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# ON INFINITE SERIES OF THREE VARIABLES INVOLVING WHITTAKER AND BESSEL FUNCTIONS

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**Abstract**. The main object of the present paper is to obtain on infinite series of three variables involving Whittaker and Bessel function into a series of Srivastava's triple hypergeometric function $F^{(3)}$. A number of known and knew transformations are also discussed as the special cases of our main result.

## 1 Introduction

Bessel function of order $\nu$ defined by Srivasta and Manocha as follows [10]:

$$J_\nu(z) = \frac{(z/2)^\nu}{\Gamma(\nu+1)}\, {}_0F_1\left[\begin{matrix} -; & \dfrac{-z^2}{4} \\ \nu+1; & \end{matrix}\right]. \tag{1.1),}$$

Modified Bessel function defined by Rainville [7] as follows:

$$I_\nu(z) = \frac{(z/2)^\nu}{\Gamma(\nu+1)}\, {}_0F_1\left[\begin{matrix} -; & \dfrac{z^2}{4} \\ \nu+1; & \end{matrix}\right]. \tag{1.2}$$

Bessel functions are of most frequent use in the theory of integral transform. The modified Bessel function $I_\nu(z)$ is related to $J_\nu(z)$ in much the same way that the hyperbolic function is related to trigonometric function, and we have

$$I_\nu(z) \;=\; i^{-\nu} J_\nu(iz). \tag{1.3}$$

Transformation of Appell's function $F_4$ into a series of Srivastava's triple hypergeometric function $F^{(3)}$ given by Khan and Pathan [6] as follows:

$$\begin{aligned} &F_4\left[\frac{a}{2}, \frac{a}{2}+\frac{1}{2}; b, c;\ \frac{4x}{(1+x+y)^2}, \frac{y^2}{(1+x+y)^2}\right] \\ &= \frac{(1+x+y)^2 2^{2a}\Gamma(a+\frac{1}{2})\Gamma(a-b+\frac{3}{2})}{\sqrt{\pi}\,\Gamma(2a-b+\frac{3}{2})} \\ &.\Bigg\{ F^{(3)}\left[\begin{matrix} a, a+\frac{1}{2} :: a-\frac{1}{2}b+\frac{3}{4}; a-\frac{1}{2}b+\frac{5}{4}; a-b+1; a-b+\frac{3}{2}; -; -; & \\ & -2y, x, 1 \\ - : - : c-\frac{1}{2}; -; \frac{b}{2}-\frac{1}{4}, \frac{b}{2}+\frac{1}{4}2c-1, a-b+1; b, \frac{1}{2} & ; \end{matrix}\right] \\ &- \frac{a(b-\frac{1}{2})}{(a-\frac{1}{2}b+\frac{3}{4})} \\ &. F^{(3)}\left[\begin{matrix} a+\frac{1}{2}, a+1, a-\frac{1}{2}b+\frac{5}{4}, a-\frac{1}{2}b+\frac{7}{4}; a-b+1; a-b+\frac{3}{2}; -; -; & \\ & -2y, x, 1 \\ - : - : -; c-\frac{1}{2}; -; \frac{b}{2}+\frac{1}{4}, \frac{b}{2}+\frac{3}{4}; 2c-1, a-b+1; b, \frac{3}{2} & ; \end{matrix}\right] \Bigg\}, \end{aligned} \tag{1.4}$$

where $F^{(3)}[x, y, z]$ is Srivastava's triple hypergeometric series defined by Srivastava and Manocha [10, p.69(39)] as follows:

$$F^{(3)}\left[\begin{array}{l} (a)::(b);(b');(b'');(e);(e');\ (e'')\ \ ; \\ \hphantom{(a)::(b);(b');(b'');(e);(e');\ (e'')\ \ ;}\quad x,y,z \\ (d)::(g);(g');(g'');(h);(h');(h'')\ ; \end{array}\right]$$

$$=\sum_{m,n,p=0}^{\infty}\frac{[(a)]_{m+n+p}[(b)]_{m+n}[(b')]_{n+p}[(b'')]_{p+m}[(e)]_m[(e')]_n[(e'')]_p}{[(d)]_{m+n+p}[(g)]_{m+n}[(g')]_{n+p}[(g'')]_{p+m}[(h)]_m[(h')]_n[(h'')]_p}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}. \tag{1.5}$$

