Palestine Journal of Mathematics

Vol. 5(1) (2016) , 6–11

Affirmative domination in graphs

B. Chaluvaraju and V. Chaitra

Communicated by I. Gotchev

MSC 2010 Classifications: 05C69, 05C70.

Keywords: Graph, affirmative domination, strongly affirmative domination.

Abstract: A function $f : V \to \{-1, 0, 1\}$ is an affirmative dominating function of graph G satisfying the conditions that for every vertex u such that $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 1$ and $\sum_{u\in N(v)} f(u) \leq 1$ for every $v \in V$. The affirmative domination number $\gamma_a(G)$ =max$\{w(f) : f$ is affirmative dominating function$\}$. In this paper, we initiate the study of affirmative and strongly affirmative dominating functions. Here, we obtain some properties of these new parameters and also determine exact values of some special classes of graph.

1 Introduction

Let $G = (V, E)$ be a simple graph with vertex set $V(G) = V$ of order $|V| = n$, edge set $E(G) = E$ of size $|E| = m$ and let v be a vertex of V. The open neighborhood of v is $N(v) = \{u \in V : uv \in E(G)\}$ and closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. For more details about the basic definition and terminologies which does not appear here, we refer to Harary [3].

Definition 1.1. A function $f : V \to \{0, 1\}$ which assigns to each vertex of a graph an element of the set $\{0, 1\}$, then f is a dominating function if $\sum_{u\in N[v]} f(u) \geq 1$ for every $v \in V$. The domination number denoted by $\gamma(G)$ is the minimum weight of the dominating function on G.

Definition 1.2. A dominating function f is said to be maximal if there exists no dominating function g such that $f \neq g$ and $g(v) \geq f(v)$ for every $v \in V$. The literature on this subject has been surveyed and detailed in the two books by Haynes et al. [5] and [6].

Allowing a negative weight of -1 motivated the definition of minus domination function in [9] as a function $f : V \to \{-1, 0, 1\}$ satisfying $\sum_{u\in N[v]} f(u) \geq 1$ for every $v \in V$. The minus domination number denoted by $\gamma^-(G)$ of graph G is the minimum weight of minus dominating function on G. There are several graphs with minus domination number which is positive, negative or zero. Many bounds for $\gamma^-(G)$ were studied in [10] and [11].

Definition 1.3. A function $f : V \to \{-1, 0, 1\}$ is a minus total dominating function of graph G, if $\sum_{u\in N(v)} f(u) \geq 1$ for every $v \in V$. The minus total domination number denoted by $\gamma_t^-(G)$ of graph G is the minimum weight of minus total dominating function on G. Harris et al. [4] introduced the concept of minus total dominating function and has been extensively studied in [2], [7] and [8].

In this paper, we initiate the study of a new graph parameter by changing " $\geq$ " to " $\leq$ " in the definition of minus total domination number with a restriction of a vertex assigned 0 being adjacent to at least one vertex assigned 1.

Definition 1.4. A function $f : V \to \{-1, 0, 1\}$ is an affirmative dominating function (ADF) of G satisfying the conditions that for every vertex u such that $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 1$ and $\sum_{u\in N(v)} f(u) = f(N(v)) \leq 1$ for every $v \in V$. The affirmative domination number $\gamma_a(G)$ =max$\{w(f) : f$ is affirmative dominating function$\}$.

In ADF, the function f instead of $v \in N(v)$, if we use $v \in N[v]$, then f is called strongly affirmative dominating function (SADF). The maximum of the values of $f(V)$, taken over all SADF of f is called a strongly affirmative domination number $\gamma_{sa}(G)$ of a graph G.

The motivation for studying these parameters may be explained through modelling perspective. Let us consider this illustration: by assigning the values -1, 0 or 1 to the vertices of a graph we can model networks of people or organizations in which global decision must be made in terms of negative, neutral or positive responses or preferences. We assume that each individual has one vote, and each has an initial opinion. We assign the values 1, 0, -1 to vertices of positive, neutral and negative opinion respectively. We assume that an individual's vote is affected by the opinion of neighboring individual and that a voter with neutral opinion is made adjacent to at least one voter with positive opinion. A voter votes **NO** if there are more vertices in its neighborhood with negative opinion than those with positive opinion and votes **YES** other wise. We look for an assignment of opinions that guarantee a unanimous decision that is in which every vertex votes **NO**. We call such an assignment an uniform negative assignment. The affirmative domination number is the maximum possible sum of all opinions with -1 for negative opinion, 0 for neutral opinion and 1 for positive opinion in a uniformly negative assignment. The affirmative domination number corresponds to the minimum number of individuals who can have negative opinions and in doing so force every individual to vote **NO**.

