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On the Conharmonic Curvature Tensor of LP-Sasakian Manifolds

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Abstract The notions of conharmonically pseudosymmetric, conharmonically ϕ-symmetric, ϕ-conharmonically flat and partially Ricci-pseudosymmetric LP-Sasakian manifolds have been introduced and the properties of these structures have been discussed.

1 Introduction

In 1989, Matsumoto [7] introduced the notion of LP-Sasakian manifolds.Then the same notion has been introduced by I. Mihai and R. Rosca independently and obtained interesting results. These manifolds have also been studied by Aqeel et al.[1], Bagewadi et al. [2], De et al. [3], Mihai et al. [9], Murathan et al. [10], Shaikh et al. [13, 14, 15] and others.

The object of the present paper is to study LP-Sasakian manifolds satisfyinag certain conditions on the conharmonc curvature tensor. Section 2 is devoted to preliminaries. In section 3 we study conharmonically pseudosymmetric LP-Sasakian manifolds and proved that every LP-Sasakian manifold is conharmonically pseudosymmetric of the form $R\cdot\tilde{C} = Q(g, \tilde{C})$. In section 4, we study conharmonically ϕ-symmetric LP-Sasakian manifolds. Section 5 is devoted to the study of ϕ-conharmonically flat LP-Sasakian manifolds. Here it is prove that, ϕ-conharmonically flat LP-Sasakian manifold is an η-Einstein manifold. In section 6, we investigate partially Ricci-pseudosymmetric LP-Sasakian manifolds and proved that such a manifold is an Einstein manifold.

2 Preliminaries

An n-dimensional differentiable manifold M is said to be an LP-Sasakian manifold [7] if it admits a $(1, 1)$ tensor field ϕ, a unit timelike contravariant vector field ξ, a 1-form η and a Lorentzian metric g which satisfy

$$\eta(\xi) = -1, \quad g(X, \xi) = \eta(X), \quad \phi^2 X = X + \eta(X)\xi, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \quad \nabla_X \xi = \phi X, \tag{2.2}$$

$$(\nabla_X \phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.3}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g.It can be easily seen that in an LP-Sasakian manifold, the following relations hold:

$$\phi\xi = 0, \ \eta(\phi X) = 0, \ rank\,\phi = n - 1. \tag{2.4}$$

Again, if we put

$$\Omega(X, Y) = g(X, \phi Y)$$

for any vector fields X, Y, then the tensor field $\Omega(X, Y)$ is a symmetric $(0, 2)$ tensor field [7]. Also, since the vector field η is closed in an LP-Sasakian manifold, we have ([7, 8])

$$(\nabla_X \eta)(Y) = \Omega(X, Y), \ \Omega(X, \xi) = 0 \tag{2.5}$$

for any vector fields X and Y.

Let M be an n-dimensional LP-Sasakian manifold with structure (ϕ, ξ, η, g). Then the following relations hold ([7]) :

$$g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) = g(Y,Z)\eta(X) - g(X,Z)\eta(Y), \tag{2.6}$$

$$R(\xi,X)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.7}$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.8}$$

$$S(X,\xi) = (n-1)\eta(X), \tag{2.9}$$

$$S(\phi X,\phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \tag{2.10}$$

for any vector fields X, Y, Z, where R is the Riemannian curvature tensor and S is the Ricci tensor of the manifold.

An LP-Sasakian manifold M is said to be an η-Einstein if its Ricci tensor S of type $(0,2)$ is of the form

$$S(X,Y) = \alpha g(X,Y) + \beta\eta(X)\eta(Y) \tag{2.11}$$

for any vector fields X, Y where α, β are smooth functions on M. In particular, if $\beta = 0$, then the manifold is said to be an Einstein manifold.

