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Recurrence Relation and Integral Representation of Generalized K- Mittag-Leffler Function $GE^{\gamma,q}_{k,\alpha,\beta}(z)$

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Abstract. In this paper author calculate the recurrence relations and six different integral representation of Generalized K- Mittag-Leffler function, $GE^{\gamma,q}_{k,\alpha,\beta}(z)$ introduced by [3]. Also find out six different integral representation of K- Mittag-Leffler function, $E^{\gamma}_{k,\alpha,\beta}(z)$ defined by [2] and several special cases have been discussed.

1 Introduction

The K-Pochhammer symbol was introduce by [1] in the form,

$$(x)_{n,k} = x(x+k)(x+2k).....(x+(n-1)k), \tag{1.1}$$

$$(x)_{(n+r)q,k} = (x)_{rq,k}(x+qrk)_{nq,k}, \tag{1.2}$$

where $x \in C$, $k \in R$ and $n \in N$.
K-Gamma function was introduce by [1] in the form,

$$\Gamma_k(x) = \int_0^\infty e^{-\frac{t^k}{k}}\, t^{x-1} dt, x \in C, k \in R, Re(x) > 0, \tag{1.3}$$

and

$$\Gamma_k(x+k) = x\Gamma_k(x). \tag{1.4}$$

The Gamma function,

$$\Gamma(n) = \int_0^\infty e^{-t}\, t^{n-1} dt. \tag{1.5}$$

The Beta function,

$$B(m,n) = \frac{\Gamma(n)\Gamma(m)}{\Gamma(n+m)} = \int_0^1 t^{n-1}\,(1-t)^{m-1} dt, Re(n) > 0, Re(m) > 0. \tag{1.6}$$

Recently in 2012, G.A. Dorrego, and R.A. Cerutti [2], introduce the K-Mittag-Leffer function $E^{\gamma}_{k,\alpha,\beta}(z)$, defined as

$$E^{\gamma}_{k,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{n,k}\, z^n}{\Gamma_k(n\alpha+\beta)(n!)}, \tag{1.7}$$

where $k \in R; \alpha, \beta, \gamma \in C; Re(\alpha) > 0, Re(\beta) > 0$.

Relation between classical Pochhammer symbol and K-Pochhammer symbol are given below (cf. [1])

Proposition 1. Let $k, s \in R$ and $\gamma \in C$, then the following identity holds

$$\Gamma_s(\gamma) = (\frac{s}{k})^{\frac{\gamma}{s}-1}\Gamma_k(\frac{k\gamma}{s}), \tag{1.8}$$

and particular case

$$\Gamma_k(\gamma) = (k)^{\frac{\gamma}{k}-1}\Gamma(\frac{\gamma}{k}). \tag{1.9}$$

Proposition 2. Let $k, s \in R, \gamma \in C \; and \; n \in N$, then the following identity holds

$$(\gamma)_{nq,s} = (\frac{s}{k})^{nq}(\frac{k\gamma}{s})_{nq,k}, \tag{1.10}$$

and particular case

$$(\gamma)_{nq,k} = (k)^{nq}(\frac{\gamma}{k})_{nq}. \tag{1.11}$$

The Generalized K- Mittag-Leffler function, introduced by [3], as

Definition 1: Let $k \in R; \alpha, \beta, \gamma \in C; Re(\alpha) > 0, Re(\beta) > 0, Re(\gamma) > 0 \; and \; q \in (0,1) \cup N$, the Generalized K- Mittag-Leffler function denoted by $GE^{\gamma,q}_{k,\alpha,\beta}(z)$ and defined as,

$$GE^{\gamma,q}_{k,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, z^n}{\Gamma_k(n\alpha+\beta)(n!)}, \tag{1.12}$$

where$(\gamma)_{nq,k}$ is the K- pochhammer symbol given by equation (1.1) and $\Gamma_k(x)$ is the K-gamma function given by equation (1.3).
([6], page 22), gives the generalized Pochhammer symbol as,

$$(\gamma)_{nq} = \frac{\Gamma(\gamma+nq)}{\Gamma(\gamma)} = q^{qn} \prod_{r=1}^{q} (\frac{\gamma+r+1}{q})_n, \; if \; q \in N. \tag{1.13}$$

Particular cases : For some particular values of the parameters
$q, k, \alpha, \beta, \gamma$ we can obtain certain Mittag-Leffler functions, defined earlier:
(a) For $q = 1$, equation (1.12) reduces in K- Mittag-Leffler functions defined by [2].

$$GE^{\gamma,1}_{k,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{n,k}\, z^n}{\Gamma_k(n\alpha+\beta)(n!)} = E^{\gamma}_{k,\alpha,\beta}(z), \tag{1.14}$$

(b) For $k = 1$, equation (1.12) reduces in Mittag-Leffler functions defined by [8].

$$GE^{\gamma,q}_{1,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq}\, z^n}{\Gamma(n\alpha+\beta)(n!)} = E^{\gamma,q}_{\alpha,\beta}(z), \tag{1.15}$$

(c) For $q = 1$ and $k = 1$, equation (1.12) reduces in Mittag-Leffler functions defined by [5].

$$GE^{\gamma,1}_{1,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n\, z^n}{\Gamma(n\alpha+\beta)(n!)} = E^{\gamma}_{\alpha,\beta}(z), \tag{1.16}$$

(d) For $q = 1, k = 1 \; and \; \gamma = 1$, equation (1.12) reduces in Mittag-Leffler functions defined by [9].

$$GE^{1,1}_{1,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha+\beta)} = E_{\alpha,\beta}(z), \tag{1.17}$$

(e) For $q = 1, k = 1, \gamma = 1 \; and \; \beta = 1$, equation (1.12) reduces in Mittag-Leffler functions defined by [4].

$$GE^{1,1}_{1,\alpha,1}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha+1)} = E_{\alpha}(z). \tag{1.18}$$

2 Recurrence Relations:

In this section we calculate the recurrence relations of $GE^{\gamma,q}_{k,\alpha,\beta}(z)$, and deduce some particular cases.