## 2 Main Transformation

In order to obtain the main transformation of the present paper, we establish an integral in the following form:

$$\int_0^\infty t^{\sigma-1}e^{-zt}\,W_{k,m}(ht)J_\nu(\alpha t)\,{}_1F_2\left(\begin{array}{l}\beta\ \ ;\\ \gamma,\delta\ ;\end{array}\ y^2t^2\right)dt$$

$$=\frac{h^{m+\frac{1}{2}}\,\alpha^\nu\,\Gamma(C)\,\Gamma(C')}{2^\nu\Gamma(\nu+1)\Gamma(E)\,(z+h/2)^C}$$

$$\cdot\Bigg\{\ F^{(3)}\left[\begin{array}{l}\frac{C}{2},\frac{C+1}{2}::\frac{C'}{2},\frac{C'+1}{2};-;-;\beta;-;\frac{D}{2},\frac{D+1}{2};\\ \frac{E}{2},\frac{E+1}{2}::-;-;-;\gamma,\delta;\nu+1;\frac{1}{2}\qquad ;\end{array}\ \frac{4y^2}{(z+h/2)^2},\frac{-\alpha^2}{(z+h/2)^2},\left(\frac{z-h/2}{z+h/2}\right)^2\right]$$

$$+\frac{CD}{E}\left(\frac{z-h/2}{z+h/2}\right)$$

$$.F^{(3)}\left[\begin{array}{l}\frac{C+1}{2},\frac{C+2}{2}::\frac{C'}{2},\frac{C'+1}{2};-;-;\beta;-;\frac{D+1}{2},\frac{D+2}{2};\\ \frac{E+1}{2},\frac{E+2}{2}::-;-;-;\gamma,\delta;\nu+1;\frac{3}{2}\qquad ;\end{array}\ \frac{4y^2}{(z+h/2)^2},\frac{-\alpha^2}{(z+h/2)^2},\left(\frac{z-h/2}{z+h/2}\right)^2\right]\ \Bigg\}, \tag{2.1}$$

where $\mathrm{C}=\sigma+\nu+\mathrm{m}+\frac{1}{2}$, $\mathrm{C}'=\sigma+\nu-\mathrm{m}+\frac{1}{2}$, $\mathrm{D}=\mathrm{m}-\mathrm{k}+\frac{1}{2}$, $\mathrm{E}=\sigma+\nu-\mathrm{k}+1$, $\mathrm{Re(C)}>0$, $\mathrm{Re(C')}>0$, $\mathrm{Re(z+h/2)}>0$ and

$$W_{k.m}(x)=\frac{\Gamma(-2m)}{\Gamma(\frac{1}{2}-m-k)}\,M_{k,m}(x)+\frac{\Gamma(2m)}{\Gamma(\frac{1}{2}+m-k)}\,M_{k,-m}(x).$$

$$M_{k,m}(x)=x^{m+1/2}e^{(-1/2)x}\,{}_1F_1\left(\frac{1}{2}+m-k,2m+1;\ x\right).$$

To obtain equation (2.1), express $J_\nu(\alpha t)$ in terms of ${}_0F_1$ with the help of the equation (1.1) and then expanding ${}_0F_1$ and ${}_1F_2$ in series and using the integral transform [2, p.216(16)], we get

$$I=\frac{(\alpha/2)^\nu\,h^{m+1/2}}{\Gamma(\nu+1)}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\frac{(-\alpha^2)^p(\beta)_q y^{2q}}{(\nu+1)_p\,(\gamma)_q\,(\delta)_q\,p!q!}$$

$$\cdot\frac{\Gamma(\sigma+\nu\pm m+\frac{1}{2}+2p+2q)}{\Gamma(\sigma+\nu-k+1+2p+2q)\,(z+h/2)^{\sigma+\nu+m+\frac{1}{2}+2p+2q}}$$

$$\cdot\,{}_2F_1\left(\begin{array}{l}\sigma+\nu+m+\frac{1}{2}+2p+2q,m-k+\frac{1}{2}\ ;\\ \sigma+\nu-k+1+2p+2q\qquad ;\end{array}\ \frac{z-h/2}{z+h/2}\right).$$

