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Power of maximal ideal

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Abstract. In this paper, we study a rings in which every maximal ideal is finitely generated provided some of its power is finitely generated. This notion is raised by Gilmer in 1971 and Roitman shows that coherent domains satisfy this property in 2001. We establish the transfer of this notion to direct products, trivial ring extensions, pullbacks, and the amalgamation of rings. Our results generate new families of examples of non-coherent rings (with zerodivisors) satisfy this condition.

1 Introduction

All rings in this paper are commutative with unity. First, we consider the following question:

Question 1: Suppose that some power M^n of the maximal ideal M of a ring R is finitely generated. Does it follow that M is finitely generated ?

This question is raised by Rober Gilmer in [11, page 74] and was mentioned in a talk given by Robert Gilmer at the AMS meeting in Auburn, Alabama in November 1971 in an integral domain. It is also listed, for the case of a quasilocal integrally closed domain, as Problem 8 in the questions list on pages 174-176 in the 1973 Notices of the AMS from the problem session organized by Graham Evans at the January 1973 AMS meeting in Dallas.

In 1999, R. Gilmer, W. Heinzer and M. Roitman gives a positive answer to Question 1 under each of the following conditions (see [14, Theorem 1.24]):
(1) M is a minimal prime over a principal ideal, in particular, $htM \leq 1$.
(2) R is integrally closed, and either M is a minimal prime over a 2-generated ideal, or $htM \leq 2$.

On the other hand, by [14, Examples 3.1 and 3.2], Gilmer, Heinzer and Roitman shows that the answer of Question 1 is negative, in general.

A ring R is coherent if every finitely generated ideal of R is finitely presented; equivalently, if $(0 : a)$ and $I \cap J$ are finitely generated for every $a \in R$ and any two finitely generated ideals I and J of R. Examples of coherent rings are Noetherian rings, Boolean algebras, von Neumann regular rings, and Prüfer/semihereditary rings. For instance see [9, 18].

Recall that Roitman shows that Question 1 hold in every coherent domain (see [24, Theorem 1.8]. At this point, we make the followwing definition:

Definition 1.1. A commutative ring R is called a Gilmer-ring (G-ring for short) if R verify a question 1.

Let A be a ring and E an $A-$module. The trivial ring extension of A by E (also called idealization of E over A) is the ring $R := A \propto E$ whose underlying group is $A \times E$ with multiplication given by $(a, e)(a', e') = (aa', ae' + ea')$.

Trivial ring extensions have been studied extensively. Considerable work, part of is summarized in Glaz's book [9] and Huckaba's book [17], has been concerned with trivial ring extension. These extensions have been useful for solving many open problems and conjectures in both commutative and non-commutative ring theory. See for instance [2, 9, 17, 18].

Let T be a domain and let K be a field which is a retract of T, that is $T := K + M$ where M is a maximal ideal of T. Each subring D of K determines a subring $R := D + M$ of T. This construction arises frequently in algebra, especially in connection with counterexamples. The original of $D + M$ construction involved a valuation domain T with $K := T/M$, where M is the maximal ideal of T and $K \subset T$. A throughout account of results about $D + M$ construction can be find in [3, 4, 9].

Let A and B be two rings with unity, J be an ideal of B and let $f : A \to B$ be a ring homomorphism. In this setting, we can consider the following subring of $A \times B$:

$$A \bowtie^f J := \{(a, f(a) + j) \mid a \in A, j \in J\}$$

called *the amalgamation of A and B along J with respect to f*. See for instance [5, 6].

In this paper, we investigate the transfer of this notion to direct products, trivial ring extensions, pullbacks, and the amalgamation of rings. Our results generate new families of examples of non-coherent rings (with zerodivisors) satisfy this condition.

2 Main results

First, we study the transfer of G-property to direct product of rings.

Theorem 2.1. *Let $(R_i)_{i=1,...,n}$ be a family of commutative rings. Then $R = \prod_{i=1}^{i=n} R_i$ is a G-ring if and only if so is R_i for each $i = 1, ..., n$.*

We need the following Lemma before proving Theorem 2.1.

Lemma 2.2. *[20, Lemma 2.5]*
Let $(R_i)_{i=1,2}$ be a family of rings and E_i an R_i-module for $i = 1, 2$. Then $E_1 \prod E_2$ is a finitely generated $R_1 \prod R_2$-module if and only if E_i is a finitely generated R_i-module for $i = 1, 2$.

