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Coefficient Estimates for Certain Subclasses of Meromorphically Bi-Univalent Functions

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Abstract. In the present investigation, we obtain the estimates on the initial Taylor-Maclaurin coefficients for functions in two new subclasses of meromorphically bi-univalent functions defined on the domain Δ given by

$$\Delta = \{z : z \in \mathbb{C} \quad \text{and} \quad 1 < |z| < \infty\}.$$

Several other closely-related earlier results are also indicated.

1 Introduction

Let $\mathcal{A}$ denote the class of functions $f(z)$ of the form:

$$f(z) = z + \sum_{k=2}^{\infty} a_k \, z^k, \tag{1.1}$$

which are analytic in the open unit open disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

We denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ which consists of functions of the form (1.1), that is, functions which are analytic and univalent in $\mathbb{U}$ and are normalized by the following conditions:

$$f(0) = 0 \qquad \text{and} \qquad f'(0) = 1.$$

A function $f \in \mathcal{S}$ is said to be starlike of order α $(0 \leqq \alpha < 1)$ in $\mathbb{U}$ if and only if

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha \qquad (z \in \mathbb{U};\ 0 \leqq \alpha < 1)$$

and convex of order α $(0 \leqq \alpha < 1)$ in $\mathbb{U}$ if and only if

$$\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha \qquad (z \in \mathbb{U};\ 0 \leqq \alpha < 1).$$

As usual, we denote these subclasses of $\mathcal{S}$ by $\mathcal{S}^*(\alpha)$ and $\mathcal{K}(\alpha)$, respectively.

Let Σ denote the class of meromorphically univalent functions $g(z)$ of the form:

$$g(z) = z + b_0 + \sum_{n=1}^{\infty} \frac{b_n}{z^n}, \tag{1.2}$$

which are defined on the domain Δ given by

$$\Delta = \{z : z \in \mathbb{C} \quad \text{and} \quad 1 < |z| < \infty\}.$$

Since $g \in \Sigma$ is univalent, it has an inverse $g^{-1} = h$ that satisfies the following condition:

$$g^{-1}\big(g(z)\big) = z \qquad (z \in \Delta)$$

and

$$g\big(g^{-1}(w)\big) = w \qquad (0 < M < |w| < \infty),$$

where

$$g^{-1}(w) = h(w) = w + B_0 + \sum_{n=1}^{\infty} \frac{B_n}{w^n} \qquad (0 < M < |w| < \infty). \tag{1.3}$$

A simple computation shows that

$$w = g\big(h(w)\big) = (b_0 + B_0) + w + \frac{b_1 + B_1}{w} + \frac{B_2 - b_1 B_0 + b_2}{w^2} + \frac{B_3 - b_1 B_1 + b_1 B_0^2 - 2b_2 B_0 + b_3}{w^3} + \cdots . \tag{1.4}$$

Comparing the initial coefficients in (1.4), we find that

$$\begin{aligned} b_0 + B_0 = 0 &\quad \Longrightarrow \quad B_0 = -b_0 \\ b_1 + B_1 = 0 &\quad \Longrightarrow \quad B_1 = -b_1 \\ B_2 - b_1 B_0 + b_2 = 0 &\quad \Longrightarrow \quad B_2 = -(b_2 + b_0 b_1) \\ B_3 - b_1 B_1 + b_1 B_0^2 - 2b_2 B_0 + b_3 = 0 &\quad \Longrightarrow \quad B_3 = -(b_3 + 2b_0 b_2 + b_0^2 b_1 + b_1^2). \end{aligned}$$

By putting these values in the equation (1.3), we get

$$g^{-1}(w) = h(w) = w - b_0 - \frac{b_1}{w} - \frac{b_2 + b_0 b_1}{w^2} - \frac{b_3 + 2b_0 b_2 + b_0^2 b_1 + b_1^2}{w^3} + \cdots . \tag{1.5}$$

A systematic study of the class $\mathbf{\Sigma}$ of bi-univalent *analytic* functions in $\mathbb{U}$, which was introduced in 1967 by Lewin [12], was revived in recent years by Srivastava *et al.* [14]. Ever since then, several authors investigated various subclasses of of the class $\mathbf{\Sigma}$ of bi-univalent analytic functions and found estimates on the initial Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions in these subclasses (see, for example, [2, 3, 4, 6, 13, 15, 17, 18]; see also [16] and the references cited therein). In our present investigation, the concept of bi-univalency is extended to the class Σ of meromorphic functions defined on Δ.

