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On class of functions related to conic regions and symmetric points

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Abstract. In this note, the concept of N-symmetric points. Janowski functions and the conic regions are combined to define a class of functions in a new interesting domain . Certain interesting results are discussed.

1 Introduction

Let $\mathcal{A}$ denote the class of functions of form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk $\mathcal{U} = \{z : z \in \mathbb{C} \ \ and \ \ |z| < 1\}$, with normalization $f(0) = 0$ and $f'(0) = 1$. A function f in $\mathcal{A}$ is said to be starlike with respect to N-symmetric points $\mathcal{S}_N^*$ if

$$\Re\left\{\frac{zf'(z)}{f_N(z)}\right\} > o, \quad z \in \mathcal{U}, N \in \mathbb{N}, \tag{1.2}$$

where

$$f_N(z) = z + \sum_{n=2}^{\infty} \lambda_N(n) a_n z^n, \tag{1.3}$$

where

$$\lambda_N(n) = \begin{cases} 1, & n = lN + 1, \quad l \in \mathbb{N}_0. \\ 0, & n \neq lN + 1. \end{cases} \tag{1.4}$$

For a positive integer N, let $\varepsilon = \exp\left(\frac{2\pi i}{N}\right)$ denote the N^{th} root of unity for $f \in \mathcal{A}$, let

$$M_{f,N}(z) = \sum_{v=1}^{N-1} \varepsilon^{-v} f(\varepsilon^v z).\frac{1}{\sum_{v=1}^{N-1} \varepsilon^{-v}}, \tag{1.5}$$

be its N-weighed mean function. It is easy to verify that

$$\frac{f(z) - M_{f,N}(z)}{N} = \frac{1}{N} \sum_{v=0}^{N-1} \varepsilon^{-v} f(\varepsilon^v z) = f_N(z).$$

The class $\mathcal{S}^*$ is the collection of functions $f \in \mathcal{A}$ such that for any r close to $r < 1$, the angular velocity of f about the point $M_{f_N(z_0)}$ positive at $z = z_0$ as z traverses the circle $|z| = r$ in the positive direction. The well-known class of starlike functions $\mathcal{S}^*$ such that $f(\mathcal{U})$ is a starlike region with respect to the origin i.e, $tw \in f(\mathcal{U})$ whenever $w \in f(\mathcal{U})$ and $t \in [0, 1]$ and the class of starlike functions with respect to symmetric points are important special cases of $\mathcal{S}_N^*$.
Further, a function $f \in \mathcal{A}$ belongs to the class $\mathcal{K}_N$ of convex functions with respect to N-symmetric points of

$$\Re\left\{\frac{(zf'(z))'}{f_N'(z)}\right\} > o, \quad z \in \mathcal{U}, N \in \mathbb{N}.$$

For $N=1$ we obtain the usual class of convex functions.

Consider conic region Ω_k, $k \geq 0$ given by

$$\Omega_k = \{u + iv : u > k\sqrt{(u-1)^2 + v^2}\}.$$

This domain represents the right half plane for $k=0$, hyperbola for $0<k<1$, a parabola for $k=1$ and ellipse for $k>1$.

The functions $p_k(z)$ play the role of extremal functions for these conic regions where

$$p_k(z) = \begin{cases} \frac{1+z}{1-z}, & k=0 \\ 1 + \frac{2}{\pi^2}\left(\log \frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2, & k=1. \\ 1 + \frac{2}{1-k^2}\sinh^2\left[\left(\frac{2}{\pi}\arccos\ k\right)\arctan h\sqrt{z}\right], & 0<k<1. \\ 1 + \frac{2}{k^2-1}\sin\left[\frac{\pi}{2R(t)}\int_0^{\frac{u(z)}{\sqrt{t}}} \frac{1}{\sqrt{1-x^2}\sqrt{1-(tx)^2}}dx\right] + \frac{1}{k^2-1}, & k>1, \end{cases} \tag{1.6}$$

where $u(z) = \frac{z-\sqrt{t}}{1-\sqrt{tx}}$, $t \in (0,1)$, $z \in \mathcal{U}$ and z is chosen such that $k = \cosh\left(\frac{\pi R'(t)}{4R(t)}\right)$, $R(t)$ is the Legendre's complete elliptic integral of the first kind and $R'(t)$ is complementary integral $R(t)$. $p_k(z) = 1 + \delta_k z + ...$, [9] where

$$\delta_k = \begin{cases} \frac{8(\arccos\ k)^2}{\pi^2(1-k^2)}, & 0 \leq k < 1 \\ \frac{8}{\pi^2}, & k=1. \\ \frac{\pi^2}{4(k^2-1)\sqrt{t}(1+t)R^2(t)}, & k>1. \end{cases} \tag{1.7}$$