Throughout this paper, if f is either ADF or SADF, then let V_1, V_0 and V_{-1} denote sets of vertices of G assigned the values 1, 0, -1 respectively.

2 Affirmative domination

Theorem 2.1. *Let f be an ADF of a graph G. Then*

(i) Every vertex of set V_0 is dominated by a vertex of V_1.

(ii) $V_1 \cup V_{-1}$ is a dominating set of G.

(iii) $\gamma_a(G) = |V_1| - |V_{-1}|$.

(iv) $|N(v) \cap V_1| \leq 1$ for some vertex $v \in V_1$; provided $\gamma(G) = \gamma_a(G)$.

(v) $|N(v) \cap V_1| \leq |N(v) \cap V_{-1}| + 1$ for any $v \in V$.

(vi) $\gamma(G) \leq \gamma_a(G)$; provided $\triangle(G) = n - 1$.

Proof. Let $f : V \to \{-1, 0, 1\}$ be ADF.
(i) By the definition of ADF, a vertex assigned 0 is adjacent to at least one vertex assigned 1. Hence (i) follows.
(ii) As vertices of V_1 dominates vertices of V_0 and may or may not dominate vertices of V_{-1}, the set $V_1 \cup V_{-1}$ is a dominating set of G.
(iii) As $V_0 \subseteq N(V_1)$, we have $\gamma_a(G) = |V_1| - |V_{-1}|$.
(iv) If $\gamma(G) = \gamma_a(G)$, then $|V_{-1}| = 0$. For any vertex $v \in V_1$, if $|N(v) \cap V_1| > 1$, then $f(N(v)) \geq 2$ which contradicts f being ADF.
(v) Assume to the contrary, that $|N(v) \cap V_1| > |N(v) \cap V_{-1}| + 1$. As $f(N(v)) = |N(v) \cap V_1| - |N(v) \cap V_{-1}|$, implies $f(N(v)) > 1$ which is a contradiction to f being ADF.
(vi) Let $\triangle(G) = n - 1$. Then $\gamma(G) = 1$. If $v \in V_1$ and all vertices adjacent to v belongs to the set V_0 such that $f(N(u)) \leq 1$ for all $u \in V$, then $\gamma_a(G) \geq f(V)$. □

Theorem 2.2. *An ADF is maximal if and only if for every vertex $v \in V$ with $f(v) = -1$, there exist a vertex $u \in N(v)$ such that $f(N(u)) = 0$ or 1.*

Proof. Suppose f is maximal ADF and assume that there is a vertex $v \in V$ with $f(v) = -1$ such that $f(N(u)) \leq -1$ for every $u \in N(v)$. Define a function $g : V \to \{-1, 0, 1\}$ such that $g(v) = f(v) + 2$ for $u = v$ and $g(u) = f(u)$ for $u \neq v$. Then for all $u \in N(v)$, $g(N(u)) = f(N(u)) + 2$, implies $g(N(u)) \leq 1$ and for $w \notin N(v)$, $g(N(v)) = f(N(v)) \leq 1$. Hence g is ADF. Since $g > f$, the maximality of f has been contradicted.

Conversely, let f be ADF such that for every $v \in V$ with $f(v) = -1$, there exists a vertex $u \in N(v)$ such that $f(N(u)) = 0$ or 1. Suppose f is not maximal ADF. Then there is an ADF, the function g with $g > f$. For every $v \in V$, $g(v) \geq f(v)$. As $f(v) \leq g(v)$, we have $f(N(v)) \leq g(N(v)) - 2$. This implies $f(N(v)) \leq -1$, which is a contradiction. Hence f is maximal ADF. □

Theorem 2.3. *For any graph G, $\gamma_a(G) = n$ if and only if $G \cong \overline{K_n}$ or K_2 or tK_2 or $\overline{K_n} \cup K_2$.*

