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New exact solitary wave solutions to the TDB and (2 +1)-DZ equations

A. Neirameh

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Abstract In this work, we investigate for finding exact solitary wave solutions of the (2 + 1)-dimensional Zoomeron equation and the Tzitzeica–Dodd–Bullough (TDB) equation by using the direct algebraic method. The direct algebraic method is promising for finding exact traveling wave solutions of nonlinear evolution equations in mathematical physics. The competence of the methods for constructing exact solutions has been established.

1 Introduction

The aim of this article is to look for new study relating to the direct algebraic method for solving the renowned Tzitzeica–Dodd– Bullough equation

$$u_{xy} - e^{-u} - e^{-2u} = 0,$$

and the (2 + 1)-dimensional Zoomeron equation

$$\left(\frac{u_{xy}}{u}\right)_{tt} - \left(\frac{u_{xy}}{u}\right)_{xx} + 2\left(u^2\right)_{xt} = 0,$$

to demonstrate the suitability and straightforwardness of the method.

The investigation of the travelling wave solutions for nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appears in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In the past several decades, new exact solutions may help to find new phenomena. A variety of powerful methods, such as inverse scattering method [1,9], Hirota bilinear tranformation[5,12], the tanh–sech method [6,11,13,8], sine–cosine method [10,2] and Exp-function method [3,7,14,4] were used to develop nonlinear dispersive and dissipative problems.

2 An Analysis of the Method

For a given partial differential equation

$$G(u, u_x, u_t, u_{xx}, u_{tt}, ...), \tag{2.1}$$

Our method mainly consists of four steps:

Step 1: We seek complex solutions of Eq. (2.1) as the following form:

$$u = u(\xi), \qquad \xi = ik(x - ct), \tag{2.2}$$

Where k and c are real constants. Under the transformation (2.2), Eq. (2.1) becomes an ordinary differential equation

$$N(u, iku', -ikcu', -k^2u'', ...), \tag{2.3}$$

Where $u' = \frac{du}{d\xi}$.

Step 2: We assume that the solution of Eq. (2.3) is of the form

$$u(\xi) = \sum_{i=0}^{n} a_i F^i(\xi), \tag{2.4}$$

Where $a_i(i = 1, 2, .., n)$ are real constants to be determined later.$F(\xi)$expresses the solution of the auxiliary ordinary differential equation

$$F'(\xi) = b + F^2(\xi), \tag{2.5}$$

Eq. (2.5) admits the following solutions:

$$\begin{aligned} F(\xi) &= \begin{cases} -\sqrt{-b}\tanh(\sqrt{-b}\xi), & b \prec 0 \\ -\sqrt{-b}\coth(\sqrt{-b}\xi), & b \prec 0 \end{cases} \\ F(\xi) &= \begin{cases} \sqrt{b}\tan(\sqrt{b}\xi), & b \succ 0 \\ -\sqrt{b}\cot(\sqrt{b}\xi), & b \succ 0 \end{cases} \\ F(\xi) &= -\tfrac{1}{\xi}, \qquad b = 0 \end{aligned} \tag{2.6}$$

Integer n in (2.4) can be determined by considering homogeneous balance [3] between the nonlinear terms and the highest derivatives of $u(\xi)$in Eq. (2.3).
Step 3: Substituting (2.4) into (2.3) with (2.5), then the left hand side of Eq. (2.3) is converted into a polynomial in $F(\xi)$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i, k, c.
Step 4: Solving the algebraic equations obtained in step 3, and substituting the results into (2.4), then we obtain the exact traveling wave solutions for Eq. (2.1).

3 The Tzitzeica–Dodd–Bullough (TDB) equation

In this sub-section, we will exert the MSE method to obtain new and more general exact solutions and then the solitary wave solutions of the Tzitzeica–Dodd–Bullough equation,

$$u_{xy} - e^{-u} - e^{-2u} = 0 \tag{3.1}$$

Using the transformation $v = e^{-u}$Eq. (3.1) transforms into the following partial differential equation,

$$vv_{xt} - v_x v_t + v^3 + v^4 = 0. \tag{3.2}$$

We use the wavetransformation$v = v(\xi)$, with wave complex variable$\xi = ik(x - ct)$, where k and c are real constants. System (3.2) takes the form as

$$ck^2vv'' - ck^2(v')^2 + v^3 + v^4 = 0. \tag{3.3}$$

Considering the homogeneous balance between vv'' and v^4in (3.3), we required that$3m = m + 2 \Rightarrow m = 1$. So the solution takes theform

$$u = a_1F + a_0, \tag{3.4}$$

Substituting (3.4) into Eq. (3.3) yields a set of algebraic equations for a_1, a_0, k, c and solving these equations with Maple package we have

$$\begin{aligned} a_1 &= \pm k\sqrt{\tfrac{1}{2k\sqrt{2(b^2+1)}}} \\ a_0 &= -\tfrac{1}{2} \\ c &= \pm\tfrac{1}{2k\sqrt{2(b^2+1)}} \end{aligned} \tag{3.5}$$

From (2.6),(3.4) and (3.5), we obtain the complex travelling wave solutions of (3.1) as follows

$$v_1 = \pm k\sqrt{\frac{1}{2k\sqrt{2(b^2+1)}}}\left[\sqrt{-b}\tanh(\sqrt{-b}ik(x\mp \frac{1}{2k\sqrt{2(b^2+1)}}t)\right] - \frac{1}{2},$$

So we have

$$u_1 = -\ln\left[\pm k\sqrt{\frac{1}{2k\sqrt{2(b^2+1)}}}\left[\sqrt{-b}\tanh(\sqrt{-b}ik(x\mp \frac{1}{2k\sqrt{2(b^2+1)}}t)\right] - \frac{1}{2}\right]$$