Theorem 2.1 For $k \in R$; $R(\alpha+p) > 0, R(\beta+s+k) > 0, R(\gamma) > 0, q \in (0,1) \cup N$, we get

$$\begin{aligned} GE^{\gamma,q}_{k,\alpha+p,\beta+s+k}(z) - k\, GE^{\gamma,q}_{k,\alpha+p,\beta+s+2k}(z) &= (\alpha+p)^2 z^2\, \ddot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z) \\ &+ [(\alpha+p)^2 + (\alpha+p)(2\beta+2s+k+1)] z\, \dot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z) \\ &+ (\beta+s)(\beta+s+k+1) GE^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z), \end{aligned} \tag{2.1}$$

where $\dot{G}E^{\gamma,q}_{k,\alpha,\beta}(z) = \frac{d}{dz} GE^{\gamma,q}_{k,\alpha,\beta}(z)$ and $\ddot{G}E^{\gamma,q}_{k,\alpha,\beta}(z) = \frac{d^2}{dz^2} GE^{\gamma,q}_{k,\alpha,\beta}(z)$.

Proof: The Generalized K- Mittag-Leffler function, from equation (1.12)

$$GE^{\gamma,q}_{k,\alpha+p,\beta+s+k}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, z^n}{\Gamma_k(n(\alpha+p)+\beta+s+k)(n!)},$$

using equation (1.4), we have

$$GE^{\gamma,q}_{k,\alpha+p,\beta+s+k}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s)\{n(\alpha+p)+\beta+s)\}(n!)},\tag{2.2}$$

and

$$GE^{\gamma,q}_{k,\alpha+p,\beta+s+2k}(z)$$
$$=\sum_{n=0}^{\infty}\frac{(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+k)\{n(\alpha+p)+\beta+s+k)\}(n!)},\tag{2.3}$$

using (1.4), we have

$$GE^{\gamma,q}_{k,\alpha+p,\beta+s+2k}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s)(n!)}$$
$$\times\frac{1}{\{n(\alpha+p)+\beta+s)\}\{n(\alpha+p)+\beta+s+k)\}},$$
$$GE^{\gamma,q}_{k,\alpha+p,\beta+s+2k}(z)=\sum_{n=0}^{\infty}\frac{1}{k}[\frac{1}{(n(\alpha+p)+\beta+s)}-\frac{1}{(n(\alpha+p)+\beta+s+k)}]$$
$$\times\frac{(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s)(n!)},$$
$$GE^{\gamma,q}_{k,\alpha+p,\beta+s+2k}(z)=\frac{1}{k}[GE^{\gamma,q}_{k,\alpha+p,\beta+s+k}(z)-S],$$
$$S=GE^{\gamma,q}_{k,\alpha+p,\beta+s+k}(z)-k\,GE^{\gamma,q}_{k,\alpha+p,\beta+s+2k}(z),\tag{2.4}$$

where

$$S=\sum_{n=0}^{\infty}\frac{(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s)\{n(\alpha+p)+\beta+s+k)\}(n!)},\tag{2.5}$$

applying the simple identity $\frac{1}{u}=\frac{k}{u(u+k)}+\frac{1}{u+k}$; for $u=n(\alpha+p)+\beta+s+k$ to equation (2.5),we obtain,

$$S=\sum_{n=0}^{\infty}\frac{k(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s)(n!)}$$
$$\times\frac{1}{\{n(\alpha+p)+\beta+s+k\}\{n(\alpha+p)+\beta+s+2k\}}$$
$$+\sum_{n=0}^{\infty}\frac{(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s)\{n(\alpha+p)+\beta+s+2k)\}(n!)},$$
$$S=\sum_{n=0}^{\infty}\frac{k\{n(\alpha+p)+\beta+s\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s)(n!)}$$
$$\times\frac{1}{\{n(\alpha+p)+\beta+s\}\{n(\alpha+p)+\beta+s+k\}\{n(\alpha+p)+\beta+s+2k\}}$$
$$+\sum_{n=0}^{\infty}\frac{\{n(\alpha+p)+\beta+s\}\{n(\alpha+p)+\beta+s+k\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s)(n!)}$$
$$\times\frac{1}{\{n(\alpha+p)+\beta+s\}\{n(\alpha+p)+\beta+s+k\}\{n(\alpha+p)+\beta+s+2k\}},$$

using (1.4) we obtain

$$S=\sum_{n=0}^{\infty}\frac{k\{n(\alpha+p)+\beta+s\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)}$$
$$+\sum_{n=0}^{\infty}\frac{\{n(\alpha+p)+\beta+s\}\{n(\alpha+p)+\beta+s+k\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)},$$

$$S=\sum_{n=0}^{\infty}\frac{\{n^2(\alpha+p)^2\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)}$$
$$+\sum_{n=0}^{\infty}\frac{\{n(\alpha+p)(2\beta+2s+k+1)\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)}$$
$$+\sum_{n=0}^{\infty}\frac{\{(\beta+s)(\beta+s+k+1)\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)}, \tag{2.6}$$

we now express each summation in the right hand side of (2.6) as follows:

$$\frac{d}{dz}[z\,GE^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)]=\sum_{n=0}^{\infty}\frac{\{(n+1)\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)},$$

$$z\dot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)+GE^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)$$
$$=\sum_{n=0}^{\infty}\frac{\{(n+1)\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)},$$

$$z\dot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)=\sum_{n=0}^{\infty}\frac{\{n\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)}, \tag{2.7}$$

