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Fuzzy Metric Spaces and Fixed Point Theorems

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Abstract. The purpose of this paper is to prove some results on common fixed point theorems for discontinuous, non compatible mappings on non complete fuzzy metric space using implicit relation by improving the condition of Sedghi and Shobe [20]. Also to prove this results we will use the concept of property (E.A) given by Aamri and Moutawakil [1]. In this paper we are taking different type of inequality.

1 Introduction

After introduction of fuzzy sets by Zadeh [22], many researchers Deng [8], Erecg[9], Kaleva and Seikkala[13], Kramosil and Michalek[14], George and Veeramani[11] have defined fuzzy metric space in different ways. They [11] have also obtained a Hausdorff topology for this kind of fuzzy metric space which have very important application in quantum particle physics, particularly in connection with both string and ε^{∞} theory [15,16]. Many authors have proved fixed and common fixed point theorems in fuzzy metric spaces [4], [5],[6],[7],[10],[12],[21]. Regan and Abbas [18] obtained some necessary and sufficient conditions for the existence of common fixed point in fuzzy metric spaces. Aliouche and Popa [2] have proved some results on common fixed point theorems for implicit relation. Also Pathak, Lopez[17] have proved some results using implicit relation and property (E.A). The aim of this paper is to obtain common fixed point of mappings using implicit relation with property (E.A) by taking different type of inequality in fuzzy metric spaces.

2 Preliminaries

Definition 2.1[19]. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if $(*, [0,1])$ is an abelian topological monoid with the unit 1 such that $a * b \leq a * c$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.
Examples of continuous t-norm are $a * b = ab$ and $a * b = min\{a, b\}$.

Definition 2.2[14]. The triplet $(X, M, *)$ is said to be fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X \times X \times [0,\infty) \to [0,1]$ satisfying the following conditions :
for all $x, y, z \in X$ and $s, t > 0$,

FM-1 $M(x, y, 0) = 0$,

FM-2 $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,

FM-3 $M(x, y, t) = M(y, x, t)$

FM-4 $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

FM-5 $M(x, y, .) : [0,\infty) \to [0,1]$ is left continuous.

Note that $M(x, y, t)$ can be considered as the degree of nearness between x and y with respect to t. We identify $x = y$ with $M(x, y, t) = 1$ for all $t > 0$.
Example 2.1 shows that every metric space induces a fuzzy metric space.

Example 2.1[11]. Let (X, d) be a metric space. Define a $*b = min\{a, b\}$ and $M(x, y, t) = \frac{t}{t+d(x,y)}, \forall x, y \in X$ and all $t > 0$. Then $(X, M, *)$ is a fuzzy metric space. It is called the fuzzy metric space induced by d.

Lemma 2.1[11]. For all $x, y \in X$, $M(x, y, .)$ is a non-decreasing function.

Lemma 2.2[11]. If for all $x, y \in X$, $t > 0$, with positive number $k \in (0,1)$ and $M(x, y, kt) \geq$

$M(x, y, t)$, then $x = y$.

Definition 2.3[3]. Let S and T be two self maps of a fuzzy metric space $(X, M, *)$, S and T are said to be compatible if $M(STx_n, TSx_n, t) \to 1$ as $n \to \infty$, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Tx_n \to z$ as $n \to \infty$, for some $z \in X$.

Definition 2.4[7]. Two self maps S and T of fuzzy metric space $(X, M, *)$ are said to be weakly compatible if they commute at their coincidence point, i.e. $STu = TSu$ whenever $Su = Tu$, $u \in X$.

The concept of weak compatibility is most general among all the commutativity concepts, clearly each pair of compatible self maps is weakly compatible but the converse is not true always.

Definition 2.5[1]. Let S and T be two self maps of a fuzzy metric space $(X, M, *)$, we say that S and T satisfy property (E.A.) if there exists a sequence $\{x_n\}$ in X such that $\lim\limits_{n\to\infty} Sx_n = \lim\limits_{n\to\infty} Tx_n = z$ for some $z \in X$.