Proof of Theorem 2.1.
By induction on n, it suffices to prove the assertion for $n = 2$. Assume that $R_1 \prod R_2$ is a G-ring and let M_1 be a maximal ideal of R_1 such that M_1^n is finitely generated ideal of R_1 for some positive integer n. Then, $M := M_1 \times R_2$ is a maximal ideal of $R_1 \prod R_2$ and $M^n := M_1^n \times R_2^n$ is a finitely generated ideal of $R_1 \prod R_2$ by Lemma 2.2. Hence, $M := M_1 \times R_2$ is a finitely generated ideal of $R_1 \prod R_2$ since $R_1 \prod R_2$ is a G-ring and so M_1 is a finitely generated ideal of R_1 by Lemma 2.2. Therefore, R_1 is a G-ring.
The same argument shows that R_2 is a G-ring.
Conversely, assume that R_1 and R_2 are G-rings and let M be a maximal ideal of $R_1 \prod R_2$ such that M^n is finitely generated ideal of $R_1 \prod R_2$ for some positive integer n. Since $M := R_1 \prod M_2$ or $M := M_1 \prod R_2$, where M_i is a maximal ideal of R_i for $i = 1, 2$, the conclusion follows easily as the above argument and from Lemma 2.2. □

Now, we study the transfer of the G-property to trivial ring extension.

Theorem 2.3. *Let A be a ring, E an $A-$module and $R := A \propto E$ be the trivial ring extension of A by E. Then, the following statements hold:*

(i) If A is a G-ring and E is a finitely generated A-module, then R is a G-ring.

(ii) Assume that E is a Noetherian A-module. Then, R is a G-ring if and only if so is A.

(iii) Assume that A is an integral domain which is not a field, $K = qf(A)$, and E is a $K-$vector space. Then, R is a G-ring if and only if so is A.

(iv) Assume that (A, M) is a local ring, E is a non-zero $A-$module with $ME = 0$. Then:
a) *Assume that M is a finitely generated ideal of A. Then:*
i) *A is a G-ring.*
ii) *R is a G-ring if and only if E is a finitely generated A-module.*
b) *Assume that M is a non finitely generated ideal of A. Then, R is a G-ring if and only if so is A.*

We need the following lemma before proving this Theorem.

Lemma 2.4. *Let A be a ring, E an $A-$module, $R := A \propto E$ be the trivial ring extension of A by E, I be an ideal of A and F be a submodule of E such that $IE \subseteq F$. Then:*

(i) $(I \propto F)^n = I^n \propto (I^{n-1}F)$ *for every positive integer n.*

(ii) If I and F are finitely generated, then $I \propto F$ is a finitely generated ideal of R.

(iii) Assume that A is an integral domain which is not a field, $K = qf(A)$, E is a $K-$vector space, and let I be a nonzero ideal of A. Then $I \propto E$ is a finitely generated ideal of R if and only if I is a finitely generated ideal of A.

Proof. Straightforward. □

Proof of Theorem 2.3.
1) Assume that A is a G-ring and E is a finitely generated A-module and let M be a maximal ideal of R such that M^n is finitely generated for a positive integer n. The ideal M has the form $M := m \propto E$, where m is a maximal ideal of A. Hence, $M^n = (m \propto E)^n = m^n \propto m^{n-1}E$ (by Lemma 2.4(1)) is a finitely generated ideal of R and so m^n is a finitely generated ideal of A. Therefore, m is a finitely generated ideal of A since A is a G-ring and so $M := m \propto E$ is a finitely generated ideal of R by Lemma 2.4(1) since E is a finitely generated A-module, as desired.

2) Assume that R is a Noetherian A-module. If A is a G-ring, then so is R by (1). Conversely, assume that R is a G-ring and let m be a maximal ideal of A such that m^n is finitely generated for some positive integer n. But $m^{n-1}E$ is a finitely generated A-module since $m^{n-1}E \subseteq E$ and E is a Noetherian A-module. Hence, $(m \propto E)^n (= m^n \propto m^{n-1}E)$ (by Lemma 2.4(1)) is a finitely generated ideal of R by Lemma 2.4(2) and so $m \propto E$ is a finitely generated ideal of R since R is a G-ring. Therefore, m is a finitely generated ideal of A and so A is a G-ring, as desired.