Now making use of a following result of Carlson [4, p.234(10)], in the above equation

$${}_2F_1\left(\begin{array}{l}a,b;\\ c\ \ ;\end{array}\ x\right)={}_4F_3\left(\begin{array}{l}\frac{a}{2},\frac{a+1}{2},\frac{b}{2},\frac{b+1}{2}\ ;\\ \frac{1}{2},\frac{c}{2},\frac{c+1}{2}\qquad ;\end{array}\ x^2\right)+\frac{ab}{c}\,x\,{}_4F_3\left(\begin{array}{l}\frac{a+1}{2},\frac{a+2}{2},\frac{b+1}{2},\frac{b+2}{2}\ ;\\ \frac{3}{2},\frac{c+1}{2},\frac{c+2}{2}\qquad ;\end{array}\ x^2\right), \tag{2.2}$$

and then expanding ${}_4F_3$'s in series and interpreting in the form of $F^{(3)}$, we arrive at the main integral (2.1).

Taking $k = 0, h = 2x, m = \nu, \beta = \mu + \frac{1}{2}, \gamma = \lambda = 2\mu + 1, \delta = \lambda + \mu$, replacing $\alpha$ by $i\alpha$, $y$ by $iy$ and using equation (1.3) and a result [3, p.432] in equation (2.1) and comparing it with a result of Saxena [11, p.131(9)], we get

$$\int_0^\infty t^{2\lambda+2\mu-1} e^{-zt}\, K_\nu(xt) I_\nu(\alpha t)\, {}_1F_2\left(\begin{matrix} \mu+\frac{1}{2} & ; & \\ & & -y^2t^2 \\ 2\mu+1, \lambda+\mu\,; & & \end{matrix}\right) dt$$

$$= \frac{2^{2(\lambda+\mu-1)}(\alpha\beta)^\nu \Gamma(\lambda+\mu+\nu)\Gamma(\lambda+\mu)}{(\alpha^2+\beta^2+2\gamma^2)^{\lambda+\mu+\nu}\Gamma(\nu+1)}$$

$$.\, F_4\left[\frac{\lambda+\mu+\nu}{2}, \frac{\lambda+\mu+\nu+1}{2}, \nu+1; \mu+1;\ \frac{4\alpha^2\beta^2}{(\alpha^2+\beta^2+2\gamma^2)^2}, \frac{4\gamma^2}{(\alpha^2+\beta^2+2\gamma^2)^2}\right], \quad (2.3)$$

$\mathrm{Re}(\lambda+\mu+\nu) > 0$, $\mathrm{Re}(\lambda+\mu) > 0$ and $\mathrm{Re}(\alpha) > \mid \mathrm{Re}(\beta) \mid + \mid Im\gamma \mid$, then we get the transformation (1.4).

## 3 Special Cases

The following special cases of the main transformation (2.1) are given below:

**(i)** On setting $z = h/2$ in equation (2.1), the integral reduces to

$$\int_0^\infty t^{\sigma-1} e^{-(h/2)t}\, W_{k,m}(ht) J_\nu(\alpha t)\, {}_1F_2\left(\begin{matrix} \beta & ; & \\ & & y^2t^2 \\ \gamma, \delta & ; & \end{matrix}\right) dt$$

$$= \frac{h^{m+\frac{1}{2}}\, \alpha^\nu\, \Gamma(C)\, \Gamma(C')}{2^\nu \Gamma(\nu+1)\Gamma(E)\, h^C}\, F^{(2)}\left[\begin{matrix} \frac{C}{2}, \frac{C+1}{2} :: \frac{C'}{2}; \frac{C'+1}{2}; -; -; \beta; -; & \\ & \frac{4y^2}{h^2}, \frac{-\alpha^2}{h^2} \\ \frac{E}{2}, \frac{E+1}{2} ::; -; \gamma, \delta, \nu+1\,; -; & \end{matrix}\right], \quad (3.1)$$

where $F^{(2)}$ is Kampe$'$ de Fe$'$riet's function defined in Burchnall and Chaundy [1] as follows:

$$F^{(2)}\left[\begin{matrix} a : b; c & ; & \\ & & x, y \\ e; f; g & ; & \end{matrix}\right] = \sum_{m=0}^\infty \sum_{n=0}^\infty \frac{(a)_{m+n}(b)_m(c)_n x^m y^n}{(e)_{m+n}(f)_m(g)_n m! n!}.$$