Proof. If G is totally disconnected graph with n vertices, then vertices of G cannot be assigned 0 as such vertex should be adjacent to at least one vertex assigned 1. Thus vertices of G should be either assigned 1 or -1. But $\gamma_a(G)$ is maximum of such assignment implies every vertex of G should be assigned 1. Hence $\gamma_a(G) = n$. If $G = K_2$, then assigning 1 to both vertices of G results in maximum weight of G. Conversely, if $\gamma_a(G) = n$, then $|V_1| - |V_{-1}| = n$. This implies $|V_1| = n + |V_{-1}|$ which is not possible. Hence $|V_{-1}| = 0$, which implies every vertex of G is assigned 1. We shall consider different cases:

Case 1. If $n = 2$, then either $G = \overline{K_2}$ or K_2 for which the converse holds.
Case 2. For $n = 1$, the converse holds.
Case 3. Let $n \geq 3$. If G is connected graph, then it leads to contradiction of f being ADF.
From all the above cases the result follows. □

Theorem 2.4. *If G is a r-regular graph of order n, then*

$$\gamma_a(G) \leq \frac{n}{r}.$$

Proof. Let f be an ADF with an affirmative domination number $\gamma_a(G)$. Consider the sum $s = \sum_{v\in V}\sum_{u\in N(v)} f(u)$. This sum counts the value $f(u)$ exactly $deg(u)$ times for each $u \in V$. Thus, $s = \sum_{u\in V} deg(u)f(u)$. As $f(N(v)) \leq 1$, the sum $\sum_{v\in V} f(N(v)) \leq n$. This implies $s \leq n$. For r-regular graph, degree of every vertex is r implies $r\sum_{v\in V} f(v) \leq n$. Hence the required result follows. □

Theorem 2.5. *For any complete graph K_n,*

$$\gamma_a(K_n) = \begin{cases} 2 & if\ n = 2, \\ 1 & if\ otherwise. \end{cases}$$

Proof. A complete graph K_n is $(n-1)$-regular graph. BY Theorem 2.4, we have $\gamma_a(K_n) \leq \frac{n}{n-1}$, that is $\gamma_a(K_n) \leq 2$. As $\gamma(K_n) = 1$ and by (iv) of Theorem 2.1, we have $\gamma_a(K_n) \geq 1$. This implies $1 \leq \gamma_a(K_n) \leq 2$. By Theorem 2.3, we have the affirmative domination number $\gamma_a(K_n) = 2$ if and only if graph is K_2 and for all other complete graphs $\gamma_a(K_n) = 1$. Thus the result follows. □

Theorem 2.6. *For any path P_n with $n \geq 2$ vertices,*

$$\gamma_a(P_n) = \begin{cases} \frac{n}{2} & if\ n \equiv 0\ (mod\ 4), \\ \frac{n+1}{2} & if\ n \equiv \pm 1\ (mod\ 4), \\ \frac{n+2}{2} & if\ n \equiv -2\ (mod\ 4). \end{cases}$$

Proof. Let f be ADF. In a path P_n with $n \geq 2$ vertices, there are two end vertices and $(n-2)$-vertices of degree 2. An end vertex and its support vertex both can belong to set V_1. But if v is not an end vertex and $v \in V_1$, then it can be adjacent to at most one vertex belonging to V_1 and the other vertex adjacent to v should belong to V_0 such that the weight is maximized. For a vertex $w \in V_0$, w can be adjacent to at most one vertex belonging to V_1. Every four vertices are assigned 1, 1, 0, 0 such that $f(N(v)) \leq 1$ for every $v \in V$. Then the following three cases are arises:
Case 1. If $n \equiv 0$ (mod 4), then n is a multiple of 4. Let $n = 4k$, where k is a positive integer. Here, vertices of P_n are assigned 1, 1, 0, 0. Thus, $\gamma_a(P_n) = 2k = \frac{n}{2}$.
Case 2. We shall consider following two subcases.
Subcase 2.1. If $n \equiv -1$ (mod 4), then $n+1$ is a multiple of 4. Let $n = 4k-1 = 4(k-1)+3$. Here, vertices of P_n are assigned 1, 1, 0, 0.... $(k-1)$ times and remaining three vertices are assigned 1, 1 and 0 such that $f(N(v)) \leq 1$ for all $v \in V$. The affirmative domination number $\gamma_a(P_n) = 2(k-1) + 2 = \frac{n+1}{2}$.
Subcase 2.2. If $n \equiv 1$ (mod 4), then $n-1$ is a multiple of 4. Let $n = 4k+1$. Here, vertices of P_n are assigned 1, 1, 0, 0.... k times and remaining one vertex is assigned 1 such that $f(N(v)) \leq 1$ for all $v \in V$. The affirmative domination number $\gamma_a(P_n) = 2k + 1 = \frac{n+1}{2}$.
Case 3. If $n \equiv -2$ (mod 4), then $n+2$ is a multiple of 4. Let $n = 4k-2 = 4(k-1)+2$. Here, vertices of P_n are assigned 1, 1, 0, 0 .... $(k-1)$ times and remaining two vertices are assigned 1. Thus, $\gamma_a(P_n) = 2(k-1) + 2 = \frac{n+2}{2}$. □