3) Assume that A is an integral domain which is not a field, $K = qf(A)$, and E is a $K-$vector space. Assume that R is a G-ring and let m be a maximal ideal of A such that m^n is a finitely generated ideal of A for some positive integer n. Hence, $(m \propto E)^n = m^n \propto E$ is a finitely generated ideal of R by Lemma 2.4(3) and so $m \propto E$ is a finitely generated ideal of R since R is a G-ring. Therefore, m is a finitely generated ideal of A and A is a G-ring, as desired.
Conversely, assume that A is a G-ring and let $M := m \propto E$ be a maximal ideal of R such that $(m \propto E)^n$ is a finitely generated ideal of R for some positive integer n, where m is a maximal ideal of A. Since $(m \propto E)^n = m^n \propto E$ is a finitely generated ideal of R, then m^n is a finitely generated ideal of A and so m is finitely generated since A is a G-ring. Therefore, $M := m \propto E$ is a finitely generated ideal of R by Lemma 2.4(3), as desired.

4) Assume that (A, M) is a local ring, E is a non-zero A module with $ME = 0$.
(a) Assume that M is a finitely generated ideal of A.
(i) Straightforward.
(ii) Assume that R is a G-ring. But $(M \propto E)^2 := M^2 \propto 0$ is a finitely generated ideal of R since M is a finitely generated ideal of A. Hence, $M \propto E$ is a finitely generated ideal of R since R is a G-ring and so E is a finitely generated A-module, as desired.
Conversely, assume that E is a finitely generated A-module. Then, $M \propto E$ is finitely generated by Lemma 2.4(2) and so R is a G-ring, as desired.
(b) Assume that M is a non finitely generated ideal of A.
Assume that R is a G-ring and assume that M^n is finitely generated for some positive integer n. Then $(M \propto E)^n := M^n \propto 0$ is finitely generated and so $M \propto E$ is finitely generated since R is a G-ring. Hence, M is finitely generated, a contradiction. Therefore, M^n is a non finitely generated ideal of A foe every positive integer n and so A is a G-ring.
Conversely, assume that A is a G-ring and $(M \propto E)^n$ is finitely generated for some positive integer n. Hence, $M^n \propto 0 := (M \propto E)^n$ is finitely generated and so M^n is finitely generated. This means that M is finitely generated since A is a G-ring, a contradiction. Therefore, $(M \propto E)^n$ is a non finitely generated ideal of R foe every positive integer n and so R is a G-ring. This completes the proof of Theorem 2.3. □

Theorem 2.3 enrichies the literature with original examples of non-coherent G-rings.

Example 2.5. Let A be a coherent domain which is not a field, $K := qf(A)$, and let $R := A \propto K$ be the trivial ring extension of A by K. Then:

(i) R is a G-ring by Theorem 2.3(3).

(ii) R is not coherent by [18, Theorem 2.8(1)].

Example 2.6. Let (A, M) be a local coherent domain such that M is a non-finitely generated ideal of A, and let $R := A \propto E$, where E is an A-module such that $ME = 0$. Then:

(i) R is an G-ring by Theorem 2.3(4).

(ii) R is not coherent by [18, Theorem 2.6] since M is a non-finitely generated ideal of A.

The following Theorem develops a result on the transfer of the G-property to pullbacks, specially $D + M$-constructions.

Theorem 2.7. *Let $T := K + M$ be a local domain, where K is a field and M is the unique maximal ideal of T; and $R := D + M$, where D is a subring of K. Then:*

(i) Assume that D is not a field. Then, R is a G-ring if and only if so is D.

(ii) Assume that D is a field with $[K : D] = \infty$. Then, R is a G-ring.

(iii) Assume that D is a field with $[K : D] < \infty$. Then, R is a G-ring if and only if so is T.

We need the following lemmas before proving Theorem 2.7.