A rank four tensor $\tilde{C}$ that remains invariant under conharmonic transformation for an $2n+1$-dimensional Riemannian manifold M^{2n+1}, is given by [6]

$$\begin{aligned} \tilde{C}(X,Y,Z,W) &= \tilde{R}(X,Y,Z,W) \\ &\quad -\frac{1}{2n-1}[g(Y,Z)S(X,W) - g(X,Z)S(Y,W) \\ &\quad +S(Y,Z)g(X,W) - S(X,Z)g(Y,W)]. \end{aligned} \tag{2.12}$$

where $\tilde{R}$ denotes the Riemannian curvature tensor of type $(0,4)$ and $\tilde{C}$ denotes the conharmonic curvature tensor of type $(0,4)$ defined by

$$\tilde{R}(X,Y,Z,W) = g(R(X,Y)Z,W),$$

$$\tilde{C}(X,Y,Z,W) = g(\tilde{C}(X,Y)Z,W)$$

.

where R is the Riemannian curvature tensor of type $(1,3)$ and S denotes the Ricci tensor of type $(0,2)$. Conharmonic curvature tensor have been studied by Siddiqui et al. [16], Praksaha et al. [12], Ghosh et al. [5], Taleshian et al. [20] and many others.

We recall the following theorem due to Taleshian et al [20].

Theorem 2.1. *A conharmonically flat LP-Sasakian manifold is locally isometric with the unit sphere $S^n(1)$, where S is a Lorentzian manifold of sectional curvature one.*

The above result will be useful in next section.

3 Conharmonically pseudosymmetric LP-Sasakian manifold

A Riemannain manifold M is called locally symmetric if its curvature tensor R is parallel, that is, $\nabla R = 0$, where ∇ denotes the Levi-Civita connection. As a proper generalization of locally symmetric manifolds the notion of semisymmetric manifolds was defined by

$$(R(X,Y)\cdot R)(U,V)W = 0, \quad X,Y,U,V,W \in \chi(M) \tag{3.1}$$

and studied by many authors [11, 21]. Here $\chi(M)$ being the Lie algebra of all differentiable vector fields on M. A complete intrinsic classification of these spaces was given by Z.I. Szabo [17]. For a $(0,k)$-tensor field T on M, $k \geq 1$, and a symmetric $(0,2)$-tensor field A on M, we define the $(0,k+2)$-tensor fields $R\cdot T$ and $Q(A,T)$ by

$$\begin{aligned} & (R\cdot T)(X_1,...,X_K;X,Y) \\ = \; & -T(R(X,Y)X_1,X_2,...,X_k) - ... - T(X_1,...,X_{k-1},R(X,Y)X_k) \\ & Q(A,T)(X_1,...,X_K;X,Y) \\ = \; & -T((X\wedge_A Y)X_1,X_2,...,X_k) - ... - T(X_1,...,X_{k-1},(X\wedge_A Y)X_k) \end{aligned}$$

where $X \wedge_A Y$ is the endomorphism given by

$$(X \wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y. \tag{3.2}$$

A Riemannian manifold M is said to be pseudosymmetric (in the sense of R. Deszcz [4]) if

$$R \cdot R = L_R Q(g,R)$$

holds on $U_R = \{x \in M | R - \frac{r}{n(n-1)} G \neq 0 \ \ at \ \ x\}$, where G is the $(0,4)$-tensor defined by $G(X_1,X_2,X_3,X_4) = g((X_1 \wedge X_2)X_3, X_4)$ and L_R is some smooth function on U_R. A Riemannian manifold M is said to be conharmonically pseudosymmetric if

$$R \cdot \tilde{C} = L_{\tilde{C}} Q(g, \tilde{C}) \tag{3.3}$$

holds on the set $U_{\tilde{C}} = \{x \in M : \tilde{C} \neq 0\}$ at x, where $L_{\tilde{C}}$ is some function on $U_{\tilde{C}}$ and $\tilde{C}$ is the conharmonic curvature tensor. Let an n-dimensional $(n > 2)$ LP-Sasakian manifold M be a conharmonically pseudosymmetric. Then from (3.3), we have

$$(R(X,\xi) \cdot \tilde{C})(U,V)W = L_{\tilde{C}}[((X \wedge_g \xi) \cdot \tilde{C})(U,V)W]. \tag{3.4}$$

Now the left-hand side of (3.4) is

$$\begin{aligned} &R(X,\xi)\tilde{C}(U,V)W - \tilde{C}(R(X,\xi)U,V)W \\ &-\tilde{C}(U,R(X,\xi)V)W - \tilde{C}(U,V)R(X,\xi)W. \end{aligned} \tag{3.5}$$