Theorem 2.7. *For any cycle C_n with $n \geq 3$ and integer $-3 \leq l \leq 0$,*

$$\gamma_a(C_n) = \frac{n+l}{2},$$

where $n \equiv l\ (mod\ 4)$.

Proof. As a cycle is 2-regular and by Theorem 2.4, we have $\gamma_a(C_n) \leq \frac{n}{2}$. If a vertex $v \in V_0$, then both vertices adjacent to v cannot belong to V_1 as $f(N[v]) \geq 1$. Thus, one vertex belongs to V_1 and other vertex belongs to V_0. For a vertex $w \in V_1$, both vertices adjacent to w cannot

belong to V_1. Thus, one vertex belongs to V_1 and other vertex belongs to V_0. Since every four consecutive vertices of G are assigned 0, 1, 1, 0. Hence the following four cases are arises:
Case 1. If $n \equiv 0 \pmod 4$, then n is a multiple of 4. Let $n = 4k$, where k is any positive integer. Here, vertices of C_n are assigned (0, 1, 1, 0..... k)- times. Thus, $\gamma_a(C_n) = 2k = \frac{n}{2}$.
Case 2. If $n \equiv -1 \pmod 4$, then $n+1$ is a multiple of 4. That is $n = 4k-1 = 4(k-1)+3$. Here, 0, 1, 1, 0 is assigned $(k-1)$- times and remaining 3 vertices are in between two vertices assigned 0. Among remaining vertices, which are adjacent to vertex assigned 0 can neither be assigned 1 as $f(N(v)) \geq 1$ nor be assigned -1 as weight of C_n will not be maximized. Thus, these 3 vertices are assigned 0, 1, 0. Thus, $\gamma_a(C_n) = 2(k-1)+1 = 2k-1 = \frac{n-1}{2}$.
Case 3. If $n \equiv -2 \pmod 4$, then $n+2$ is a multiple of 4. That is $n = 4k-2 = 4(k-1)+2$. Here, 0, 1, 1, 0 is assigned $(k-1)$ times and remaining two vertices are assigned 0 such that $f(N(v)) \leq 1$ for every $v \in V$. Thus, $\gamma_a(C_n) = 2(k-1) = \frac{n-2}{4}$.
Case 4. If $n \equiv -3 \pmod 4$, then $n+3$ is a multiple of 4. That is $n = 4k-3 = 4(k-1)+1$. Here, 0, 1, 1, 0 is assigned to vertices $(k-1)$ times. And remaining one vertex is adjacent to two vertices assigned 0. Hence this vertex cannot be assigned 1 or 0. This implies remaining vertex belongs to -1. Thus, $\gamma_a(C_n) = 2(k-1)-1 = 2k-3 = \frac{n-3}{2}$. □

Theorem 2.8. *For any complete multipartite graph $G \cong K_{r_1,r_2,.....r_t}$ with $t \geq 2$,*

$$\gamma_a(G) = 2.$$

Proof. Let A_1, A_2,.....,A_t denote the partite sets of G. Let $p_i = \{v \in A_i : f(v) = 1\}$ and $q_i = \{v \in A_i : f(v) = -1\}$ for $1 \leq i \leq t$. There exists an integer i such that $|p_i| - |q_i| < 2$ where $1 \leq i \leq t$. For $u \in A_i$, $f(N(u)) = \sum_{v \in V - A_i} f(v) \leq 1$. This implies that $\gamma_a(G) = f(V) = \sum_{v \in V} f(N(v)) = f(N(u)) + \sum_{v \in A_i} f(v) \leq 1 + |p_i| - |q_i|$. Hence $\gamma_a(G) \leq 2$.