Using the concept of starlike and convex functions with respect to N-symmetric points and conic regions we define the following:

Definition 1.1. A function $f \in \mathcal{A}$ is said to be in the class $k-UB(\alpha,\beta,\gamma,N)$, for $k \geq 0$, $\alpha \geq 0$, $0 \leq \beta, < 1$, $0 \leq \gamma < 1$ if and only if

$$\Re(J(\alpha,\beta,\gamma,N,f(z)) > |J(\alpha,\beta,\gamma,N,f(z)) - 1|,$$

where

$$J(\alpha,\beta,\gamma,N,f(z)) = \frac{1-\alpha}{1-\beta}\left(\frac{zf'(z)}{f_N(z)} - \beta\right) + \frac{\alpha}{1-\gamma}\left(\frac{(zf'(z))'}{f_N'(z)} - \gamma\right),$$

and $f_N(z)$ is defined by (1.3),

Or equivalently

$$J(\alpha,\beta,\gamma,N,f(z)) \prec p_k(z),$$

where $p_k(z)$ is defined by (1.6).

This class generalizes for Khalida Inayat Noor and Sarfraz Nawaz Malik in [6], Kanas and Winsiowska [3,10], Shams and Kulkarni [4], Kanas [7], Mocaun [1], Goodman [5].

2 Main results

Theorem 2.1. *A function $f \in \mathcal{A}$ and of the form* (1.1) *is in the class $k-UB(\alpha,\beta,\gamma,N)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty} \psi_n(k;\alpha,\beta,\gamma,N) < (1-\beta)(1-\gamma), \tag{2.1}$$

where

$$\begin{aligned}\psi_n(k;\alpha,\beta,\gamma,N) &= (1-\beta)(1-\gamma)\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)|a_j a_{n+1-j}| \\ &+ (k+1)|(1-\alpha)(1-\gamma)(1+\lambda_N(n))n - [(1-\gamma)+\alpha(\gamma-\beta)](n+1)\lambda_N(n) + \alpha(1-\beta)(n^2+\lambda_N(n))||a_n| \\ &+ \sum_{j=2}^{n-1}(k+1)|(j(1-\alpha)(1-\gamma) - \lambda_N(j)[(1-\gamma)+\alpha(\gamma-\beta)])(n+1-j)\lambda_N(n+1-j)a_j a_{n+1-j}| \\ &+ \sum_{j=2}^{n-1}(k+1)|\alpha(1-\beta)(n+1-j)^2\lambda_N(j)a_j a_{n+1-j}| + (1-\beta)(1-\gamma)(n+1)\lambda_N(n)|a_n|\end{aligned}$$

,

where $N \geq 2$, $k \geq 0$, $\alpha \geq 0$, $0 \leq \beta, < 1$, $0 \leq \gamma < 1$ *and* $\lambda_N(n)$ *is defined by* (1.4).

Proof. Assuming that (2.1) holds, then it suffices to show that

$$k|J(\alpha,\beta,\gamma,N,f(z))-1|-\Re(J(\alpha,\beta,\gamma,N,f(z))-1)<1.$$

Now consider $|J(\alpha,\beta,\gamma,N,f(z))-1|$, then

$$\left|\frac{1-\alpha}{1-\beta}\left(\frac{zf'(z)}{f_N(z)}-\beta\right)+\frac{\alpha}{1-\gamma}\left(\frac{(zf'(z))'}{f'_N(z)}-\gamma\right)-1\right|$$

$$\left|\frac{(1-\alpha)(1-\gamma)zf'(z)f_N(z)-[(1-\gamma)+\alpha(\gamma-\beta)]f_N(z)f'_N(z)+\alpha(1-\beta)\{(zf'(z))'f_N(z)\}}{(1-\beta)(1-\gamma)f_N(z)f'_N(z)}\right|. \quad (2.2)$$