Definition 2.6. Let $(X, M, *)$ be a fuzzy metric space. M is said to be continuous function on $X^2 \times (0, \infty)$ if

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t)$$

whenever a sequence $M(x_n, y_n, t_n)$ in $X^2 \times (0, \infty)$ convergence to a point $(x, y, t) \in X^2 \times (0, \infty)$ i.e. $\lim\limits_{n\to\infty} M(x_n, x, t) = \lim\limits_{n\to\infty} M(y_n, y, t) = 1$ and $\lim\limits_{n\to\infty} M(x, y, t_n) = M(x, y, t)$.

Lemma 2.3. Let $(X, M, *)$ be a fuzzy metric space. Then M is continuous function on $X^2 \times (0, \infty)$.

A class of implicit relation

Let Ω denotes a family of mappings such that each $\Phi \in \Omega$, $\Phi : [0, 1]^3 \to [0, 1]$, and Φ is continuous and increasing in each co-ordinate variable. Also $\Phi(s, s, s) > s$ for every $s \in [0, 1)$.

Example 2.3. Let $\Phi : [0, 1]^3 \to [0, 1]$ is defined by

- **(i)** $\Phi(x_1, x_2, x_3) = \{min(x_i)\}^h$ for some $0 < h < 1$.
- **(ii)** $\Phi(x_1, x_2, x_3) = \{x_1\}^h$ for some $0 < h < 1$.
- **(iii)** $\Phi(x_1, x_2, x_3) = max\{{x_1}^{\alpha_1}, {x_2}^{\alpha_2}, {x_3}^{\alpha_3}\}$.

3 Main Results

Theorem 3.1. Let $(X, M, *)$ be a fuzzy metric space. Let A, B, S and T be maps from X into itself such that

- **(i)** there exists $\Psi, \Phi \in \Omega$ such that for all $x, y \in X$,

$$M^{2p}(Ax, By, t) \geq a(s)\Phi^{2p}\begin{pmatrix} M(Sx, Ty, kt), M(Ax, Sx, kt), \\ M(By, Ty, kt) \end{pmatrix}$$
$$+ b(s)\Psi^{p}\begin{pmatrix} M^2(Sx, Ty, kt), M(Sx, Ax, kt)M(Ty, By, kt), \\ M(Sx, By, kt) \vee M(Ty, Ax, kt) \end{pmatrix}$$

  for some $k > 1$,where $a, b : [0, 1] \to [0, 1]$ are two continuous functions such that $a(s) + b(s) = 1$ for every $s = M(x, y, t)$.

- **(ii)** the pairs (A, S) and (B, T) are weakly compatible pairs such that $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$.
- **(iii)** (A, S) or (B, T) satisfies property (E.A).

if any of the ranges of A, B, T and S are a complete subspace of X, then A, B, S and T have a unique common fixed point.

Proof: Suppose that (B, T) satisfies the property (E.A), therefore there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z \text{ for some } z \in X.$$

We will show that $\lim_{n\to\infty} Ay_n = z$, for some $z \in X$.
From (i) we have

$$M^{2p}(Ay_n, Bx_n, t) \geq a(s)\Phi^{2p}\begin{pmatrix} M(Sy_n, Tx_n, kt), M(Ay_n, Sy_n, kt), \\ M(Bx_n, Tx_n, kt) \end{pmatrix} + b(s)\Psi^{p}\begin{pmatrix} M^2(Sy_n, Tx_n, kt), M(Sy_n, Ay_n, kt)M(Tx_n, Bx_n, kt), \\ M(Sy_n, Bx_n, kt) \vee M(Tx_n, Ay_n, kt) \end{pmatrix}$$

Taking $n \to \infty$ and by continuity of M and Φ, we get

$$\lim_{n\to\infty} M^{2p}(Ay_n, z, t) \geq a(s)\Phi^{2p}\lim_{n\to\infty}\begin{pmatrix} M(z, z, kt), M(Ay_n, z, kt), \\ M(z, z, kt) \end{pmatrix} + b(s)\Psi^{p}\lim_{n\to\infty}\begin{pmatrix} M^2(z, z, kt), M(Ay_n, z, kt)M(z, z, kt), \\ M(z, z, kt) \vee M(z, Ay_n, kt) \end{pmatrix}$$