On the other hand, let v_1 and v_2 be two vertices in partitions A_1 and A_2 respectively. Also, let $g : V \to \{-1, 0, 1\}$ such that vertices $v_1, v_2 \in V_1$ and remaining vertices that is $V - \{v_1, v_2\} \in V_0$. This implies that g is an ADF on graph G with $g(V) = 2$. This implies that $\gamma_a(G) \geq g(V) = 2$. Hence $\gamma_a(G) = 2$. □

3 Strongly affirmative domination

Theorem 3.1. *Let f be a SADF of a graph G. Then*

(i) Every vertex of set V_0 is dominated by a vertex of V_1.

(ii) $V_1 \cup V_{-1}$ is a dominating set of G.

(iii) $\gamma_{sa}(G) = |V_1| - |V_{-1}|$.

(vi) $\gamma(G) \leq \gamma_{sa}(G)$; provided $\triangle(G) = n-1$.

(v) $|N(v) \cap V_1| < 1$ for a vertex $v \in V_1$; provided $\gamma(G) = \gamma_{sa}(G)$.

(vi) $|N(v) \cap V_1| \leq |N(v) \cap V_{-1}|$ for any vertex $v \in V_1$.

Proof. Let f be a SADF on graph G. Results (i) to (iv) are same as in ADF
(v) If $\gamma(G) = \gamma_{sa}(G)$, then $|V_{-1}| = 0$. For any vertex $v \in V_1$, if $|N(v) \cap V_1| \geq 1$, then $f(N[u]) \geq 2$ for every vertex $u \in V$, which is a contradiction. Thus, (v) follows.
(vi) Assume to the contrary, that for any vertex $v \in V_1$, $|N(v) \cap V_1| > |N(v) \cap V_{-1}|$. As $f(N[v]) = f(v) + |N(v) \cap V_1| - |N(v) \cap V_{-1}|$. $f(N[v]) > 1$ which is a contradiction. Hence $|N(v) \cap V_1| \leq |N(v) \cap V_{-1}|$ follows. □

Theorem 3.2. *For any graph G,*

$$\gamma_a(G) \geq \gamma_{sa}(G).$$

Proof. By Theorems 2.1 and 3.1, the desired result follows. Also, when $G \cong \overline{K_n}$ and $K_n (\neq K_2)$, the equality holds. □

Theorem 3.3. *An SADF is maximal if and only if for every vertex $v \in V$ with $f(v) = -1$, there exist a vertex $u \in N[v]$ such that $f(N[u]) = 0$ or 1.*

Proof. Proof follows as in Theorem 2.2. □

Theorem 3.4. *For any graph G, $\gamma_{sa}(G) = n$ if and only if $G \cong \overline{K_n}$.*

Proof. If G is totally disconnected graph with n vertices, then vertices of G cannot be assigned 0 as they should be adjacent to at least one vertex assigned 1. Thus, vertices of G should be either assigned 1 or -1. But $\gamma_{sa}(G)$ is maximum of such assignment, every vertex should be assigned 1. Hence $\gamma_{sa}(G) = n$. Conversely, if $\gamma_{sa}(G) = n$, then $|V_1| - |V_{-1}| = n$. This implies $|V_1| = n + |V_{-1}|$ which is not possible. Hence $|V_{-1}| = 0$, implies every vertex of G is assigned 1. If G is connected graph with $n > 1$, then it contradicts to f being SADF. Hence the result. □