In view of (2.7) the above expression becomes

$$\begin{aligned} &\big[g(\xi,\tilde{C}(U,V)W)X - g(X,\tilde{C}(U,V)W)\xi \\ &-\eta(U)\tilde{C}(X,V)W + g(X,U)\tilde{C}(\xi,V)W - \eta(V)\tilde{C}(U,X)W \\ &+g(X,V)\tilde{C}(U,\xi)W - \eta(W)\tilde{C}(U,V)X + g(X,W)\tilde{C}(U,V)\xi\big]. \end{aligned} \tag{3.6}$$

Next the right hand side of (3.4) is

$$\begin{aligned} &L_{\tilde{C}}\big[(X \wedge_g \xi)\tilde{C}(U,V)W - \tilde{C}((X \wedge_g \xi)U,V)W \\ &-\tilde{C}(U,(X \wedge_g \xi)V)W - \tilde{C}(U,V)(X \wedge_g \xi)W\big]. \end{aligned}$$

By virtue of (3.2) the above expression reduces to

$$\begin{aligned} &L_{\tilde{C}}\big[g(\xi,\tilde{C}(U,V)W)X - g(X,\tilde{C}(U,V)W)\xi \\ &-\eta(U)\tilde{C}(X,V)W + g(X,U)\tilde{C}(\xi,V)W - \eta(V)\tilde{C}(U,X)W \\ &+g(X,V)\tilde{C}(U,\xi)W - \eta(W)\tilde{C}(U,V)X + g(X,W)\tilde{C}(U,V)\xi\big]. \end{aligned} \tag{3.7}$$

Using the expressions (3.6) and (3.7) in (3.4), we obtain

$$\begin{aligned} &(1 - L_{\tilde{C}})\big[g(\xi,\tilde{C}(U,V)W)X - g(X,\tilde{C}(U,V)W)\xi \\ &-\eta(U)\tilde{C}(X,V)W + g(X,U)\tilde{c}(\xi,V)W - \eta(V)\tilde{C}(U,X)W \\ &+g(X,V)\tilde{C}(U,\xi)W - \eta(W)\tilde{C}(U,V)X + g(X,W)\tilde{c}(U,V)\xi\big] = 0, \end{aligned}$$

which implies either $L_{\tilde{c}} = 1$ or

$$\begin{aligned} &\big[g(\xi,\tilde{C}(U,V)W)X - g(X,\tilde{C}(U,V)W)\xi \\ &-\eta(U)\tilde{C}(X,V)W + g(X,U)\tilde{C}(\xi,V)W - \eta(V)\tilde{C}(U,X)W \\ &+g(X,V)\tilde{C}(U,\xi)W - \eta(W)\tilde{C}(U,V)X + g(X,W)\tilde{C}(U,V)\xi\big] = 0. \end{aligned} \tag{3.8}$$

Taking innerproduct of (3.8) with ξ and using (2.1) we get

$$\begin{aligned} &[\eta(\tilde{C}(U,V)W)\eta(X) + \tilde{C}(U,V,W,X) \\ &-\eta(U)\eta(\tilde{C}(X,V)W) + g(X,U)\eta(\tilde{C}(\xi,V)W) \\ &-\eta(V)\eta(\tilde{C}(U,X)W) + g(X,V)\eta(\tilde{C}U,\xi)W) \\ &-\eta(W)\eta(\tilde{C}(U,V)X) + g(X,W)\eta(\tilde{C}(U,V)\xi)] = 0. \end{aligned} \tag{3.9}$$

In view of (2.12) we have

$$\eta(\tilde{C}(X,\xi)Z) = \eta(\tilde{C}(X,Y)\xi) = \eta(\tilde{C}(\xi,Y)Z) = 0. \tag{3.10}$$

Using (3.10) in (3.9), we get

$$\begin{aligned} 0 = \eta(\tilde{C}(U,V)W)\eta(X) + \tilde{C}(U,V,W,X) - \eta(U)(\tilde{C}(X,V)W) \\ -\eta(V)\eta(\tilde{C}(U,X)W) - \eta(W)\eta(\tilde{C}(U,V)X). \end{aligned} \tag{3.11}$$

Finally, by simplifying we get

$$\tilde{C}(U,VW,X) = 0,$$

which implies that M is conharmonically flat.