We claim that $\lim_{n\to\infty} Ay_n = z$, for if $\lim_{n\to\infty} Ay_n \neq z$. Then we have $\lim_{n\to\infty}(Ay_n, z, t) < 1$,
from above inequality we have

$$\lim_{n\to\infty} M^{2p}(Ay_n, z, t) \geq a(s)\Phi^{2p}\lim_{n\to\infty}\begin{pmatrix} M(Ay_n, z, kt), M(Ay_n, z, kt), \\ M(Ay_n, z, kt) \end{pmatrix} + b(s)\Psi^{p}\lim_{n\to\infty}\{M^2(Ay_n, z, kt), M(Ay_n, z, kt)M(Ay_n, z, kt), 1\}$$

Therefore

$$\lim_{n\to\infty} M^{2p}(Ay_n, z, t) > a(s)\lim_{n\to\infty} M^{2p}(Ay_n, z, kt) + b(S)\lim_{n\to\infty} M^{2p}(Ay_n, z, kt)$$

Which gives $\lim_{n\to\infty} M^{2p}(Ay_n, z, t) > \lim_{n\to\infty} M^{2p}(Ay_n, z, kt)$.
Which is contradiction. Hence

$$\lim_{n\to\infty} M^{2p}(Ay_n, z, t) = 1$$

i.e $\lim_{n\to\infty} Ay_n = z$.
As $S(X)$ is a complete subspace of X, then there exists some point u in X such that $z = Su$.
Therefore we have

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Tx_n = z = Su$$

Now we will show that $Au = Su$, from (i) we have

$$M^{2p}(Au, Bx_n, t) \geq a(s)\Phi^{2p}\begin{pmatrix} M(Su, Tx_n, kt), M(Au, Su, kt), \\ M(Bx_n, Tx_n, kt) \end{pmatrix} + b(s)\Psi^{p}\begin{pmatrix} M^2(Su, Tx_n, kt), M(Su, Au, kt)M(Tx_n, Bx_n, kt), \\ M(Su, Bx_n, kt) \vee M(Tx_n, Au, kt) \end{pmatrix}$$

Taking limit $n \to \infty$,

$$M^{2p}(Au, z, t) \geq a(s)\Phi^{2p}\begin{pmatrix} M(z, z, kt), M(Au, z, kt), \\ M(z, z, kt) \end{pmatrix} + b(s)\Psi^{p}\begin{pmatrix} M^2(z, z, kt), M(Au, z, kt)M(z, z, kt), \\ M(z, z, kt) \vee M(z, Au, kt) \end{pmatrix}$$

We claim that $Au = z$, for if $Au \neq z$, then $(Au, z, t) < 1$,
from above inequality we have

$$\begin{aligned} M^{2p}(Au, z, t) &> a(s)\Phi^{2p}\begin{pmatrix} M(Au, z, kt), M(Au, z, kt), \\ M(Au, z, kt) \end{pmatrix} \\ &\quad + b(s)\Psi^{p}\{M^2(Au, z, kt), M(Au, z, kt)M(Au, z, kt), 1\} \\ &= M^{2p}(Au, z, kt) \end{aligned}$$

Which is a contradiction. Hence $Au = z$, therefore $Au = Su$.
The weak compatibility of A and S implies that $ASu = SAu$ and then

$$AAu = ASu = SAu = SSu$$

Since $AX \subseteq TX$, there exists $v \in X$ such that $Au = Tv$, so

$$\begin{aligned}
M^{2p}(Au, Bv, t) &\geq a(s)\Phi^{2p}\begin{pmatrix} M(Su, Tv, kt), M(Au, Su, kt), \\ M(Bv, Tv, kt) \end{pmatrix} \\
&\quad + b(s)\Psi^{p}\begin{pmatrix} M^2(Su, Tv, kt), M(Su, Au, kt)M(Tv, Bv, kt), \\ M(Su, Bv, kt) \vee M(Tv, Au, kt) \end{pmatrix} \\
&= a(s)\Phi^{2p}\{1, 1, M(Bv, z, kt)\} + b(s)\Psi^{p}\{1, M(z, Bv, kt), 1\}
\end{aligned}$$