Theorem 3.5. *If G is a r-regular graph, then*

$$\gamma_{sa}(G) \leq \frac{n}{r+1}.$$

Proof. Let f be a SADF with a strongly affirmative domination number $\gamma_{sa}(G)$. Consider the sum $s = \sum_{v\in V} \sum_{u\in N[v]} f(u)$. This sum counts the value $f(u)$ exactly $deg(u)+1$ times for each $u \in V$. The sum $s = \sum_{u\in V}[deg(u) + 1]f(u)$. As $f(N[v]) \leq 1$, $\sum_{v\in V} f(N[v]) \leq n$, the sum $s \leq n$. For r-regular graph, degree of every vertex is r. This implies $(r+1)\sum_{v\in V} f(v) \leq n$. □

Theorem 3.6. *For any complete graph K_n,*

$$\gamma_{sa}(K_n) = 1.$$

Proof. As a complete graph K_n is $(n-1)$-regular graph. By Theorem 3.5, we have $\gamma_{sa}(K_n) \leq 1$. Also $\gamma(K_n) = 1$ and by Theorem 3.1, we have the strongly affirmative domination number $\gamma_{sa}(K_n) \geq 1$. Thus the result follows. □

Theorem 3.7. *For any path P_n with integers $n \geq 1$ and $-2 \leq l \leq 0$,*

$$\gamma_{sa}(P_n) = \frac{n-l}{3},$$

where $n \equiv l\ (mod\ 3)$.

Proof. In a path P_n with $n \geq 1$ vertices, there are two end vertices and $n-2$ vertices of degree 2. If v is end vertex and w is its support vertex, then both vertices cannot belong to the same set. That is if $v \in V_1$, then $w \in V_0$ and vice versa. If v is neither an end vertex nor a support vertex, then both vertices adjacent to v may belong to V_0 or V_1 provided $v \in V_1$ or $v \in V_{-1}$ respectively. Every three consecutive vertices are assigned 0, 1, 0 such that $f(N[v]) \leq 1$ for every $v \in V$. We shall prove in following three cases:

Case 1. If $n \equiv 0\ (mod\ 3)$, then n is a multiple of 3. Let $n = 3k$, where k is a positive integer. Here, vertices of P_n are assigned 0, 1, 0. The strongly affirmative domination number $\gamma_{sa}(P_n) = k = \frac{n}{3}$.

Case 2. If $n \equiv -1\ (mod\ 3)$, then $n + 1$ is a multiple of 3. Let $n = 3k - 1 = 3(k - 1) + 2$. Here, vertices of P_n are assigned 0, 1, 0.... $(k-1)$ times and remaining two vertices are assigned 0 and 1 such that $f(N[v]) \leq 1$ for all $v \in V$. The strongly affirmative domination number $\gamma_{sa}(P_n) = k = \frac{n+1}{3}$.

Case 3. If $n \equiv -2\ (mod\ 3)$, then $n+2$ is a multiple of 3. Let $n = 3k-2 = 3(k-1)+1$. Here, vertices of P_n are assigned 1, 0, 0,.... $(k-1)$ times and remaining one vertex is assigned 1 such that $f(N[v]) \leq 1$ for all $v \in V$. The strongly affirmative domination number $\gamma_{sa}(P_n) = \frac{n+2}{3}$. □

Theorem 3.8. *For any cycle C_n with integers $n \geq 3$ and $-2 \leq l \leq 0$*

$$\gamma_{sa}(C_n) = \frac{n+2l}{3},$$

where $n \equiv l\ (mod\ 3)$.

Proof. As a cycle is 2-regular, by Theorem 3.5, we have $\gamma_{sa}(C_n) \leq \frac{n}{3}$. If a vertex $v \in V_1$, then both the vertices adjacent to v cannot belong to V_1 as $f(N[v]) \geq 1$. Thus, one neighbor of v belongs to V_1 and other neighbor of v belongs to V_{-1}. Hence every three consecutive vertices of G are assigned -1, 1, 1 in order. We consider following three cases:

Case 1. If $n \equiv 0\ (mod\ 3)$, then n is a multiple of 3. Let $n = 3k$, where k is any positive integer. Here, vertices of C_n are assigned -1, 1, 1, -1, 1, 1,....k times. The strongly affirmative domination number $\gamma_{sa}(C_n) = k = \frac{n}{3}$.