Now from (1.1) and (1.4) we get

$$zf'(z)f'_N(z)=z\left[\sum_{n=0}^{\infty}na_nz^{n-1}\right]\left[\sum_{n=0}^{\infty}n\lambda_N(n)a_nz^{n-1}\right],\ \ a_0=\lambda_N(0)=0,a_1=\lambda_N(1)=1,$$

$$=\frac{1}{z}\left[\sum_{n=0}^{\infty}na_nz^{n}\right]\left[\sum_{n=0}^{\infty}n\lambda_N(n)a_nz^{n}\right]=\frac{1}{z}\sum_{n=0}^{\infty}\left[\sum_{j=0}^{n}j(n-j)\lambda_N(n-j)a_ja_{n-j}\right]z^n$$

$$=\sum_{n=0}^{\infty}\left[\sum_{j=0}^{n}j(n-j)\lambda_N(n-j)a_ja_{n-j}\right]z^{n-1}=z+\sum_{n=3}^{\infty}\left[\sum_{j=0}^{n}j(n-j)\lambda_N(n-j)a_ja_{n-j}\right]z^{n-1}$$

$$=z+\sum_{n=2}^{\infty}\left[(1+\lambda_N(n))na_n+\sum_{j=2}^{n-1}j(n+1-j)\lambda_N(n+1-j)a_ja_{n+1-j}\right]z^n.$$

Similarly , we get

$$f_N(z)f'_N(z)=z+\sum_{n=2}^{\infty}\left[(n+1)\lambda_N(n)a_n+\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+-j}\right]z^n,$$

and

$$f_N(z)(zf'(z))'=z+\sum_{n=2}^{\infty}\left[(n^2+\lambda_N(n))a_n+\sum_{j=2}^{n-1}(n+1-j)^2\lambda_N(j)a_ja_{n+1-j}\right]z^n,$$

Using the above equalities in (2.2), we get

$$\frac{\sum_{n=2}^{\infty}\left[(1-\alpha)(1-\gamma)(1+\lambda_N(n))n-[(1-\gamma)+\alpha(\gamma-\beta)](n+1)\lambda_N(n)+\alpha(1-\beta)(n^2+\lambda_N(n))\right]a_nz^n}{(1-\beta)(1-\gamma)\left[z+\sum_{n=2}^{\infty}\left[(n+1)\lambda_N(n)a_n+\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+-j}\right]z^n\right]}$$

$$+\frac{\sum_{n=2}^{\infty}\left[\sum_{j=2}^{n-1}\left(j(1-\alpha)(1-\gamma)\lambda_N(n+1-j)\right)\right](n+1-j)a_ja_{n+1-j}z^n}{(1-\beta)(1-\gamma)\left[z+\sum_{n=2}^{\infty}\left[(n+1)\lambda_N(n)a_n+\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+-j}\right]z^n\right]} \quad (2.3)$$

$$-\frac{\sum_{n=2}^{\infty}\left[\sum_{j=2}^{n-1}\left([(1-\gamma)+\alpha(\gamma-\beta)]\lambda_N(j)\lambda_N(n+1-j)\right)\right](n+1-j)a_ja_{n+1-j}z^n}{(1-\beta)(1-\gamma)\left[z+\sum_{n=2}^{\infty}\left[(n+1)\lambda_N(n)a_n+\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+-j}\right]z^n\right]}$$

$$+\frac{\sum_{n=2}^{\infty}\left[\sum_{j=2}^{n-1}\left(\alpha(1-\beta)(n+1-j)\lambda_N(j)\right)\right](n+1-j)a_ja_{n+1-j}z^n}{(1-\beta)(1-\gamma)\left[z+\sum_{n=2}^{\infty}\left[(n+1)\lambda_N(n)a_n+\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+-j}\right]z^n\right]}$$

$$\leq\frac{\sum_{n=2}^{\infty}\left|(1-\alpha)(1-\gamma)(1+\lambda_N(n))n-[(1-\gamma)+\alpha(\gamma-\beta)](n+1)\lambda_N(n)+\alpha(1-\beta)(n^2+\lambda_N(n))\right||a_n|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)\lambda_N(n)|a_n|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+1-j}\right|\right]}$$