Lemma 2.8. *Let T and R be as in Theorem 2.7. Then, every maximal ideal of R contain M.*

Proof. Let P be a maximal ideal of R. Two cases are then possible:
Case 1: $P \subseteq M$. In this cases, $P = M$ since P is a maximal ideal of R and so D is a field.
Case 2: $P \nsubseteq M$. Let $d + m \in P - M$, where $d(\neq 0) \in D$ and $m \in M$. Hence, $(d + m)M = (d + m)TM = TM = M$ since $d + m$ is invertible in T (since (T, M) is local and $d + m \notin M$). Therefore, $M = (d + m)M \subseteq (d + m)R \subseteq P$ since $d + m \in P$, as desired. □

Lemma 2.9. *Let T, D, K, M, and R be as in Theorem 2.7. Assume that D is not a field or D is a field and $[K : D] = \infty$. Then, M^n is never a finitely generated ideal of R for every positive integer n.*

Proof. Assume that M^n is a finitely generated ideal of R for some positive integer n. Then, $M^n/M^{n+1} := M^n \otimes_R (R/M)$ is a finitely generated D-module. On the other hand, $M^n/M^{n+1} := M^n \otimes_T (T/M) = (T/M)^{(I)}$ is a K-vector space. Also, $M^n/M^{n+1} \neq 0$ by Nakayama Lemma since $M \subseteq J(R)$ (by Lemma 2.8). Therefore, $(T/M)^{(I)}$ and so $K := T/M$ is a finitely generated D-module and so D is a field and $[K : D] < \infty$, a contradiction. Hence, M^n is never a finitely generated ideal of R for every positive integer n, as desired. □

Proof of Theorem 2.7.
1) Assume that D is not a field. Then, any maximal ideal P of R has the form $P := P_0 + M$ $(=P_0R)$ by Lemma 2.8, where P_0 is a nonzero maximal ideal of D (since D is not a field).
Assume that R is a G-ring and let P_0 be a nonzero maximal ideal of D such that P_0^n is finitely generated for some positive integer n. Then, $P := P_0 + M$ $(=P_0R)$ is a maximal ideal of R and $P^n := (P_0 + M)^n = P_0^n + M = P_0^nR)$ is a finitely generated ideal of R. Hence, $P := P_0 + M$ is a finitely generated ideal of R since R is a G-ring and so P_0 is a finitely generated ideal of D. Therefore, D is a G-ring.
Conversely, assume that D is a G-ring and let $P := P_0 + M$ $(=P_0R)$ be a maximal ideal of R such that $P^n := (P_0 + M)^n = P_0^n + M$ is a finitely generated ideal of R. Hence, P_0^n is a finitely generated ideal of D and so P_0 is finitely generated since D is a G-ring. Therefore, $P := P_0R$ is a finitely generated ideal of R, as desired.

2) Assume that D is a field and $[K : D] = \infty$. Then, M is the only maximal ideal of R by Lemma 2.8. On the other hand, M^n is never a finitely generated ideal of R by Lemma 2.9. Therefore, R is a G-ring.

3) Assume that D is a field with $[K : D] < \infty$. Then, M is the only maximal ideal of R by Lemma 2.8.
Assume that R is a G-ring and assume that M^n is a finitely generated ideal of T for some positive

integer n. Then, M^n is a finitely generated ideal of R (since T is a finitely generated R-module) and so M is a finitely generated ideal of R since R is a G-ring. Therefore, M is a finitely generated ideal of T, as desired.
Conversely, assume that T is a G-ring. Then, R is a G-ring as argue above and this completes the proof of Theorem 2.7. □

Theorem 2.7 enrichies the literature with original examples of non-coherent G-rings.

Example 2.10. Let $R := \mathbb{Z} + X\mathbb{R}[[X]]$ and $T := \mathbb{R}[[X]]$. Then:

(i) R is a G-ring by Theorem 2.7(1).

(ii) R is not coherent by [9, Theorem 5.2.3].

The last Theorem develops a result on the transfer of the G-property to amalgamation of rings $A \bowtie^f J$.

Theorem 2.11. *Let A and B be a pair of rings, $f : A \to B$ be a ring homomorphism and J be a proper ideal of B. Then, the following statements hold:*

(i) Assume that $J \subseteq Rad(B)$ and J is a finitely generated ideal of $f(A) + J$. If A is a G-ring, then so is $A \bowtie^f J$.

(ii) Assume that (A, M) is a local ring, $J^2 = 0$, and $f(M) \subseteq J$. If $A \bowtie^f J$ is a G-ring, then so is A.

(iii) Assume that (A, M) is a local ring, $J^2 = 0$, $f(M) \subseteq J$, and J is a finitely generated ideal of $f(A) + J$. Then, $A \bowtie^f J$ is a G-ring if and only if so is A.