We claim that $Au = z$, for if $Au \neq z$, then $(Au, z, t) < 1$,
from above inequality we have

$$M^{2p}(Bv, z, t) > M^{2p}(Bv, z, kt)$$

Which is a contradiction, so $Bv = z = Au$. Hence $Au = Su = Tv = Bv = z$.
Since (B, T) is weakly compatible, so $BTv = TBv$ and then

$$BBv = BTv = TBv = TTv,$$

i.e. $Bz = Tz$.
Let us show that Au is the common fixed point of A, B, S and T, in view of (i) it follows

$$\begin{aligned}
M^{2p}(AAu, Bv, t) &\geq a(s)\Phi^{2p}\begin{pmatrix} M(SAu, Tv, kt), M(AAu, SAu, kt), \\ M(Bv, Tv, kt) \end{pmatrix} \\
&\quad + b(s)\Psi^{p}\begin{pmatrix} M^2(SAu, Tv, kt), M(SAu, AAu, kt)M(Tv, Bv, kt), \\ M(SAu, Bv, kt) \vee M(Tv, AAu, kt) \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
M^{2p}(AAu, Au, t) &\geq a(s)\Phi^{2p}\{M(AAu, Au, kt), 1, 1\} \\
&\quad + b(s)\Psi^{p}\{M^2(AAu, Au, kt), 1, M(AAu, Au, kt)\}
\end{aligned}$$

We claim that $AAu = Au$, for if $AAu \neq Au$, then $(AAu, Auz, t) < 1$, so we get

$$M^{2p}(AAu, Au, t) > M^{2p}(AAu, Au, kt)$$

Which is a contradiction, so $AAu = Au$. Therefore $SAu = AAu = Au$.
Hence Au is the common fixed point of A and S.
Similarly we can prove that Bv is the common fixed point of B and T i.e. $BBv = TBv = Bv$.
Since $Au = Bv$, therefore $BAu = TAu = Au$.
Hence Au is the common fixed point of A, B, S and T.
Now we will show that the common fixed point is unique.
Let z and z' be two distinct common fixed point of A, B, S and T then

$$\begin{aligned}
M^{2p}(z, z', t) = M^{2p}(Az, Bz', t) &\geq a(s)\Phi^{2p}\begin{pmatrix} M(Sz, Tz', kt), M(Az, Sz, kt), \\ M(Bz', Tz', kt) \end{pmatrix} \\
&\quad + b(s)\Psi^{p}\begin{pmatrix} M^2(Sz, Tz', kt), M(Sz, Az, kt)M(Tz', Bz', kt), \\ M(Sz, Bz', kt) \vee M(Tz', Az, kt) \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
M^{2p}(Az, Bz', t) &\geq a(s)\Phi^{2p}\{M(z, z', kt), 1, 1\} \\
&\quad + b(s)\Psi^{p}\{M^2(z, z', kt), 1, M(z, z', kt)\}
\end{aligned}$$

so in this way we get

$$M^{2p}(z, z', t) > M^{2p}(z, z', kt)$$

Which is a contradiction, therefore $z = z'$.
Hence A, B, S and T have a unique common fixed point.

Remark 3.1 When $T(X)$ is assumed to be a complete subspace of X, the proof of the above theorem will be same.

In our next result, function $\Phi : [0,1]^4 \to [0,1]$, is continuous and increasing in each co-ordinate variable. Also $\Phi(s,s,s,s) > s$ for every $s \in [0,1)$.

Theorem 3.2. Let $(X, M, *)$ be a fuzzy metric space. Let A, B, S and T be maps from X into itself such that

(i) there exists $\Psi, \Phi \in \Omega$ such that for all $x, y \in X$,

$$M(Ax, By, t) \geq \Phi \begin{pmatrix} M(Sx, Ty, kt), M(Ax, Sx, kt), M(By, Ty, kt), \\ M(Ax, Ty, kt) \vee M(By, Sx, kt) \end{pmatrix}$$

for every $x, y \in X$, $k > 1$, and $\Phi \in \Omega$

(ii) the pairs (A, S) and (B, T) are weakly compatible pairs such that $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$.

(iii) (A, S) or (B, T) satisfies property (E.A).

then A, B, S and T have a unique common fixed point.