Thus in view of Theorem 2.1, manifold is locally isometric to the unit sphere $S^n(1)$.Therefore we can state the following :

Theorem 3.1. *Let M be an n-dimensional $(n > 2)$ LP-Sasakian manifold. If M is conharmonically pseudosymmetric then M is either conharmonically flat, in which case M is locally isometric to the unit sphere $S^n(1)$ or $L_{\tilde{C}} = 1$ holds on M.*

If $L_{\tilde{C}} = 0$ on $U_{\tilde{C}}$, then a conharmonically pseudosymmetric manifold is conharmonically semisymmetric. Thus we can state the following corollary.

Corollary 3.2. *Let M be an n-dimensional $(n > 2)$ LP-Sasakian manifold. If M is conharmonically semisymmetric, then M is locally isometric to the unit sphere $S^n(1)$.*

But $L_{\tilde{C}}$ need not be zero, in general and hence there exists conharmonically pseudosymmetric manifolds which are not conharmonic semisymmetric. Thus the class of conharmonic pseudosymmetric manifolds is a natural extension of the class of conharmonic semisymmetric manifolds. Thus, if $L_{\tilde{C}} \neq 0$ then it is easy to see that $R \cdot \tilde{C} = Q(g, \tilde{C})$, which implies that the pseudosymmetric function $L_{\tilde{C}} = 1$. Therefore, we able to state the following result:

Theorem 3.3. *Every LP-Sasakian manifold is conharmonically pseudosymmetric of the form $R \cdot \tilde{C} = Q(g, \tilde{C})$.*

4 Conharmonically ϕ-symmetric LP-Sasakian manifold

Definition 4.1. An LP-Sasakian manifold M is said to be conharmonically ϕ-symmetric if the conharmonic curvature tensor $\tilde{C}$ satisfies

$$\phi^2((\nabla_X\tilde{C})(U,V)W) = 0. \tag{4.1}$$

for all vector fields $U, V, W and\ X \in \chi(M)$.

Let an n-dimensional $(n > 2)$ LP-Sasakian manifold M be conharmonically ϕ-symmetric. Then by virtue of (4.1) and (2.1), we have

$$(\nabla_X\tilde{C})(U,V)W + \eta((\nabla_X\tilde{C})(U,V)W)\xi = 0. \tag{4.2}$$

From (4.2) it follows that

$$\begin{aligned} & g((\nabla_X R)(U,V)W,Y) - \frac{1}{(n-2)}[g(U,Y)(\nabla_X S)(V,W) \\ & -g(V,Y)(\nabla_X S)(U,W) + g(V,W)g((\nabla_X Q)U,Y) \\ & -g(U,W)g((\nabla_X Q)V,Y)] + \eta((\nabla_X R)(U,V)W)\eta(Y) \\ & -\frac{1}{(n-2)}[(\nabla_X S)(V,W)\eta(U)\eta(Y) - (\nabla_X S(U,W)\eta(Y)\eta(V) \\ & +g(V,W)\eta((\nabla_X Q)U)\eta(Y) - g(U,W)\eta((\nabla_X Q)V)\eta(Y)] = 0. \end{aligned} \tag{4.3}$$

Putting $U = Y = e_i$, where $\{e_i\}, i = 1, 2, ..., n$, is an orthonormal basis of the tangent space at each point of the manifold, and taking summation over i, we get

$$\begin{aligned} & \eta((\nabla_X R)(e_i,V)W)\eta(e_i) - \frac{1}{(n-2)}[g((\nabla_X Q)e_i,e_i) + \eta((\nabla_X Q)e_i)\eta(e_i)]g(V,W) + \\ & \frac{1}{(n-2)}[g((\nabla_X Q)V,W) + (\nabla_X S)(\xi,W)\eta(V) + \eta((\nabla_X Q)V)\eta(W)] = 0. \end{aligned} \tag{4.4}$$