The function $g(z) \in \Sigma$ given by (1.2) is said to be *meromorphically* bi-univalent in Δ if both g and its inverse $g^{-1} = h$ are meromorphically univalent in Δ. The class of all meromorphically bi-univalent functions is denoted by Σ_B.

Estimates on the coefficients of meromorphically univalent functions were widely investigated in the literature on *Geometric Function Theory*. Recently, several researchers such as (for example) Halim *et al.* [8], Janani and Murugusundaramoorthy [11] and Hamidi *et al.* [9, 10] introduced new subclasses of meromorphically bi-univalent functions and obtained estimates on the initial coefficients for functions in each of these subclasses.

Babalola [1] defined the class $\mathscr{L}_\lambda(\beta)$ of λ-pseudo-starlike functions of order β as follows:

Definition 1 (see [1]). Let $f \in \mathcal{A}$ and suppose that $0 \leqq \beta < 1$ and $\lambda \geqq 1$. Then $f(z) \in \mathscr{L}_\lambda(\beta)$ of λ-pseudo-starlike functions of order β in $\mathbb{U}$ if and only if

$$\Re\left(\frac{z\,[f'(z)]^\lambda}{f(z)}\right) > \beta \qquad (z \in \mathbb{U};\ 0 \leqq \beta < 1;\ \lambda \geqq 1). \tag{1.6}$$

In particular, Babalola [1] proved that all λ-pseudo-starlike functions are Bazilevič of type $1-\frac{1}{\lambda}$ and order $\beta^{\frac{1}{\lambda}}$ and are univalent in open unit disk $\mathbb{U}$.

Motivated by the aforecited works, in our present investigation, we introduce two new subclasses of the class Σ_B of meromorphically bi-univalent functions and obtained the estimates for the initial coefficients $|b_0|$ and $|b_1|$ of functions in these subclasses.

In order to derive our main results, we recall here the following lemma.

Carathéodory's Lemma (see, for example, [7]; see also [5, p. 41]). *If* $\mathfrak{p} \in \mathcal{P}$, *then*

$$|\mathfrak{p}_j| \leqq 2 \qquad (j \in \mathbb{N}),$$

where $\mathcal{P}$ *is the family of all functions* $\mathfrak{p}(z)$, *analytic in* Δ, *for which*

$$\Re\big(\mathfrak{p}(z)\big) > 0 \qquad (z \in \Delta),$$

where

$$\mathfrak{p}(z) = 1 + \frac{\mathfrak{p}_1}{z} + \frac{\mathfrak{p}_2}{z^2} + \frac{\mathfrak{p}_3}{z^3} + \cdots \qquad (z \in \Delta),$$

$\mathbb{N}$ *being the set of positive integers.*

2 Coefficient Bounds for the Function Class $\Sigma_{B,\lambda^*}(\alpha)$

We begin by defining the function class $\Sigma_{B,\lambda^*}(\alpha)$ as follows.

Definition 2. A function $g(z) \in \varsigma_B$ given by (1.2) is said to be in the class $\Sigma_{B,\lambda^*}(\alpha)$ if the following conditions are satisfied:

$$\left|\arg\left(\frac{z[g'(z)]^{\lambda}}{g(z)}\right)\right| < \frac{\alpha\pi}{2} \qquad (z \in \Delta;\ 0 < \alpha \leqq 1;\ \lambda \geqq 1) \tag{2.1}$$

and

$$\left|\arg\left(\frac{w[h'(w)]^{\lambda}}{h(w)}\right)\right| < \frac{\alpha\pi}{2} \qquad (w \in \Delta;\ 0 < \alpha \leqq 1;\ \lambda \geqq 1), \tag{2.2}$$

where the function h is inverse of the function g given by

$$h(w) = w - b_0 - \frac{b_1}{w} - \frac{b_2 + b_0 b_1}{w^2} - \frac{b_3 + 2b_0 b_2 + b_0^2 b_1 + b_1^2}{w^3} + \cdots.$$

We call $\Sigma_{B,\lambda^*}(\alpha)$ the class of meromorphically strongly λ-pseudo-starlike bi-univalent functions of order α in Δ.

The estimates on the coefficients $|b_0|$ and $|b_1|$ for the function class $\Sigma_{B,\lambda^*}(\alpha)$ are given by Theorem 2.1 below.