Proof: Suppose that (B, T) satisfies the property (E.A), therefore there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z \text{ for some } z \in X.$$

As $S(X)$ is a complete subspace of X, every convergent sequence of points of $S(X)$ has a limit point in $S(X)$.
Therefore $\lim_{n\to\infty} Sy_n = Su$ for some $u \in X$, then $z = Su$.
Subsequently we have

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sy_n = Su$$

We will show that $\lim_{n\to\infty} Ay_n = z$, for some $z \in X$.
From (i) we have

$$M(Ay_n, Bx_n, t) \geq \Phi \begin{pmatrix} M(Sy_n, Tx_n, kt), M(Ay_n, Sy_n, kt), M(Bx_n, Tx_n, kt), \\ M(Ay_n, Tx_n, kt) \vee M(Bx_n, Sy_n, kt) \end{pmatrix}$$

Taking limit $n \to \infty$, and by continuity of M and Φ, we get

$$M(Ay_n, z, t) \geq \Phi\{1, M(Ay_n, z, kt), 1, 1\}$$

We claim that $\lim_{n\to\infty} Ay_n = z$, for if $\lim_{n\to\infty} Ay_n \neq z$, then $M(Ay_n, z, t) < 1$,
from above inequality we have

$$M(Ay_n, z, t) > \Phi\{M(Ay_n, z, kt), M(Ay_n, z, kt), M(Ay_n, z, kt), M(Ay_n, z, kt)\}$$

So, we have

$$M(Ay_n, z, t) > M(Ay_n, z, kt)$$

Which is contradiction. Hence $M(Ay_n, z, t) = 1$ i.e $\lim_{n\to\infty} Ay_n \neq z$.
Therefore we have

$$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Tx_n = z = Su.$$

Now we will show that $Au = Su$

$$M(Au, Bx_n, t) \geq \Phi \begin{pmatrix} M(Su, Tx_n, kt), M(Au, Su, kt), M(Bx_n, Tx_n, kt), \\ M(Au, Tx_n, kt) \vee M(Bx_n, Su, kt) \end{pmatrix}$$

Taking limit $n \to \infty$, we have

$$M(Au, z, t) \geq \Phi \begin{pmatrix} M(z, z, kt), M(Au, z, kt), M(z, z, kt), \\ M(Au, z, kt) \vee M(z, z, kt) \end{pmatrix}$$

We claim that $Au = z$, for if $Au \neq z$, then $M(Au, z, t) < 1$,
from above inequality, we have

$$M(Au, z, t) > M(Au, z, kt)$$

Which is a contradiction. Hence $Au = z$, therefore $Au = Su$.
The weak compatibility of A and S implies that $ASu = SAu$ and then

AAu = ASu = SAu = SSu

Since $AX \subseteq TX$, there exists $v \in X$ such that $Au = Tv$, so

$$M(Au, Bv, t) \geq \Phi \begin{pmatrix} M(Su, Tv, kt), M(Au, Su, kt), M(Bv, Tv, kt), \\ M(Au, Tv, kt) \vee M(Bv, Su, kt) \end{pmatrix}$$

This implies that

$$M(Bv, z, t) \geq \Phi\{1, 1, M(Bv, z, t), 1\}$$

We claim that $Bv = z$, for if $Bv \neq z$, then $(Bv, z, t) < 1$,
from above inequality we have

$$M(Bv, z, t) > M(Bv, z, kt)$$

Which is a contradiction, so $Bv = z = Au$. Hence

$$Au = Su = Tv = Bv = z.$$

Since (B, T) is weakly compatible, so $BTv = TBv$ and then

$$BBv = BTv = TBv = TTv, \text{ i.e. } Bz = Tz.$$

Let us show that Au is the common fixed point of A, B, S and T, in view of (i) it follows

$$M(AAu, Bv, t) \geq \Phi \begin{pmatrix} M(SAu, Tv, kt), M(AAu, SAu, kt), M(Bv, Tv, kt), \\ M(AAu, Tv, kt) \vee M(Bv, SAu, kt) \end{pmatrix}$$

$$M(AAu, Bv, t) \geq \Phi\{M(AAu, Au, kt), 1, 1, M(AAu, Tv, kt)\}$$

We claim that $AAu = Au$, for if $AAu \neq Au$, then $(AAu, Auz, t) < 1$, so we get

$$M(AAu, Au, t) > M(AAu, Au, kt)$$

Which is a contradiction, so $AAu = Au$. Therefore $SAu = AAu = Au$.
Hence Au is the common fixed point of A and S.
Similarly we can prove that Bv is the common fixed point of B and T i.e. $BBv = TBv = Bv$.
Since $Au = Bv$, therefore $BAu = TAu = Au$.
Hence Au is the common fixed point of A, B, S and T.
Easily we can show that the common fixed point is unique.
Hence A, B, S and T have a unique common fixed point.