Putting $W = \xi$, we obtain

$$\begin{aligned}&\eta((\nabla_X R)(e_i, V)\xi)\eta(e_i) \\ &-\frac{1}{(n-2)}[dr(X)\eta(V) + \eta((\nabla_X Q)e_i)\eta(e_i) - (\nabla_X S)(\xi, \xi)] = 0.\end{aligned} \tag{4.5}$$

Now,

$$\begin{aligned}\eta((\nabla_X Q)e_i)\eta(e_i) &= g((\nabla_X Q)e_i, \xi)g(e_i, \xi) \\ &= g((\nabla_X Q)\xi, \xi) \\ &= 0,\end{aligned} \tag{4.6}$$

$$\eta((\nabla_X R)(e_i, Y)\xi)\eta(e_i) = 0, \tag{4.7}$$

and

$$(\nabla_X S)(\xi, \xi) = 0. \tag{4.8}$$

By the use of (4.6) - (4.8) from (4.5) we obtain

$$dr(W) = 0.$$

This implies r is constant. Hence we state the following theorem :

Theorem 4.2. *Let M be an n-dimensional LP-Sasakian manifold. If M is conharmonically ϕ-symmetric then the scalar curvature r is constant.*

5 ϕ-Conharmonically flat LP-Sasakian manifold

Definition 5.1. A LP-Sasakian manifold is said to be ϕ-conharmonically flat if it satisfies

$$\phi^2 \tilde{C}(\phi U, \phi V)\phi W = 0 \tag{5.1}$$

for any vector fields $U, V and\ W \in \chi(M)$

The notion of ϕ-conformally flat for K-contact manifolds was first introduced by G. Zhen [22]. In a recent paper [13] Shaikh et al studied ϕ-conformally flat LP-Sasakian manifolds.

Let an n-dimensional $(n > 2)$ LP-Sasakian manifold be ϕ-conharmonically flat. Then (5.1) holds. By virtue of (2.1) and (2.4), (5.1) yields for any $X \in \chi(M)$

$$g(\tilde{C}(\phi U, \phi V)\phi W, \phi X) = 0.$$

By using the definition of conharmonic curvature tensor, the above relation implies

$$\begin{aligned}g(R(\phi U, \phi V)\phi W, \phi X) &= \frac{1}{n-2}[S(\phi V, \phi W)g(\phi U, \phi X) - S(\phi U, \phi W)g(\phi V, \phi X) \\ &\quad + g(\phi V, \phi W)S(\phi U, \phi X) - g(\phi U, \phi W)S(\phi V, \phi X)].\end{aligned} \tag{5.2}$$

Let $\{e_1, ..., e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields in M. Using that $\{\phi e_1, ..., \phi e_{n-1}, \xi\}$ is also a local orthonormal basis, if we put $U = X = e_i$ in (5.2) and summing up with respect to i, we have

$$\begin{aligned}\sum_{i=1}^{n-1} g(R(\phi e_i, \phi V)\phi W, \phi e_i) &= \frac{1}{n-2}\sum_{i=1}^{n-1}[S(\phi V, \phi W)g(\phi e_i, \phi e_i) \\ &\quad - S(\phi e_i, \phi W)g(\phi V, \phi e_i) \\ &\quad + g(\phi V, \phi W)S(\phi e_i, \phi e_i) \\ &\quad - g(\phi e_i, \phi W)S(\phi V, \phi e_i)].\end{aligned} \tag{5.3}$$

It can be easily verify that

$$\sum_{i=1}^{n-1} g(R(\phi e_i, \phi V)\phi W, \phi e_i) = S(\phi V, \phi W) + g(\phi V, \phi W), \tag{5.4}$$

$$\sum_{i=1}^{n-1} S(\phi e_i, \phi e_i) = r + (n-1), \tag{5.5}$$

$$\sum_{i=1}^{n-1} g(\phi e_i, \phi W) S(\phi V, \phi e_i) = S(\phi V, \phi W), \tag{5.6}$$

$$\sum_{i=1}^{n-1} g(\phi e_i, \phi e_i) = (n+1). \tag{5.7}$$

So by virtue of (5.4)-(5.7) the equation (5.3) can be written as

$$\begin{aligned} S(\phi V, \phi W) + g(\phi V, \phi W) &= \frac{1}{n-2}[(n+1)S(\phi V, \phi W) - S(\phi V, \phi W) \\ &\quad +(r + (n-1))S(\phi V, \phi W) - S(\phi V, \phi W)]. \end{aligned}$$