Again

$$\frac{d^2}{dz^2}[z^2\,GE^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)]=\sum_{n=0}^{\infty}\frac{\{(n+1)(n+2)\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)}, \tag{2.8}$$

and

$$\frac{d^2}{dz^2}[z^2\,GE^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)]$$
$$=z^2\ddot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)+4z\dot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)+2GE^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z), \tag{2.9}$$

from equation (2.8) and (2.9) we have

$$\sum_{n=0}^{\infty}\frac{\{n^2\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)}=z^2\ddot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)$$
$$+4z\dot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)-3\sum_{n=0}^{\infty}\frac{\{n\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)},$$

using equation (2.7), we have

$$\sum_{n=0}^{\infty}\frac{\{n^2\}(\gamma)_{nq,k}\,z^n}{\Gamma_k(n(\alpha+p)+\beta+s+3k)(n!)}$$
$$=z^2\ddot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)+z\dot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z), \tag{2.10}$$

applying equation (2.7) and (2.10) to (2.6), we get

$$S=(\alpha+p)^2z^2\,\ddot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)+[(\alpha+p)^2+(\alpha+p)(2\beta+2s+k+1)]z$$
$$\times\dot{G}E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z)+(\beta+s)(\beta+s+k+1)GE^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z),$$

Hence.

Corollary 2.1.1 For $k\in R; R(\alpha+p)>0, R(\beta+s+k>0, R(\gamma)>0$ and put $q=1$ in equation (2.1), we get

$$E^{\gamma}_{k,\alpha+p,\beta+s+k}(z)-k\,E^{\gamma}_{k,\alpha+p,\beta+s+2k}(z)=(\alpha+p)^2z^2\,\ddot{E}^{\gamma}_{k,\alpha+p,\beta+s+3k}(z)$$
$$+[(\alpha+p)^2+(\alpha+p)(2\beta+2s+k+1)]z\,\dot{E}^{\gamma}_{k,\alpha+p,\beta+s+3k}(z)$$
$$+(\beta+s)(\beta+s+k+1)E^{\gamma,q}_{k,\alpha+p,\beta+s+3k}(z), \tag{2.11}$$

where $\dot{E}^{\gamma}_{k,\alpha,\beta}(z) = \frac{d}{dz}E^{\gamma}_{k,\alpha,\beta}(z)$ and $\ddot{E}^{\gamma}_{k,\alpha,\beta}(z) = \frac{d^2}{dz^2}E^{\gamma}_{k,\alpha,\beta}(z)$
which is the new recurrence relation for K-Mittag-Leffler function $E^{\gamma}_{k,\alpha,\beta}(z)$, definded by [2].

Corollary 2.1.2 For $R(\alpha+p) > 0, R(\beta+s+k) > 0, R(\gamma) > 0, q \in (0,1) \cup N$, and put $k = 1$ in equation (2.1), we get

$$E^{\gamma,q}_{\alpha+p,\beta+s+1}(z) - GE^{\gamma,q}_{\alpha+p,\beta+s+2}(z) = (\alpha+p)^2 z^2\, \ddot{E}^{\gamma,q}_{\alpha+p,\beta+s+3}(z)$$
$$+[(\alpha+p)^2 + (\alpha+p)(2\beta+2s+2)]z\, \dot{E}^{\gamma,q}_{\alpha+p,\beta+s+3}(z)$$
$$+(\beta+s)(\beta+s+2)E^{\gamma,q}_{\alpha+p,\beta+s+3}(z), \qquad (2.12)$$

where $\dot{E}^{\gamma,q}_{\alpha,\beta}(z) = \frac{d}{dz}E^{\gamma,q}_{\alpha,\beta}(z)$ and $\ddot{E}^{\gamma,q}_{\alpha,\beta}(z) = \frac{d^2}{dz^2}E^{\gamma,q}_{\alpha,\beta}(z)$.
Remark: The result (2.12) is well known result obtained by ([7],equation (2.1, page 134).

Theorem 2.2 For $r \in N, k \in R; \alpha,\beta,\gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0$ and $q \in (0,1) \cup N$, then

$$(\gamma)_{qr,k}\, GE^{\gamma+qrk,q}_{k,\alpha,\beta+\alpha r}(z) = \frac{d^r}{dz^r}[GE^{\gamma,q}_{k,\alpha,\beta}(z) - \sum_{n=0}^{r-1}\frac{(\gamma)_{qn,k}\, z^n}{\Gamma_k(n\alpha+\beta)(n!)}]. \qquad (2.13)$$

Proof: Consider the right hand side,

$$A \equiv \frac{d^r}{dz^r}[GE^{\gamma,q}_{k,\alpha,\beta}(z) - \sum_{n=0}^{r-1}\frac{(\gamma)_{qn,k}\, z^n}{\Gamma_k(n\alpha+\beta)(n!)}],$$

$$A \equiv \frac{d^r}{dz^r}[\sum_{n=r}^{\infty}\frac{(\gamma)_{qn,k}\, z^n}{\Gamma_k(n\alpha+\beta)(n!)}],$$

$$A \equiv \frac{d^r}{dz^r}[\sum_{n=0}^{\infty}\frac{(\gamma)_{(n+r)q,k}\, z^{n+r}}{\Gamma_k((n+r)\alpha+\beta)(n+r)!}],$$

using equation (1.2), we have

$$A \equiv \frac{d^r}{dz^r}[\sum_{n=0}^{\infty}\frac{(\gamma)_{rq,k}(\gamma+qrk)_{nq,k}\, z^n}{\Gamma_k(n\alpha+\beta+\alpha r)(n!)}]$$

$$A \equiv (\gamma)_{qr,k}\, GE^{\gamma+qrk,q}_{k,\alpha,\beta+\alpha r}(z).$$

Hence.