$$+\frac{\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}\left(j(1-\alpha)(1-\gamma)\lambda_N(n+1-j)\right)(n+1-j)a_ja_{n+1-j}\right|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)\lambda_N(n)|a_n|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+1-j}\right|\right]}$$

$$-\frac{\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}\left[[(1-\gamma)+\alpha(\gamma-\beta)]\lambda_N(j)\lambda_N(n+1-j)\right](n+1-j)a_ja_{n+1-j}\right|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)\lambda_N(n)|a_n|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+1-j}\right|\right]}$$

$$+\frac{\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}\left(\alpha(1-\beta)(n+1-j)^2\lambda_N(j)\right)a_ja_{n+1-j}\right|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)\lambda_N(n)|a_n|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+1-j}\right|\right]}.$$

Since

$$k|J(\alpha,\beta,\gamma,N,f(z))-1|-\Re\{J(\alpha,\beta,\gamma,N,f(z))-1\}\leq(k+1)|J(\alpha,\beta,\gamma,N,f(z))-1|,$$

then

$$\leq \frac{(k+1)\sum_{n=2}^{\infty}\left|(1-\alpha)(1-\gamma)(1+\lambda_N(n))n-[(1-\gamma)+\alpha(\gamma-\beta)](n+1)\lambda_N(n)+\alpha(1-\beta)(n^2+\lambda_N(n))\right||a_n|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)\lambda_N(n)|a_n|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+1-j}\right|\right]}$$

$$+\frac{(k+1)\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}\left(j(1-\alpha)(1-\gamma)-\lambda_N(j)[(1-\gamma)+\alpha(\gamma-\beta)]\right)(n+1-j)\lambda_N(n+1-j)a_ja_{n+1-j}\right|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)\lambda_N(n)|a_n|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+1-j}\right|\right]}$$

$$+\frac{(k+1)\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}\left(\alpha(1-\beta)(n+1-j)^2\lambda_N(j)\right)a_ja_{n+1-j}\right|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)\lambda_N(n)|a_n|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)a_ja_{n+1-j}\right|\right]}.$$

The last expression is bounded by 1 if

$$\sum_{n=2}^{\infty}(k+1)\left|(1-\alpha)(1-\gamma)(1+\lambda_N(n))n-[(1-\gamma)+\alpha(\gamma-\beta)](n+1)\lambda_N(n)+\alpha(1-\beta)(n^2+\lambda_N(n))\right||a_n|$$

$$+\sum_{n=2}^{\infty}\left\{\sum_{j=2}^{n-1}(k+1)\left|\left(j(1-\alpha)(1-\gamma)-\lambda_N(j)[(1-\gamma)+\alpha(\gamma-\beta)]\right)(n+1-j)\lambda_N(n+1-j)a_ja_{n+1-j}\right|\right\}$$

$$+\sum_{n=2}^{\infty}\left\{\sum_{j=2}^{n-1}(k+1)\left|\left(\alpha(1-\beta)(n+1-j)^2\lambda_N(j)\right)a_ja_{n+1-j}\right|+(1-\beta)(1-\gamma)(n+1)\lambda_N(n)|a_n|\right\}$$

$$+\sum_{n=2}^{\infty}\left\{(1-\beta)(1-\gamma)\sum_{j=2}^{n-1}(n+1-j)\lambda_N(j).\lambda_N(n+1-j)\left|a_ja_{n+1-j}\right|\right\}<(1-\beta)(1-\gamma).$$

This completes the proof. □

When $N=1$, we have the following known result, proved by Khalida Inayat Noor and Sarfraz Nawaz Malik in [6].