Theorem 2.1. *Let* $g(z)$ *given by* (1.2) *be in the class* $\Sigma_{B,\lambda^*}(\alpha)$. *Then*

$$|b_0| \leqq 2\alpha \tag{2.3}$$

and

$$|b_1| \leqq \frac{2\sqrt{5}\,\alpha^2}{1+\lambda}. \tag{2.4}$$

Proof. Let $g \in \Sigma_{B,\lambda^*}(\alpha)$. Then, by Definition 2 of meromorphically bi-univalent function class $\Sigma_{B,\lambda^*}(\alpha)$, the conditions (2.1) and (2.2) can be rewritten as follows:

$$\frac{z[g'(z)]^{\lambda}}{g(z)} = [p(z)]^{\alpha} \tag{2.5}$$

and

$$\frac{w[h'(w)]^{\lambda}}{h(w)} = [q(w)]^{\alpha}, \tag{2.6}$$

respectively. Here, and in what follows, the functions $p(z) \in \mathcal{P}$ and $q(w) \in \mathcal{P}$ have the following forms:

$$p(z) = 1 + \frac{p_1}{z} + \frac{p_2}{z^2} + \frac{p_3}{z^3} + \cdots \qquad (z \in \Delta)$$

and

$$q(w) = 1 + \frac{q_1}{w} + \frac{q_2}{w^2} + \frac{q_3}{w^3} + \cdots \qquad (w \in \Delta).$$

Clearly, we have

$$\begin{aligned}[p(z)]^{\alpha} = 1 &+ \frac{\alpha\, p_1}{z} + \frac{\frac{1}{2}\,\alpha(\alpha-1)\, p_1^2 + \alpha\, p_2}{z^2} \\ &+ \frac{\frac{1}{6}\,\alpha(\alpha-1)(\alpha-2)\, p_1^3 + \alpha(\alpha-1)\, p_1 p_2 + \alpha\, p_3}{z^3} + \cdots\end{aligned}$$

and

$$\begin{aligned}[q(w)]^{\alpha} = 1 &+ \frac{\alpha\, q_1}{w} + \frac{\frac{1}{2}\,\alpha(\alpha-1)\, q_1^2 + \alpha\, q_2}{w^2} \\ &+ \frac{\frac{1}{6}\,\alpha(\alpha-1)(\alpha-2)\, q_1^3 + \alpha(\alpha-1)\, q_1 q_2 + \alpha\, q_3}{w^3} + \cdots.\end{aligned}$$

We also find that

$$\begin{aligned}\frac{z[g'(z)]^{\lambda}}{g(z)} = 1 &- \frac{b_0}{z} + \frac{b_0^2 - (1+\lambda)b_1}{z^2} \\ &+ \frac{b_0^3 - (2+\lambda)b_0 b_1 + (1+2\lambda)b_2}{z^3} + \cdots\end{aligned}$$

and

$$\begin{aligned}\frac{w[h'(w)]^{\lambda}}{h(w)} = 1 &+ \frac{b_0}{w} + \frac{b_0^2 + (1+\lambda)b_1}{w^2} \\ &+ \frac{b_0^3 + 3(1+\lambda)b_0 b_1 + (1+2\lambda)b_2}{w^3} + \cdots.\end{aligned}$$

Now, equating the coefficients in (2.5) and (2.6), we get

$$-\, b_0 = \alpha p_1, \tag{2.7}$$

$$b_0^2 - (1+\lambda)b_1 = \frac{1}{2}\,\alpha(\alpha-1)\, p_1^2 + \alpha\, p_2, \tag{2.8}$$

$$b_0 = \alpha q_1 \tag{2.9}$$

and

$$b_0^2 + (1+\lambda)b_1 = \frac{1}{2}\,\alpha(\alpha-1)\, q_1^2 + \alpha\, q_2. \tag{2.10}$$

From (2.7) and (2.9), we find that

$$p_1 = -q_1 \tag{2.11}$$

and

$$2b_0^2 = \alpha^2\left(p_1^2 + q_1^2\right), \tag{2.12}$$

that is,

$$b_0^2 = \frac{\alpha^2(p_1^2 + q_1^2)}{2}.$$

Applying Carathéodory's Lemma for the coefficients p_1 and q_1, we immediately have

$$|b_0^2| \leqq \frac{\alpha^2(4+4)}{2} \quad \Longrightarrow \quad |b_0| \leqq 2\alpha.$$

This gives the bound on $|b_0|$ as asserted in (2.3).