Where $b \prec 0$and k is an arbitrary real constant. And

$$u_2 = -\ln\left[\pm k\sqrt{\frac{1}{2k\sqrt{2(b^2+1)}}}\left[\sqrt{-b}\coth(\sqrt{-b}ik(x\mp \frac{1}{2k\sqrt{2(b^2+1)}}t)\right] - \frac{1}{2}\right],$$

Where $b \prec 0$and k is an arbitrary real constant.

$$u_3 = -\ln\left[\pm k\sqrt{\frac{1}{2k\sqrt{2(b^2+1)}}}\left[\sqrt{b}\tan(\sqrt{b}ik(x\mp\frac{1}{2k\sqrt{2(b^2+1)}}t)\right]-\frac{1}{2}\right],$$

Where $b \succ 0$and k is an arbitrary real constant.

$$u_4 = -\ln\left[\pm k\sqrt{\frac{1}{2k\sqrt{2(b^2+1)}}}\left[\sqrt{b}\cot(\sqrt{b}ik(x\mp\frac{1}{2k\sqrt{2(b^2+1)}}t)\right]-\frac{1}{2}\right],$$

Where $b \succ 0$and k is an arbitrary real constant.For $b = 0$

$$u_5 = -\ln\left[\mp\sqrt{\frac{1}{2\sqrt{2}k}}\frac{i}{x\mp\frac{1}{2\sqrt{2}k}t}-\frac{1}{2}\right],$$

In these cases if assume$u_{1,2,3,4,5} = \ln[D]$, D Must be greaterthan zero (or $D > 0$).

4 The (2 +1)-dimensional Zoomeron equation

Let us consider the Zoomeron equation

$$\left(\frac{u_{xy}}{u}\right)_{tt} - \left(\frac{u_{xy}}{u}\right)_{xx} + 2\left(u^2\right)_{xt} = 0 \tag{4.1}$$

where$u(x, y, t)$ is the amplitude of the relative wave mode. The traveling wave transformation

$$u = u(\xi), \qquad \xi = ik(x + y - \omega t) \tag{4.2}$$

Reduces Eq. (4.1) into the following ODE:

$$k^2(1-\omega^2)u'' - 2\omega u^3 + R = 0. \tag{4.3}$$

where$R$ is a constant of integration.Balancing the highest order derivative u'' and nonlinear term of the highest order u^3, yields $m = 1$.So the solution takes the form

$$u = a_1 F + a_0, \tag{4.4}$$

Substituting (4.4) into Eq. (4.3) yields a set of algebraic equations for a_1, a_0, k, c and solving these equations with Maple package we have

$$\begin{aligned} a_1 &= \pm k\sqrt{\tfrac{1-\omega^2}{\omega}} \\ a_0 &= \pm k\sqrt{\tfrac{1-\omega^2}{3\omega}} \end{aligned} \tag{4.5}$$

From (2.6),(4.4) and (4.5), we obtain the complex travelling wave solutions of (4.1) as follows

$$u_1 = \pm k\sqrt{\frac{1-\omega^2}{\omega}}\left[\sqrt{-b}\tanh(\sqrt{-b}ik(x+y-\omega\,t)\right] \pm k\sqrt{\frac{1-\omega^2}{3\omega}},$$

Where $b \prec 0$and k is an arbitrary real constant. And

$$u_2 = \pm k\sqrt{\frac{1-\omega^2}{\omega}}\left[\sqrt{-b}\coth(\sqrt{-b}ik(x+y-\omega\,t)\right] \pm k\sqrt{\frac{1-\omega^2}{3\omega}},$$

Where $b \prec 0$and k is an arbitrary real constant.

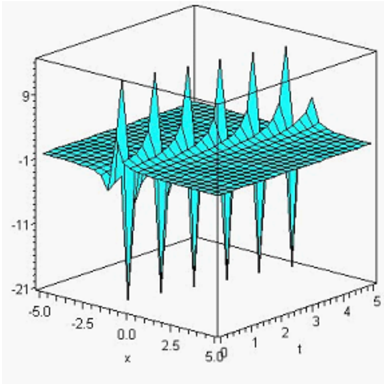
$$u_3 = \pm k\sqrt{\frac{1-\omega^2}{\omega}}\left[\sqrt{b}\tan(\sqrt{b}ik(x+y-\omega\,t)\right] \pm k\sqrt{\frac{1-\omega^2}{3\omega}},$$

Where $b \succ 0$and k is an arbitrary real constant.

$$u_4 = \pm k\sqrt{\frac{1-\omega^2}{\omega}}\left[\sqrt{b}\cot(\sqrt{b}ik(x+y-\omega\,t)\right] \pm k\sqrt{\frac{1-\omega^2}{3\omega}},$$

Where $b \succ 0$and k is an arbitrary real constant.For $b = 0$

$$u_5 = \mp k\sqrt{\frac{1-\omega^2}{\omega}}\frac{i}{k(x+y-\omega t)} \pm k\sqrt{\frac{1-\omega^2}{3\omega}},$$



Solitary complex wave solutions for (2 +1)-dimensional Zoomeron equation in (1+1)-dimentional: y is constant(in this figure y=1)

5 Conclusion

In this work direct algebraic method applied successfully for solving the system of non-linear evolution equations. The performance of this method is reliable and effective and gives more solutions. This method has more advantages: it is direct and concise. Thus, we deduce that the proposed method can be extended to solve many systems of non-linear fractional partial differential equations.

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Author information

A. Neirameh, Department of Mathematics, Faculty of sciences, Gonbad Kavous University, Gonbad, Iran.
E-mail: a.neirameh@gonbad.ac.ir

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