Corollary 2.2.1 For $r \in N, k \in R; \alpha,\beta,\gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0$, then equation (2.13) gives the result for $q = 1$,

$$(\gamma)_{r,k}\, E^{\gamma+rk}_{k,\alpha,\beta+\alpha r}(z) = \frac{d^r}{dz^r}[E^{\gamma}_{k,\alpha,\beta}(z) - \sum_{n=0}^{r-1}\frac{(\gamma)_{n,k}\, z^n}{\Gamma_k(n\alpha+\beta)(n!)}], \qquad (2.14)$$

which is the new recurrence relation for K-Mittag-Leffler function $E^{\gamma}_{k,\alpha,\beta}(z)$, definded by [2].

Corollary 2.2.2 For $r \in N; \alpha,\beta,\gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0$, then equation (2.13) gives the result for $k = 1$,

$$(\gamma)_{qr}\, E^{\gamma+qr,q}_{\alpha,\beta+\alpha r}(z) = \frac{d^r}{dz^r}[E^{\gamma,q}_{\alpha,\beta}(z) - \sum_{n=0}^{r-1}\frac{(\gamma)_{qn}\, z^n}{\Gamma(n\alpha+\beta)(n!)}], \qquad (2.15)$$

which is the new recurrence relation for Mittag-Leffler function $E^{\gamma,q}_{\alpha,\beta}(z)$, definded by [8].

3 Integral Representation:

In this section we calculate six different integral representations of Generalized K- Mittag-Leffler function, $GE^{\gamma,q}_{k,\alpha,\beta}(z)$. Also evaluate six different integral representations of K- Mittag-Leffler function, $E^{\gamma}_{k,\alpha,\beta}(z)$ defined by [2] and several special cases have been discussed.

Theorem 3.1 For $k \in R; R(\alpha+p) > 0, R(\beta+s+k) > 0, R(\gamma) > 0, q \in (0,1) \cup N$, we get

$$\int_0^1 t^{\beta+s+k-1} GE^{\gamma,q}_{k,\alpha+p,\beta+s}(t^{\alpha+p})dt$$

$$= GE^{\gamma,q}_{k,\alpha+p,\beta+s+k}(1) - k\, GE^{\gamma,q}_{k,\alpha+p,\beta+s+2k}(1). \tag{3.1}$$

Proof: Put $z = 1$ in equations (2.4) and (2.5), we have
$S = GE^{\gamma,q}_{k,\alpha+p,\beta+s+k}(1) - k\, GE^{\gamma,q}_{k,\alpha+p,\beta+s+2k}(1)$

$$= \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(n(\alpha+p)+\beta+s)\{n(\alpha+p)+\beta+s+k\}(n!)}, \tag{3.2}$$

now consider the integral,

$$A \equiv \int_0^z t^{\beta+s+k-1} GE^{\gamma,q}_{k,\alpha+p,\beta+s}(t^{\alpha+p})dt$$

$$A \equiv \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(n(\alpha+p)+\beta+s)(n!)} \int_0^z t^{n(\alpha+p)\beta+s+k-1}dt,$$

$$A \equiv \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, z^{n(\alpha+p)+\beta+s+k}}{\Gamma_k(n(\alpha+p)+\beta+s)\{n(\alpha+p)+\beta+s+k\}(n!)},$$

for $z = 1$

$$A \equiv \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(n(\alpha+p)+\beta+s)\{n(\alpha+p)+\beta+s+k\}(n!)},$$

from equation (3.2), we have

$$A \equiv GE^{\gamma,q}_{k,\alpha+p,\beta+s+k}(1) - k\, GE^{\gamma,q}_{k,\alpha+p,\beta+s+2k}(1).$$

Hence.

Corollary 3.1.1 For $k \in R; R(\alpha+p) > 0, R(\beta+s+k) > 0, R(\gamma) > 0$, then equation (3.1) gives the result for $q = 1$,

$$\int_0^1 t^{\beta+s+k-1} E^{\gamma}_{k,\alpha+p,\beta+s}(t^{\alpha+p})dt = E^{\gamma}_{k,\alpha+p,\beta+s+k}(1) - k\, E^{\gamma}_{k,\alpha+p,\beta+s+2k}(1). \tag{3.3}$$

which is the new Integral representation for K-Mittag-Leffler function $E^{\gamma}_{k,\alpha,\beta}(z)$, definded by [2].

Corollary 3.1.2 For $R(\alpha+p) > 0, R(\beta+s+k) > 0, R(\gamma) > 0, q \in (0,1) \cup N$, then equation (3.1) gives the result for $k = 1$,

$$\int_0^1 t^{\beta+s} E^{\gamma,q}_{\alpha+p,\beta+s}(t^{\alpha+p})dt = E^{\gamma,q}_{\alpha+p,\beta+s+1}(1) - E^{\gamma,q}_{\alpha+p,\beta+s+2}(1). \tag{3.4}$$

Remark : Which is well known result obtained for Mittag-Leffler function $E^{\gamma,q}_{\alpha,\beta}(z)$, by ([7], equation (3.1), page 137).