Remark 3.2. When $T(X)$ is assumed to be a complete subspace of X, the proof of the theorem 3.2 will be same.

References

[1] M. Aamri and D. El Moutawakil: Some new common fixed point theorem under strict contractive conditions, J. Math. Anal. Appl., 270(2002)181-188.

[2] A. Aliouche and V. Popa: Common fixed point theorem for occasionally weakly compatible mappings via implicit relations, Faculty of Sciences and Mathematics University of Nis, Serbia 22(2) (2008), 99-107.

[3] I. Altun and D. Turkoglu : Some fixed point theorems on fuzzy metric spaces with implicit relations, Commun. Korean Math. Soc. 23 (2008), No. 1, 111-124.

[4] R. Badard : Fixed point theorems for fuzzy numbers, Fuzzy sets and systems, 13(1984), 291-302.

[5] D. Butnariu : Fixed point for fuzzy mappings, Fuzzy Sets and Systems, 7(1982),191-207.

[6] S. S. Chang, Y.J. Cho, B.S. Lee, J.S. Jung, S. M. Kang : Coincidence point and minimization theorems in fuzzy metric spaces, Fuzzy Sets and Systems, 88(1) (1997), 119-128.

[7] Y. J. Cho, S. Sedghi, and N. Shobe: Generalized fixed point theorems for Compatible mappings with some types in fuzzy metric spaces, Chaos, Solitons and Fractals, 39 (2009), 2233-2244.

[8] Z. Deng : Fuzzy pseudo-metric space, J. Math. Anal. Appl., 86(1982), 74-95.

[9] M. A. Erceg : Metric space in fuzzy set theory, J. Math. Anal. Appl., 69(1979), 205-230.

[10] J. X. Fang : On fixed point theorems in fuzzy metric spaces, Fuzzy Sets and Systems,46(1992),107-113.

[11] A. George and P. Veeramani : On some results in fuzzy metric spaces, Fuzzy Sets and Systems, 64 (1994), 395-399.

[12] O. Hadzic : Fixed point theorems for multi-valued mappings in some classes of fuzzy metric spaces, Fuzzy Sets and Systems, 29(1989), 115-125.

[13] Kaleva, O. and Seikkala, S., On fuzzy metric spaces, Fuzzy Sets and Systems 12(1984), 215-229.

[14] O. Kramosil and J. Michalek: Fuzzy metric and statistical metric spaces, Kybernetika 11 (1975), 326-334.

[15] M. S. El Naschie : On the uncertainty of Cantorian geometry and the two-slit experiment, Chaos Solitons Fractals, 9 (1998), no. 3, 517-529.

[16] M. S. El Naschie : A review of E-infnity theory and the mass spectrum of high energy particle physics, Chaos, Solitons and Fractals, 19 (2004), 209-236.

[17] H. K. Pathak, RosanaRodrt'iguez-Lt'opez and R. K. Verma : A common fixed point theorem using implicit relations and property (E.A) in metric spaces, Faculty of Sciences and Mathematics, University of Nis, Serbia, 21(2) (2007), 211-234.

[18] D. O' Regan and M. Abbas : Necessary and sufficient conditions for common fixed point theorems in fuzzy metric spaces, Demonstratio Mathematica, Vol. 42, No. 4, (2009), 887-900..

[19] B. Schweizer and A. Sklar : Statistical metric spaces, Pacific J. Math., 10 (1960), 314-334.

[20] Shaban Sedgi and Nabi Shobe : Common fixed point theorem for weakly compatible of four mappings, Commun. Korean Math. Soc. 22(2007)No. 3, 429-440.

[21] R. Vasuki : Common fixed point theorems in fuzzy metric spaces, Fuzzy Sets and Systems 97(1998), 395-397.

[22] L.A. Zadeh, Fuzzy sets, Inform and Control 8 (1965), 338-353.

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