Next, in order to find the bound on $|b_1|$, by using the equation (2.8) and the equation (2.10), we get

$$\begin{aligned}&\left[b_0^2 - (1+\lambda)b_1\right] \cdot \left[b_0^2 + (1+\lambda)b_1\right]\\ &\qquad = \left(\frac{1}{2}\alpha(\alpha-1)p_1^2 + \alpha p_2\right) \cdot \left(\frac{1}{2}\alpha(\alpha-1)q_1^2 + \alpha q_2\right),\end{aligned}$$

$$\begin{aligned}b_0^4 - (1+\lambda)^2\, b_1^2 = \frac{1}{4}\,\alpha^2(\alpha-1)^2 p_1^2 q_1^2&\\ + \frac{1}{2}\,\alpha^2(\alpha-1)(p_2 q_1^2 + p_1^2 q_2) + \alpha^2 p_2 q_2,&\end{aligned}$$

$$\begin{aligned}(1+\lambda)^2\, b_1^2 = (b_0^2)^2 - \frac{1}{4}\,\alpha^2(\alpha-1)^2 p_1^2 q_1^2&\\ - \frac{1}{2}\,\alpha^2(\alpha-1)(p_2 q_1^2 + p_1^2 q_2) - \alpha^2 p_2 q_2&\end{aligned}$$

and

$$\begin{aligned}(1+\lambda)^2\, b_1^2 = \frac{1}{4}\,\alpha^4(p_1^2 + q_1^2)^2 - \frac{1}{4}\,\alpha^2(\alpha-1)^2 p_1^2 q_1^2&\\ - \frac{1}{2}\,\alpha^2(\alpha-1)(p_2 q_1^2 + p_1^2 q_2) - \alpha^2 p_2 q_2.&\end{aligned}$$

Applying Carathéodory's Lemma once again for the coefficients p_1, q_1, p_2 and q_2, we get

$$\begin{aligned}(1+\lambda)^2 |b_1^2| \leqq \frac{1}{4}\,\alpha^4(4+4)^2 + \frac{1}{4}\alpha^2(\alpha-1)^2(16)&\\ + \frac{1}{2}\,\alpha^2(\alpha-1)(8+8) + \alpha^2(4),&\end{aligned}$$

that is,

$$|b_1^2| \leqq \frac{20\alpha^4}{(1+\lambda)^2} \quad \Longrightarrow \quad |b_1| \leqq \frac{2\sqrt{5}\,\alpha^2}{1+\lambda},$$

which evidently completes the proof of Theorem 1. □

3 Coefficient Bounds for the Function Class $\Sigma_{B^*}(\lambda, \beta)$

We first introduce the function class $\Sigma_{B^*}(\lambda, \beta)$ as follows.

Definition 3. A function $g(z) \in \Sigma_B$ given by (1.2) is said to be in the class $\Sigma_{B^*}(\lambda, \beta)$ if the following conditions are satisfied:

$$\Re\left(\frac{z[g'(z)]^\lambda}{g(z)}\right) > \beta \qquad (z \in \Delta;\ 0 \leqq \beta < 1;\ \lambda \geqq 1) \tag{3.1}$$

and

$$\Re\left(\frac{w[h'(w)]^\lambda}{h(w)}\right) > \beta \qquad (w \in \Delta;\ 0 \leqq \beta < 1;\ \lambda \geqq 1), \tag{3.2}$$

where the function h is inverse of the function g given by

$$h(w) = w - b_0 - \frac{b_1}{w} - \frac{b_2 + b_0 b_1}{w^2} - \frac{b_3 + 2b_0 b_2 + b_0^2 b_1 + b_1^2}{w^3} + \cdots .$$

We call $\Sigma_{B^*}(\lambda, \beta)$ the class of meromorphically λ-pseudo-starlike bi-univalent functions of order α.

We now derive the estimates on the coefficients $|b_0|$ and $|b_1|$ for the meromorphically bi-univalent function class $\Sigma_{B^*}(\lambda, \beta)$.

Theorem 3.1. *Let $g(z)$ given by (1.2) be in the class $\Sigma_{B^*}(\lambda, \beta)$. Then*

$$|b_0| \leqq 2(1-\beta) \tag{3.3}$$

and

$$|b_1| \leqq \frac{2(1-\beta)\sqrt{4\beta^2 - 8\beta + 5}}{1+\lambda}. \tag{3.4}$$