Theorem 3.2 For $k \in R; \alpha, \beta, \gamma, \delta \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0, R(\delta) > 0 and q \in N$, then

$$k^{\delta}\, GE^{\gamma,q}_{k,\alpha,\beta+\delta k}(z) = \frac{1}{\Gamma(\delta)} \int_0^1 u^{\frac{\beta}{k}-1}(1-u)^{\delta-1} GE^{\gamma,q}_{k,\alpha,\beta}(z\, u^{\frac{\alpha}{k}})du. \tag{3.5}$$

Proof : Consider the right side integral and using equation (1.12), we have

$$A \equiv \frac{1}{\Gamma(\delta)} \int_0^1 u^{\frac{\beta}{k}-1}(1-u)^{\delta-1} GE^{\gamma,q}_{k,\alpha,\beta}(zu^{\frac{\alpha}{k}})du,$$

$$A \equiv \frac{1}{\Gamma(\delta)} \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, z^n}{\Gamma_k(n\alpha+\beta)(n!)} \int_0^1 u^{\frac{\alpha n+\beta}{k}-1}(1-u)^{\delta-1}du,$$

using the definition of Beta function (1.6), we have

$$A \equiv \frac{1}{\Gamma(\delta)} \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, z^n}{\Gamma_k(n\alpha+\beta)(n!)} \frac{\Gamma(\frac{\alpha n+\beta}{k})\Gamma(\delta)}{\Gamma(\frac{\alpha n+\beta}{k}+\delta)},$$

applying equation (1.9), we have

$$A \equiv \sum_{n=0}^{\infty} \frac{k^{\delta}\,(\gamma)_{nq,k}\, z^n}{\Gamma_k(n\alpha+\beta+\delta k)(n!)} = k^{\delta}\, GE^{\gamma,q}_{k,\alpha,\beta+\delta k}(z),$$

Hence.

Corollary 3.2.1 For $k \in R; \alpha,\beta,\gamma,\delta \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0, R(\delta) > 0$, then equation (3.5) gives the result for $q = 1$,

$$k^{\delta}\, E^{\gamma}_{k,\alpha,\beta+\delta k}(z) = \frac{1}{\Gamma(\delta)} \int_0^1 u^{\frac{\beta}{k}-1}(1-u)^{\delta-1} E^{\gamma}_{k,\alpha,\beta}(z\, u^{\frac{\alpha}{k}})du. \tag{3.6}$$

which is new integral representation of K-Mittag-Leffler function $E^{\gamma}_{k,\alpha,\beta}(z)$, definded by [2].

Corollary 3.2.2 For $\alpha,\beta,\gamma,\delta \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0, R(\delta) > 0$ and $q \in N$, then equation (3.5) gives the result for $k = 1$,

$$E^{\gamma,q}_{\alpha,\beta+\delta}(z) = \frac{1}{\Gamma(\delta)} \int_0^1 u^{\beta-1}(1-u)^{\delta-1} E^{\gamma,q}_{\alpha,\beta}(z\, u^{\alpha})du. \tag{3.7}$$

Remark : Which is well known result obtained for Mittag-Leffler function $E^{\gamma,q}_{\alpha,\beta}(z)$, by ([8], equation (2.4.1), page 803).

Theorem 3.3 For $k \in R; \beta,\gamma \in C; R(\beta) > 0, R(\gamma) > 0\; and\; \alpha, q \in N$, then

$$GE^{\gamma,q}_{k,k\alpha,\beta}(z) = \frac{1}{\Gamma_k(\beta)} \prod_{i=1}^{q}\prod_{j=1}^{\alpha} \frac{\Gamma(b_j)}{\Gamma(a_i)\Gamma(b_j-a_i)}$$

$$\times \int_0^1 u^{a_i-1}(1-u)^{b_j-a_i-1} e^{(\frac{k^{(q-\alpha)}q^q}{\alpha^{\alpha}})uz} du, \tag{3.8}$$

where $a_i = \dfrac{\frac{\gamma}{k}+i-1}{q}$ and $b_j = \dfrac{\frac{\beta}{k}+j-1}{\alpha}$.

Proof : Using definition of Generalized K-Mittag- Leffler function, from equation (1.12),

$$A \equiv GE^{\gamma,q}_{k,k\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, z^n}{\Gamma_k(nk\alpha+\beta)(n!)},$$

using relation ([1], page 183);$(x)_{n,k} = \dfrac{\Gamma_k(x+nk)}{\Gamma_k(x)}$, we have

$$A \equiv \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, z^n}{(\beta)_{n\alpha,k}\, \Gamma_k(\beta)(n!)} = \sum_{n=0}^{\infty} D \frac{z^n}{\Gamma_k(\beta)(n!)}, \tag{3.9}$$

where $D \equiv \dfrac{(\gamma)_{nq,k}}{(\beta)_{n\alpha,k}}$,
using equation (1.11), we have

$$D \equiv \frac{(\gamma)_{nq,k}}{(\beta)_{n\alpha,k}} = \frac{k^{qn}(\frac{\gamma}{k})_{qn}}{k^{\alpha n}(\frac{\beta}{k})_{\alpha n}},$$