We need the following lemmas before proving this Theorem 2.11.

Lemma 2.12. *Let (A, B) be a pair of rings, $f : A \to B$ be a ring homomorphism and J be a proper ideal of B such that $J \subseteq Rad(B)$. Then, $Max(A \bowtie^f J) = \{m \bowtie^f J/m \in Max(A)\}$.*

Proof. Assume that $J \subseteq Rad(B)$. Then J is contained in Q for all $Q \in Max(B)$. Consequently, the set $\{\overline{Q}^f\}$ in [6, Proposition 2.6 (5)] is empty and so $Max(A \bowtie^f J) = \{m \bowtie^f J/m \in Max(A)\}$, as desired. □

Lemma 2.13. *Let (A, B) be a pair of rings, $f : A \to B$ be a ring homomorphism and J be a proper ideal of B. Assume that J is a finitely generated ideal of $f(A) + J$ and let I be a finitely generated ideal of A. Then, $I \bowtie^f J$ is a finitely generated ideal of $A \bowtie^f J$.*

Proof. Assume that $I := \sum_{i=1}^{i=n} Ax_i$ is a finitely generated ideal of A, where $x_i \in I$ for all $i \in \{1, ...n\}$ and $J := \sum_{i=1}^{i=m}(f(A) + J)e_i$ is a finitely generated ideal of $f(A) + J$, where $e_i \in J$ for all $i \in \{1, ...m\}$. It is clear that $I \bowtie^f J = \sum_{i=1}^{i=n}(A \bowtie^f J)(x_i, f(x_i)) + \sum_{i=1}^{i=m}(A \bowtie^f J)(0, e_i)$, as desired. □

Proof of Theorem 2.11.
1) Assume that $J \subseteq Rad(B)$, J is a finitely generated ideal of $f(A) + J$ and let $M := m \bowtie^f J$ be a maximal ideal of $A \bowtie^f J$ (by Lemma 2.12) such that $M^n(:= (m \bowtie^f J)^n)$ is a finitely generated ideal of $A \bowtie^f J$ for some positif integer n. Hence, m^n is a finitely generated ideal of A and so m is finitely generated since A is a G-ring. Therefore, $M := m \bowtie^f J$ is a finitely generated ideal of $A \bowtie^f J$ by Lemma 2.13, as desired.

2) Assume that (A, M) is a local ring, $J^2 = 0$, $f(M) \subseteq J$, $A \bowtie^f J$ is a G-ring, and let m be a maximal ideal of A such that m^n is a finitely generated ideal of A for some positif integer n. Then, $M := m \bowtie^f J$ is a maximal ideal of $A \bowtie^f J$ and $M^n(:= (m^n \bowtie^f O))$ (since $f(M) \subseteq J$ and $J^2 = 0$) which is a finitely generated ideal of $A \bowtie^f J$. Therefore, $M := m \bowtie^f J$ is a finitely generated ideal of $A \bowtie^f J$ and so m is a finitely generated ideal of A, as desired.

3) By 1) and 2) and this completes the proof of Theorem 2.11. □

Theorem 2.11 enrichies the literature with original examples of non-coherent G-rings.

Example 2.14. Let $f : A \to B$ be a ring homomorphism, where A is a non-coherent G-ring, and let J be a finitely generated proper ideal of B. Then:

(i) $A \bowtie^f J$ is a G-ring by Theorem 2.11.

(ii) $A \bowtie^f J$ is a non-coherent ring by [9, Theorem 4.1.5] since A is a module retract of $A \bowtie^f J$ and A is non-coherent.

Finally, we show that the hypothesis "J is a finitely generated ideal of $f(A) + J$" cannot be removed in Theorem 2.11(1).

Example 2.15. Let $A := K$ be a field, $B := K \propto E$ be the trivial ring extension of K by E (where E is a K-vector space with infinite rank), $J := 0 \propto E$, and $f : A \to B$ such that $f(a) = (a, 0)$. Then:

(i) $A \bowtie^f J$ is a non-G-ring since $(0 \bowtie^f J)^2 = 0$, $0 \bowtie^f J$ is a non finitely generated ideal of $A \bowtie^f J$, and $0 \bowtie^f J$ is a maximal ideal of $A \bowtie^f J$.

(ii) A is a G-ring since it is a field.

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