Case 2. If $n \equiv -1\ (mod\ 3)$, then $n + 1$ is a multiple of 3. That is $n = 3k - 1 = 3(k - 1) + 2$. Here, -1, 1, 1 is assigned $(k-1)$ - times and out of remaining two vertices one vertex adjacent to vertex assigned 1 should be assigned -1 and other vertex is assigned 1. The strongly affirmative domination number $\gamma_{sa}(C_n) = (k - 1) + 1 - 1 = \frac{n-2}{3}$.

Case 3. If $n \equiv -2\ (mod\ 3)$, then $n + 2$ is a multiple of 3. That is $n = 3k - 2 = 3(k - 1) + 1$. Here, -1, 1, 1 is assigned $(k - 1)$-times and remaining one vertex is assigned -1 as it cannot be assigned 1 or 0. The strongly affirmative domination number $\gamma_{sa}(C_n) = (k - 1) - 1 = \frac{n-4}{3}$. □

Theorem 3.9. *For any complete multipartite graph $G \cong K_{r_1,r_2,....,r_t}$,*

$$\gamma_{sa}(G) = 0.$$

Proof. Let A_1, A_2,.....,A_t be t partitions of G. Also, let $p_i = \{v \in A_i : f(v) = 1\}$ and $q_i = \{v \in A_i : f(v) = -1\}$ for $1 \le i \le t$. There exists an integer i such that $|p_i| - |q_i| < 1$ where $1 \le i \le t$. For $u \in A_i$, $f(N[u]) = f(u) + \sum_{v \in V - A_i} f(v) \le 1$. Also, $\gamma_{sa}(G) = f(V) = \sum_{v \in V} f(N[v]) \le 0$.

On the other hand, let $v_1, v_2 \in A_1$ and $v_3, v_4 \in A_2$ such that $v_1, v_3 \in V_1$ and $v_2, v_4 \in V_{-1}$ and remaining vertices $V - \{v_1, v_2, v_3, v_4\} \in V_0$. Then g is a SADF with $g(V) = 0$. The strongly affirmative domination number $\gamma_{sa}(G) \ge g(V) = 0$. Hence the result follows. □

References

[1] B. Chaluvaraju and K. A. Vidya, *Generalized perfect domination in graphs*, J. Comb. Optim. Springer, 27(2)(2014) 292-301.

[2] Erfang Shan and T.C.E. Cheng, *Remarks on the minus (signed) total domination in graphs*, Discrete Mathematics 308(2008) 3373-3380.

[3] F. Harary, *Graph theory*, Addison-Wesley, Reading Mass (1969).

[4] L. Harris and J. H. Hattingh, *The algorithm complexity of certain functional variations of total domination in graphs*, Australas. J. Combin., 29(2004) 143-156.

[5] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, *Fundamentals of domination in graphs*, Marcel Dekker, Inc., New York (1998).

[6] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, *Domination in graphs: Advanced topics*, Marcel Dekker, Inc., New York (1998).

[7] Hong Yan, Xiaoqi Yang and Erfang Shan, *Upper minus total domination in small-degree regular graphs*, Discrete Mathematics, 307(2007) 2453-2463.

[8] Hua-Ming Xing and Hai-Long Liu, *Minus total domination in graphs*, Czechoslovak Mathematical Journal, 59(134)(2009) 861-870.

[9] Jean Dunbar, W. Goddard, S.T. Hedetniemi, M.A. Henning and A. McRae, *The algorithm complexity of minus domination in graphs*, Discrete Appl. Math., 68 (1996) 73-84.

[10] Jean Dunbar, S. T. Hedetniemi, M. A. Henning and A. McRae, *Minus domination in graphs*, Discrete Mathematics, 199(1999) 35-47.

[11] Peter Damaschke, *Minus domination in small-degree graphs*, Discrete Applied Mathematics, 108(2001) 53-64.

[12] C. Wang, *The negative decision number in graphs*, The Australasian Journal of Combinatorics, 41(2008) 263-272.

[13] C. Wang, *The modified negative decision number in graphs*, International Journal of Mathematics and Mathematical Sciences, Volume 2011, Article ID 135481, 9 pages.

Author information

B. Chaluvaraju and V. Chaitra, Department of Mathematics, Central College Campus, Bangalore University, Bangalore-560 001,, INDIA.
E-mail: `bchaluvaraju@gmail.com`

Received: March 13, 2015.

Accepted: June 12, 2015