using the relation given by equation (1.13), we have

$$D \equiv \frac{k^{(q-\alpha)n} q^{qn} \prod_{i=1}^{q}(\frac{\frac{\gamma}{k}+i-1}{q})_n}{\alpha^{\alpha n} \prod_{j=1}^{\alpha}(\frac{\frac{\beta}{k}+j-1}{\alpha})_n},$$

$$let\ a_i = \frac{\frac{\gamma}{k}+i-1}{q}\ and\ b_j = \frac{\frac{\beta}{k}+j-1}{\alpha},$$

$$D \equiv (\frac{k^{(q-\alpha)}q^q}{\alpha^\alpha})^n \prod_{i=1}^{q}\prod_{j=1}^{\alpha} \frac{(a_i)_n}{(b_j)_n},$$

$$D \equiv (\frac{k^{(q-\alpha)}q^q}{\alpha^\alpha})^n \prod_{i=1}^{q}\prod_{j=1}^{\alpha} \frac{\Gamma(a_i+n)\Gamma(b_j)}{\Gamma(b_j+n)\Gamma(a_i)},$$

$$D \equiv (\frac{k^{(q-\alpha)}q^q}{\alpha^\alpha})^n \prod_{i=1}^{q}\prod_{j=1}^{\alpha} \frac{\Gamma(b_j)}{\Gamma(b_j-a_i)\Gamma(a_i)} \frac{\Gamma(a_i+n)\Gamma(b_j-a_i)}{\Gamma(b_j-a_i+a_i+n)},$$

using the definition of Beta function, equation (1.6), we have

$$D \equiv (\frac{k^{(q-\alpha)}q^q}{\alpha^\alpha})^n \prod_{i=1}^{q}\prod_{j=1}^{\alpha} \frac{\Gamma(b_j)}{\Gamma(b_j-a_i)\Gamma(a_i)} \int_0^1 u^{a_i+n-1}(1-u)^{b_j-a_i-1}du, \tag{3.10}$$

from equation (3.9) and (3.10), we have

$$A \equiv \frac{1}{\Gamma_k(\beta)} \prod_{i=1}^{q}\prod_{j=1}^{\alpha} \frac{\Gamma(b_j)}{\Gamma(b_j-a_i)\Gamma(a_i)} \int_0^1 u^{a_i-1}(1-u)^{b_j-a_i-1}$$

$$\times \sum_{n=0}^{\infty} \frac{z^n}{n!}(\frac{k^{(q-\alpha)}q^q}{\alpha^\alpha}u)^n du,$$

$$A \equiv \frac{1}{\Gamma_k(\beta)} \prod_{i=1}^{q}\prod_{j=1}^{\alpha} \frac{\Gamma(b_j)}{\Gamma(a_i)\Gamma(b_j-a_i)}$$

$$\times \int_0^1 u^{a_i-1}(1-u)^{b_j-a_i-1} e^{(\frac{k^{(q-\alpha)}q^q}{\alpha^\alpha})uz} du.$$

Hence.

Corollary 3.3.1 For $k \in R; \beta, \gamma \in C; R(\beta) > 0, R(\gamma) > 0\ and\ \alpha \in N$, then equation (3.8), gives the result for $q = 1$,

$$E^{\gamma}_{k,k\alpha,\beta}(z) = \frac{1}{\Gamma_k(\beta)} \prod_{j=1}^{\alpha} \frac{\Gamma(b_j)}{\Gamma(a_1)\Gamma(b_j-a_1)}$$

$$\times \int_0^1 u^{a_1-1}(1-u)^{b_j-a_1-1} e^{(\frac{k^{(1-\alpha)}}{\alpha^\alpha})uz} du, \tag{3.11}$$

where $a_1 = \frac{\gamma}{k}$ and $b_j = \frac{\frac{\beta}{k}+j-1}{\alpha}$,
which is new integral representation of K-Mittag-Leffler function $E^{\gamma}_{k,\alpha,\beta}(z)$, definded by [2].

Corollary 3.3.2 For $\beta,\gamma \in C; R(\beta) > 0, R(\gamma) > 0 \; and \; \alpha, q \in N$, then equation (3.8) gives the result for $k = 1$,

$$E^{\gamma,q}_{\alpha,\beta}(z) = \frac{1}{\Gamma(\beta)} \prod_{i=1}^{q} \prod_{j=1}^{\alpha} \frac{\Gamma(b_j)}{\Gamma(a_i)\Gamma(b_j - a_i)} \times \int_0^1 u^{a_i-1}(1-u)^{b_j-a_i-1} e^{(\frac{q^q}{\alpha^\alpha})uz} du, \tag{3.12}$$

where $a_i = \frac{\gamma+i-1}{q}$ and $b_j = \frac{\beta+j-1}{\alpha}$.
which is new integral representation of Mittag-Leffler function $E^{\gamma,q}_{\alpha,\beta}(z)$, definded by [8].

