphs:

Definition 4.1. [2] A graph G is said to be set-semigraceful if $\gamma(G) = \lceil \log_2(|E|+1) \rceil$.

Obviously, all set-graceful graphs are set-semigraceful.

By Theorem 3.6, it can be easily seen that, the cycles C_k; $2^n - 1 \leq k \leq 2^n + 2^{n-1} - 2$, $n \geq 3$ are set-semigraceful while by Theorem 3.5, C_{2^n-2}; $n \geq 3$ are not set-semigraceful. Further, C_3 and C_4 are set-semigraceful by Theorem 2.11 and Theorem 2.12 respectively. But C_5 is not set-semigraceful by Theorem 2.13.

Also set-semigraceful paths have been characterized in [19]:

Theorem 4.2. *The paths P_n is set-semigraceful if and only if $n \neq 2^m$; $m > 1$.*

Again by Theorem 3.21, Theorem 3.22 and Theorem 2.14, the wheels W_k; $2^n - 1 \leq k \leq 2^n + 2^{n-1} - 2$, $n \geq 2$ are set-semigraceful.

Further, the k-suns; $2^n - 1 \leq k \leq 2^n + 2^{n-1} - 2$, $n \geq 2$ are also set-semigraceful by Theorem 3.13, Theorem 3.23 and Theorem 2.14.

Now we shall look into set-semigraceful double stars.

Definition 4.3. [22] The double star graph $ST(m,n)$ is the graph formed by two stars $K_{1,m}$ and $K_{1,n}$ by joining their centers by an edge.

Theorem 4.4. [20] *For a double star graph $ST(m,n)$ with $|V| = 2^l$; $l \geq 2$*

$$\gamma(ST(m,n)) = \begin{cases} l & \text{if } m \text{ is even,} \\ l+1 & \text{if } m \text{ is odd.} \end{cases}$$

Theorem 4.5. [20] $\gamma(ST(m,n))$ = $l+1$ *if* $2^l < |V| < 2^{l+1}$.

Theorem 4.6. *A double star is set-semigraceful if its order is not a power of 2.*

Proof. Let $ST(m,n)$ be a double star whose order is not a power of 2. Then there exists a positive integer $l \geq 2$ such that $2^l < |V| < 2^{l+1}$. By Theorem 4.5, $\gamma(ST(m,n))$ = $\lceil \log_2 |V| \rceil$ = $\lceil \log_2(|E|+1) \rceil$, since $ST(m,n)$ is a tree. Therefore, $ST(m,n)$ is set-semigraceful. □

Remark 4.7. The converse of Theorem 4.6 is not true. For example consider the double star $ST(m,n)$; $m = 4$ and $n = 2$. By Theorem 4.4, $\gamma(ST(m,n))$ = 3 = $\lceil \log_2(|E|+1) \rceil$ so that $ST(m,n)$ is set-semigraceful.

Theorem 4.8. *The double star $ST(m,n)$ is set-semigraceful if m is even.*

Proof. Since $o(ST(m,n)) \geq 4$, there exists a positive integer $l \geq 2$ such that $2^l \leq |V| \leq 2^{l+1}-1$. If $|V| = 2^l$, then by Theorem 4.4, $\gamma(ST(m,n)) = l = \lceil \log_2 |V| \rceil = \lceil \log_2(|E|+1) \rceil$ so that $ST(m,n)$ is set-semigraceful. Otherwise, the required result follows from Theorem 4.6. □

Remark 4.9. The converse of the above Theorem is not true as the double star $ST(5,7)$ is set-semigraceful by Theorem 4.6.

The following theorem characterizes the set-semigraceful double stars.

Theorem 4.10. *The double star $ST(m,n)$ is set-semigraceful if and only if either the order is not a power of 2 or m is even.*

Proof. For a double star $ST(m,n)$, suppose the order is a power of 2 and m is odd. Then by Theorem 4.4, $\gamma(ST(m,n)) = \lceil \log_2 |V| \rceil + 1 = \lceil \log_2(|E|+1) \rceil + 1 > \lceil \log_2(|E|+1) \rceil$.
$\Rightarrow ST(m,n)$ is not set-semigraceful. Consequently, if $ST(m,n)$ is set-semigraceful, then either the order is not a power of 2 or m is even.

The converse part follows from Theorem 4.6 and Theorem 4.8. □

The following theorem gives a necessary condition for the set-semigracefulness of a graph.

Theorem 4.11. *If G is a (p,q)-graph such that $p > 2^n > q$ for some n, then G is not set-semigraceful.*

Proof. By Theorem 2.5, $\gamma(G) \geq \lceil \log_2 p \rceil$
$> \lceil \log_2(q+1) \rceil$, by hypothesis.
Thus, $\gamma(G) > \lceil \log_2(|E|+1) \rceil$ so that G is not set-semigraceful. □

Remark 4.12. The above Theorem provides a class of disconnected graphs that are not set-semigraceful. However, there are disconnected graphs that are not set-semigraceful and do not belong to the above class. For instance, $K_8 \cup K_1$ and $K_{10} \cup K_1$. Further, the converse of Theorem 4.11 is not true as there are infinitely many connected graphs that are not set-semigraceful by Theorem 4.2.

The following theorem identifies the set-semigraceful spanning subgraphs of stars.

Theorem 4.13. *Let $G = (V, E)$ be a spanning subgraph of $K_{1,n}$; $n \geq 2$. Then G is set-semigraceful if and only if $2^{\lceil \log_2 n \rceil - 1} \leq |E| \leq 2^{\lceil \log_2 n \rceil} - 1$.*

Proof. Suppose G is set-semigraceful so that $\gamma(G) = \lceil \log_2(|E|+1) \rceil$. By Theorem 2.7 and Theorem 2.8, $\lceil \log_2 n \rceil = \gamma(N_{n+1})$
$\leq \gamma(G)$, by Theorem 2.6
$\leq \gamma(K_{1,n})$, again by Theorem 2.6
$= \lceil \log_2 n \rceil$, by Theorem 2.7
$\Rightarrow \lceil \log_2(|E|+1) \rceil = \lceil \log_2 n \rceil$
$\Rightarrow 2^{\lceil \log_2 n \rceil - 1} < |E| + 1 \leq 2^{\lceil \log_2 n \rceil}$
$\Rightarrow 2^{\lceil \log_2 n \rceil - 1} \leq |E| \leq 2^{\lceil \log_2 n \rceil} - 1$.

Conversely, suppose $2^{\lceil \log_2 n \rceil - 1} \leq |E| \leq 2^{\lceil \log_2 n \rceil} - 1$. By Theorem 2.7 and Theorem 2.8, $\lceil \log_2 n \rceil = \gamma(N_{n+1})$
$\leq \gamma(G)$, by Theorem 2.6
$\leq \gamma(K_{1,n})$, again by Theorem 2.6
$= \lceil \log_2 n \rceil$, by Theorem 2.7.
But, $\lceil \log_2(|E|+1) \rceil = \lceil \log_2 n \rceil = \gamma(G)$ so that G is set-semigraceful. □

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