Corollary 2.2. *A function $f\in\mathcal{A}$ and form* (1.1) *in the class $k-(\alpha,\beta,\gamma)$, for $-1\leq\beta,\gamma<1$, $\alpha\geq0$, $k\geq0$ if it satisfies the condition*

$$\sum_{n=2}^{\infty}\psi_n(k;\alpha,\beta,\gamma)<(1-\beta)(1-\gamma), \tag{2.4}$$

where

$$\psi_n(k;\alpha,\beta,\gamma)=(k+1)\{(n-1)(1-\alpha)(1-\gamma)+n\alpha(1-\beta)(n-1)\}|a_n|$$

$$+(k+1)\sum_{j=2}^{n-1}\{(j-1)(1-\alpha)(1-\gamma)+\alpha(1-\beta)(n-j)\}(n+1-j)|a_ja_{n+1-j}|$$

$$+(1-\beta)(1-\gamma)(n+1)|a_n|+(1-\beta)(1-\gamma)\sum_{j=2}^{n-1}(n+1-j)|a_ja_{n+1-j}|.$$

For $N=1,\alpha=0$, we have following result due to Shams and Kulkarni [4].

Corollary 2.3. *A function $f\in\mathcal{A}$ and form* (1.1) *in the class $SD(k,\beta)$, if it satisfies the condition*

$$(1-\beta)(1-\gamma)>\sum_{n=2}^{\infty}\left\{(k+1)(n-1)(1-\gamma)|a_n|+(k+1)\sum_{j=2}^{n-1}(j-1)(1-\gamma)(n+1-j)|a_ja_{n+1-j}\right\}$$

$$+\sum_{n=2}^{\infty}\left\{(1-\beta)(1-\gamma)|a_n|+(1-\beta)(1-\gamma)\sum_{j=2}^{n-1}(n+1-j)|a_ja_{n+1-j}\right\}$$

$$> (1-\gamma)\sum_{n=2}^{\infty}\{(k+1)(n-1)+(1-\beta)\}|a_n|.$$

This implies that

$$\sum_{n=2}^{\infty}\{n(k+1)-(k+\beta)\}|a_n| < 1-\beta$$

.

For $N=1,\ \alpha=1$ we arrive at Shams and Kulkarni et result in [4].

Corollary 2.4. *A function $f\in\mathcal{A}$ and form* (1.1) *in the class $KD(k,\gamma)$, if it satisfies the condition*

$$(1-\beta)(1-\gamma) > \sum_{n=2}^{\infty}\left\{n(k+1)(n-1)(1-\beta)|a_n| + (k+1)\sum_{j=2}^{n-1}(n-j)(n+1-j)(1-\beta)|a_j a_{n+1-j}\right\}$$

$$+\sum_{n=2}^{\infty}\left\{n(1-\beta)(1-\gamma)|a_n| + (1-\beta)(1-\gamma)\sum_{j=2}^{n-1}(n+1-j)|a_j a_{n+1-j}\right\}$$

$$> (1-\beta)\sum_{n=2}^{\infty}n\{(k+1)(n-1)+(1-\gamma)\}|a_n|.$$

This implies that

$$\sum_{n=2}^{\infty}n\{n(k+1)-(k+\gamma)\}|a_n| < 1-\gamma$$

.

Also for $N=1,\ \beta=0,\ \ \gamma=0$ then we get the well- known Kanas's result [7].

Corollary 2.5. *A function $f\in\mathcal{A}$ and form* (1.1) *in the class $UM(\alpha,k)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\psi_n(k;\alpha) < 1,$$

where

$$\psi_n(k;\alpha) = (k+1)(n-1)(1-\alpha+n\alpha)|a_n| + (n+1)|a_n| + \sum_{j=2}^{n-1}(n+1-j)|a_j a_{n+1-j}|$$

$$+(k+1)\sum_{j=2}^{n-1}\{(j-1)(1-\alpha)+\alpha(n-j)\}(n+1-j)|a_j a_{n+1-j}|.$$

For $N=1,\ \ \alpha=0,\ \ \beta=0$, then we get result proved by Kanas and Wisniowska in [3]

Corollary 2.6. *A function $f\in\mathcal{A}$ and form* (1.1) *in the class $k-ST$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{n+k(n-1)\}|a_n| < 1.$$

Also for $N=1,\ \ k=0,\ \ \alpha=0$, then we have the following known result, proved by Silverman in [8]

Corollary 2.7. *A function $f\in\mathcal{A}$ and form* (1.1) *in the class $\mathcal{S}^*(\beta)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}(n-\beta)|a_n| < 1-\beta.$$

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