Theorem 3.4 For $k \in R; \alpha,\beta,\gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0 \; and \; q \in N$, then

$$GE^{\gamma,q}_{k,\alpha,\beta}(z) = \frac{1}{\Gamma(\frac{\gamma}{k})} \int_0^\infty e^{-t}\, t^{(\frac{\gamma}{k}-1)} GE^{1,0}_{k,\alpha,\beta}(zt^q k^q) dt. \tag{3.13}$$

Proof: Using definition of Generalized K- Mittag-Leffler function, equation (1.12), we have

$$GE^{\gamma,q}_{k,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, z^n}{\Gamma_k(n\alpha+\beta)(n!)},$$

using equation (1.11), we have

$$GE^{\gamma,q}_{k,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma_k(n\alpha+\beta)(n!)} \frac{k^{qn}\Gamma(\frac{\gamma}{k}+qn)}{\Gamma(\frac{\gamma}{k})},$$

using equation (1.5), we have

$$GE^{\gamma,q}_{k,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma_k(n\alpha+\beta)(n!)} \frac{k^{qn}}{\Gamma(\frac{\gamma}{k})} \int_0^\infty e^{-t}\, t^{(\frac{\gamma}{k}+qn-1)} dt,$$

$$GE^{\gamma,q}_{k,\alpha,\beta}(z) = \frac{1}{\Gamma(\frac{\gamma}{k})} \int_0^\infty e^{-t}\, t^{(\frac{\gamma}{k}-1)} \sum_{n=0}^{\infty} \frac{z^n\, k^{qn}\, t^{qn}}{\Gamma_k(n\alpha+\beta)(n!)} dt,$$

$$GE^{\gamma,q}_{k,\alpha,\beta}(z) = \frac{1}{\Gamma(\frac{\gamma}{k})} \int_0^\infty e^{-t}\, t^{(\frac{\gamma}{k}-1)} GE^{1,0}_{k,\alpha,\beta}(zt^q k^q) dt.$$

Hence.

Corollary 3.4.1 For $k \in R; \alpha,\beta,\gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0$, then equation (3.13) gives the result for $q = 1$,

$$E^{\gamma}_{k,\alpha,\beta}(z) = \frac{1}{\Gamma(\frac{\gamma}{k})} \int_0^\infty e^{-t}\, t^{(\frac{\gamma}{k}-1)} E^{1}_{k,\alpha,\beta}(ztk) dt, \tag{3.14}$$

which is new integral representation for K-Mittag-Leffler function $E^{\gamma}_{k,\alpha,\beta}(z)$, definded by [2].

Corollary 3.4.2 For $\alpha,\beta,\gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0$ and $q \in N$ then equation (3.13) gives the result for $k = 1$,

$$E^{\gamma,q}_{\alpha,\beta}(z) = \frac{1}{\Gamma(\gamma)} \int_0^\infty e^{-t}\, t^{(\gamma-1)} E^{1,0}_{\alpha,\beta}(z\, t^q) dt, \tag{3.15}$$

which is new integral representation for Mittag-Leffler function $E^{\gamma,q}_{\alpha,\beta}(z)$, definded by [8].

Theorem 3.5 For $k \in R; \alpha, \beta, \gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0\ and\ q \in N$, then

$$GE^{\gamma,q}_{k,kq,\beta}(z) = \frac{k^{1-\frac{\beta}{k}}}{\Gamma(\frac{\gamma}{k})\Gamma(\frac{\beta-\gamma}{k})} \int_0^1 u^{\frac{\gamma}{k}-1} (1-u)^{(\frac{\beta-\gamma}{k}-1)} e^{(u^q z)} du. \tag{3.16}$$

Proof: Using definition of Generalized K- Mittag-Leffler function, equation (1.12), we have

$$A \equiv GE^{\gamma,q}_{k,kq,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, z^n}{\Gamma_k(nkq+\beta)(n!)},$$

using relation ([1], page 183);$(x)_{n,k} = \dfrac{\Gamma_k(x+nk)}{\Gamma_k(x)}$, we have

$$A \equiv \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, z^n}{(\beta)_{nq,k}\,\Gamma_k(\beta)(n!)} = \sum_{n=0}^{\infty} D \frac{z^n}{\Gamma_k(\beta)(n!)}, \tag{3.17}$$

where $D \equiv \dfrac{(\gamma)_{nq,k}}{(\beta)_{nq,k}}$,

using equation (1.11), we have

$$D \equiv \frac{(\gamma)_{nq,k}}{(\beta)_{nq,k}} = \frac{k^{qn}(\frac{\gamma}{k})_{nq}}{k^{nq}(\frac{\beta}{k})_{nq}},$$

$$D \equiv \frac{\Gamma(\frac{\gamma}{k}+nq)\Gamma(\frac{\beta}{k})}{\Gamma(\frac{\beta}{k}+nq)\Gamma(\frac{\gamma}{k})} = \frac{\Gamma(\frac{\beta}{k})}{\Gamma(\frac{\gamma}{k})\Gamma(\frac{\beta}{k}-\frac{\gamma}{k})} \frac{\Gamma(\frac{\gamma}{k}+nq)\Gamma(\frac{\beta}{k}-\frac{\gamma}{k})}{\Gamma(\frac{\gamma}{k}+qn+\frac{\beta}{k}-\frac{\gamma}{k})},$$

using the definition of Beta function, equation (1.6), we have

$$D \equiv \frac{\Gamma(\frac{\beta}{k})}{\Gamma(\frac{\gamma}{k})\Gamma(\frac{\beta-\gamma}{k})} \int_0^1 u^{\frac{\gamma}{k}+qn-1} (1-u)^{(\frac{\beta-\gamma}{k}-1)} du, \tag{3.18}$$

using equation (3.18) in (3.17), we have

$$A \equiv \sum_{n=0}^{\infty} \frac{z^n}{\Gamma_k(\beta)(n!)} \frac{\Gamma(\frac{\beta}{k})}{\Gamma(\frac{\gamma}{k})\Gamma(\frac{\beta-\gamma}{k})} \int_0^1 u^{\frac{\gamma}{k}+qn-1} (1-u)^{(\frac{\beta-\gamma}{k}-1)} du,$$