Proof. Let $g \in \Sigma_{B^*}(\lambda, \beta)$. Then, Definition 3 of the meromorphically bi-univalent function class $\Sigma_{B^*}(\lambda, \beta)$, the conditions (3.1) and (3.2) can be rewritten as follows:

$$\frac{z[g'(z)]^\lambda}{g(z)} = \beta + (1-\beta)p(z) \tag{3.5}$$

and

$$\frac{w[h'(w)]^\lambda}{h(w)} = \beta + (1-\beta)q(w), \tag{3.6}$$

respectively. Here, just as in our proof of Theorem 1, the functions $p(z) \in \mathcal{P}$ and $q(w) \in \mathcal{P}$ have the following forms:

$$p(z) = 1 + \frac{p_1}{z} + \frac{p_2}{z^2} + \frac{p_3}{z^3} + \cdots \qquad (z \in \Delta)$$

and

$$q(w) = 1 + \frac{q_1}{w} + \frac{q_2}{w^2} + \frac{q_3}{w^3} + \cdots \qquad (w \in \Delta).$$

Clearly, we have

$$\beta + (1-\beta)p(z) = 1 + \frac{(1-\beta)p_1}{z} + \frac{(1-\beta)p_2}{z^2} + \frac{(1-\beta)p_3}{z^3} + \cdots$$

and

$$\beta + (1-\beta)q(w) = 1 + \frac{(1-\beta)q_1}{w} + \frac{(1-\beta)q_2}{w^2} + \frac{(1-\beta)q_3}{w^3} + \cdots .$$

We also find that

$$\frac{z[g'(z)]^\lambda}{g(z)} = 1 - \frac{b_0}{z} + \frac{b_0^2 - (1+\lambda)b_1}{z^2} + \frac{b_0^3 - (2+\lambda)b_0 b_1 + (1+2\lambda)b_2}{z^3} + \cdots$$

and

$$\frac{w[h'(w)]^\lambda}{h(w)} = 1 + \frac{b_0}{w} + \frac{b_0^2 + (1+\lambda)b_1}{w^2} + \frac{b_0^3 + 3(1+\lambda)b_0 b_1 + (1+2\lambda)b_2}{w^3} + \cdots .$$

Now, equating the coefficients in (3.5) and (3.6), we get

$$-b_0 = (1-\beta)p_1, \tag{3.7}$$

$$b_0^2 - (1+\lambda)b_1 = (1-\beta)p_2, \tag{3.8}$$

$$b_0 = (1-\beta)q_1 \tag{3.9}$$

and

$$b_0^2 + (1+\lambda)b_1 = (1-\beta)q_2. \tag{3.10}$$

From (3.7) and (3.9), we obtain

$$p_1 = -q_1 \tag{3.11}$$

and

$$2\, b_0^2 = (1-\beta)^2 \left(p_1^2 + q_1^2\right), \tag{3.12}$$

which readily yields

$$b_0^2 = \frac{(1-\beta)^2 \left(p_1^2 + q_1^2\right)}{2}.$$

Applying Carathéodory's Lemma for the coefficients p_1 and q_1, we immediately have

$$|b_0^2| \leqq \frac{(1-\beta)^2(4+4)}{2} \quad \Longrightarrow \quad |b_0| \leqq 2(1-\beta).$$

This gives the bound on $|b_0|$ as asserted in (3.3).

Next, in order to find the bound on $|b_1|$, by using the equation (3.8) and the equation (3.10), we get

$$\left[b_0^2 - (1+\lambda)b_1\right] \cdot \left[b_0^2 + (1+\lambda)b_1\right] = \left[(1-\beta)p_2\right] \cdot \left[(1-\beta)q_2\right],$$

$$b_0^4 - (1+\lambda)^2\, b_1^2 = (1-\beta)^2\, p_2 q_2,$$

$$(1+\lambda)^2\, b_1^2 = \left(b_0^2\right)^2 - (1-\beta)^2\, p_2 q_2$$

and

$$(1+\lambda)^2\, b_1^2 = \frac{1}{4}\,(1-\beta)^4 \left(p_1^2 + q_1^2\right)^2 - (1-\beta)^2\, p_2 q_2.$$

Applying Carathéodory's Lemma once again for the coefficients p_1, q_1, p_2 and q_2, we get

$$(1+\lambda)^2\,|b_1^2| \leqq \frac{1}{4}\,(4+4)^2\cdot(1-\beta)^4+4\cdot(1-\beta)^2,$$

which readily yields the following inequality:

$$|b_1| \leqq \frac{2(1-\beta)\sqrt{4\beta^2-8\beta+5}}{1+\lambda}.$$

This completes the proof of Theorem 2. □

Remark. By suitably specializing the various parameters involved in the assertions of Theorem 1 and Theorem 2, we can deduce the corresponding coefficient estimates for several simpler meromorphically bi-univalent function classes. The details involved are being left for the interested reader.

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