This implies that

$$S(\phi V, \phi W) = -(r+1)g(\phi V, \phi W). \tag{5.8}$$

Using (2.2) and (2.10) in (5.8), we obtain

$$S(V, W) = -(r+1)g(V, W) - (r+n)\eta(V)\eta(W). \tag{5.9}$$

Contracting (5.9) we have

$$r = 0. \tag{5.10}$$

Thus (5.9) turns into

$$S(V, W) = -g(V, W) - n\,\eta(V)\eta(W),$$

that is, M is an η-Einstein manifold. This leads us to state the following :

Theorem 5.2. *A ϕ-conharmonically flat LP-Sasakian manifold $M(n > 2)$ is an η-Einstein manifold.*

6 Partially Ricci-pseudo symmetric LP-Sasakian manifold

Definition 6.1. An LP-Sasakian manifold M is said to be partially Ricci-pseudosymmetric if and only if the relation

$$R \cdot S = f(p)Q(g, S) \tag{6.1}$$

holds on the set $A = \{x \in M : Q(g, S) \neq 0 \;\; at \;\; x\}$, where $f \in C^\infty(A)$ for $p \in A$, $R \cdot S$ and $Q(g, S)$ are respectively defined by

$$(R(X, Y) \cdot S)(U, V) = -S(R(X, Y)U, V) - S(U, R(X, Y)V), \tag{6.2}$$

and

$$Q(g, S) = ((X \wedge_g Y) \cdot S)(U, V) \tag{6.3}$$

for all $X, Y, U and\ V \in \chi(M)$.

Let an n-dimensional $(n > 2)$ LP-Saskian manifold be partially Ricci-pseudosymmetric. Then we have the relation (6.1), which can be written by virtue of (6.3)

$$(R(X, Y) \cdot S)(U, V) = f(p)[(X \wedge_g Y) \cdot S)(U, V)],$$

for all $X, Y, U, V \in \chi(M)$. From the above relation, it follows that

$$\begin{aligned} &S(R(X, Y)U, V) + S(U, R(X, Y)V) \\ &= f(p)[S((X \wedge_g Y)U, V) + S(U, (X \wedge_g Y)V)]. \end{aligned} \tag{6.4}$$

Taking the restriction $Y = V = \xi$ in (6.4), we have

$$\begin{aligned} & S(R(X,\xi)U,\xi) + S(U, R(X,\xi)\xi) \\ = \quad & f(p)[S((X \wedge_g \xi)U,\xi) + S(U,(X \wedge_g \xi)\xi)]. \end{aligned}$$

Applying (2.7), (2.9) and (3.2) we obtain

$$\begin{aligned} & (n-1)\eta(R(X,\xi)U) - S(U,X) - S(U,\xi)\eta(X) \\ = \quad & f(p)[\eta(U)S(X,\xi - g(X,U)S(\xi,\xi) - S(U,X) - \eta(X)S(U,\xi)]. \end{aligned} \tag{6.5}$$

Using (2.1) and (2.9) in (6.5), we get

$$\begin{aligned} & (n-1)\{\eta(X)\eta(U) + g(X,U)\} - S(U,X) - (n-1)\eta(X)\eta(U) \\ & = f(p)[(n-1)\eta(X)\eta(U) + g(X,U) - \eta(X)\eta(U) - S(U,X)] \end{aligned}$$

This can be written as

$$[S(X,U) - (n-1)g(X,U)] = f(p)[S(X,U) - (n-1)g(X,U)]$$

Thus, we have

$$[f(p) - 1][S(X,U) - (n-1)g(X,U)] = 0.$$

This can be hold only if either :
(a) $f(p) = 1$ or
(b)$S(X,U) = (n-1)g(X,U)$.
However (b) means that M is an Einstein manifold. Hence we have the following:

Theorem 6.2. *A partially Ricci-pseudosymmetric LP-Sasakian manifold is an Einstein manifold with* $f(p) \neq 1$.

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