$$A \equiv \frac{\Gamma(\frac{\beta}{k})}{\Gamma_k(\beta)\Gamma(\frac{\gamma}{k})\Gamma(\frac{\beta-\gamma}{k})} \int_0^1 u^{\frac{\gamma}{k}-1} (1-u)^{(\frac{\beta-\gamma}{k}-1)} \sum_{n=0}^{\infty} \frac{(u^q z)^n}{(n!)} du,$$

$$A \equiv \frac{k^{1-\frac{\beta}{k}}}{\Gamma(\frac{\gamma}{k})\Gamma(\frac{\beta-\gamma}{k})} \int_0^1 u^{\frac{\gamma}{k}-1} (1-u)^{(\frac{\beta-\gamma}{k}-1)} e^{(u^q z)} du.$$

Hence.

Corollary 3.5.1 For $k \in R; \alpha, \beta, \gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0$, then equation (3.16) gives the result for $q = 1$,

$$E^{\gamma}_{k,k,\beta}(z) = \frac{k^{1-\frac{\beta}{k}}}{\Gamma(\frac{\gamma}{k})\Gamma(\frac{\beta-\gamma}{k})} \int_0^1 u^{\frac{\gamma}{k}-1} (1-u)^{(\frac{\beta-\gamma}{k}-1)} e^{(uz)} du. \tag{3.19}$$

which is new integral representation for K-Mittag-Leffler function $E^{\gamma}_{k,\alpha,\beta}(z)$, definded by [2].

Corollary 3.5.2 For $\alpha, \beta, \gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0\ and\ q \in N$, then equation (3.16) gives the result for $k = 1$,

$$E^{\gamma,q}_{q,\beta}(z) = \frac{1}{\Gamma(\gamma)\Gamma(\beta-\gamma)} \int_0^1 u^{\gamma-1}\,(1-u)^{(\beta-\gamma-1)} e^{(u^q z)} du. \tag{3.20}$$

which is new integral representation for Mittag-Leffler function $E^{\gamma,q}_{\alpha,\beta}(z)$, definded by [8].

Theorem 3.6 For $k \in R; \alpha, \beta, \gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0\ and\ q \in N$, then

$$z^{\beta} k^{-\frac{\beta}{k}} GE^{\gamma,q}_{k,0,1}(z^{\alpha}) = \int_0^{\infty} u^{\beta-1}\, e^{-(\frac{u}{z})^k} GE^{\gamma,q}_{k,\alpha,\beta}(u^{\alpha}\, k^{(\frac{\alpha}{k})}) du. \tag{3.21}$$

Proof. Consider the right side integral

$$A \equiv \int_0^{\infty} u^{\beta-1}\, e^{-(\frac{u}{z})^k} GE^{\gamma,q}_{k,\alpha,\beta}(u^{\alpha}\, k^{(\frac{\alpha}{k})}) du,$$

using definition of Generalized K- Mittag-Leffler function equation (1.12), we have

$$A \equiv \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, k^{\frac{\alpha n}{k}}}{\Gamma_k(n\alpha+\beta)(n!)} \int_0^{\infty} u^{\beta+\alpha n-1}\, e^{-(\frac{u}{z})^k} du,$$

put $(\frac{u}{k})^k = t$, we have

$$A \equiv \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, k^{\frac{\alpha n}{k}}}{\Gamma_k(n\alpha+\beta)(n!)} \frac{z^{\beta+\alpha n}}{k} \int_0^{\infty} t^{\frac{\beta+\alpha n}{k}-1}\, e^{-t} dt,$$

from definition of Gamma function, equation (1.5), we have

$$A \equiv \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, k^{\frac{\alpha n}{k}}}{\Gamma_k(n\alpha+\beta)(n!)} \frac{z^{\beta+\alpha n}}{k} \Gamma(\frac{\beta+\alpha n}{k}),$$

using equation (1.9), we have

$$A \equiv \frac{z^{\beta}}{k^{\frac{\beta}{k}}} \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}\, z^{\alpha n}}{(n!)} = z^{\beta} k^{-\frac{\beta}{k}} GE^{\gamma,q}_{k,0,1}(z^{\alpha}),$$

Hence.

Corollary 3.6.1 For $k \in R; \alpha, \beta, \gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0$, then equation (3.21) gives the result for $q = 1$,

$$z^{\beta} k^{-\frac{\beta}{k}} E^{\gamma}_{k,0,1}(z^{\alpha}) = \int_0^{\infty} u^{\beta-1}\, e^{-(\frac{u}{z})^k} E^{\gamma}_{k,\alpha,\beta}(u^{\alpha}\, k^{(\frac{\alpha}{k})}) du, \tag{3.22}$$

which is new integral representation for K-Mittag-Leffler function $E^{\gamma}_{k,\alpha,\beta}(z)$, definded by [2].

Corollary 3.6.2 For $\alpha, \beta, \gamma \in C; R(\alpha) > 0, R(\beta) > 0, R(\gamma) > 0\ and\ q \in N$, then equation (3.21) gives the result for $k = 1$,

$$z^{\beta} E^{\gamma,q}_{0,1}(z^{\alpha}) = \int_0^{\infty} u^{\beta-1}\, e^{-(\frac{u}{z})} E^{\gamma,q}_{\alpha,\beta}(u^{\alpha}) du, \tag{3.23}$$

which is new integral representation for Mittag-Leffler function $E^{\gamma,q}_{\alpha,\beta}(z)$, definded by [8].

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