**(ii)** On setting $k = m + 1/2$ in equation (2.1) and using the relation [4, p.32], then the integral reduces to

$$\int_0^\infty t^{\sigma+m+1/2-1} e^{-(z+h/2)t}\, J_\nu(\alpha t)\, {}_1F_2\left(\begin{matrix} \beta & ; & \\ & & y^2t^2 \\ \gamma, \delta & ; & \end{matrix}\right) dt$$

$$= \frac{\alpha^\nu\, \Gamma(C)}{2^\nu \Gamma(\nu+1)\, (z+h/2)^C}\, F^{(2)}\left[\begin{matrix} \frac{C}{2}, \frac{C+1}{2} :::; - : \beta \quad ; -; & \\ & \frac{4y^2}{(z+h/2)^2}, \frac{-\alpha^2}{(z+h/2)^2} \\ - :: - : \gamma, \delta, \nu+1\,; -; & \end{matrix}\right]. \quad (3.2)$$

**(iii)** On setting $k = 0, h = 2x, m = \nu + 1/2, \nu = \mu$ and $y = 0$ in equation (2.1) and using a relation [4, p.32] and integrating with the help of the integral transform [3, p.198(27)], we get

$$\int_0^\infty t^{\sigma+\mu+2r-1/2} e^{-zt}\, K_{\nu+1/2}(xt) J_\mu(\alpha t) dt$$

$$= \frac{\alpha^\mu}{2x\, 2^\mu \Gamma(\mu+1)}\, \Gamma(\sigma+\mu-\nu+2r)\, \Gamma(\sigma+\mu+\nu+1+2r) \sum_{r=0}^\infty \frac{(-1)^r \alpha^{2r}}{2^{2r}(\mu+1)_r\, r!}$$

$$.s^{-(\sigma+\mu+2r)}\ P^{-(\sigma+\mu+2r)}(z/x), \tag{3.3}$$

where $P^{\mu}_{\nu}(z)$ is Legendre function [2, p.122(3)] and $\mathrm{Re}(\sigma+\mu+\nu+2\mathrm{r})>-1, Re(\sigma+\mu-\nu+2r)>0$ and $Re(z)>-Re(x)$.

**(iv)** On setting $\gamma=\beta+\frac{1}{2},\ \delta=2\beta-1$ in equation (2.1) and using a known result [7, p.595(6)], we get

$$\Gamma(\beta-\frac{1}{2})\Gamma(\beta+\frac{1}{2})\left(\frac{y}{2}\right)^{2-2\beta}\int_0^\infty t^{\sigma+2-2\beta-1}e^{-zt}\ W_{k,m}(ht)J_\nu(\alpha t)[I_{\beta-3/2}(yt)I_{\beta-1/2}(yt)]dt$$

$$=\frac{h^{m+\frac{1}{2}}\ \alpha^\nu\ \Gamma(C)\ \Gamma(C')}{2^\nu\Gamma(\nu+1)\Gamma(E)\left(z+h/2\right)^C}$$

$$\left\{\quad F^{(3)}\left[\begin{array}{l}\frac{C}{2},\frac{C+1}{2}::\frac{C'}{2},\frac{C'+1}{2};-;-;\beta;-;\frac{D}{2},\frac{D+1}{2}\quad;\\ \frac{E}{2},\frac{E+1}{2}::-;-;-;\beta+\frac{1}{2},\ 2\beta-1;\nu+1;\frac{1}{2};\end{array}\frac{4y^2}{(z+h/2)^2},\frac{-\alpha^2}{(z+h/2)^2},\left(\frac{z-h/2}{z+h/2}\right)^2\right]\right.$$

$$+\frac{CD}{E}\left(\frac{z-h/2}{z+h/2}\right)$$

$$\left.F^{(3)}\left[\begin{array}{l}\frac{C+1}{2},\frac{C+2}{2}::\frac{C'}{2},\frac{C'+1}{2};-;-;\beta;-;\frac{D+1}{2},\frac{D+2}{2}\quad;\\ \frac{E+1}{2},\frac{E+2}{2}::-;-;-;\beta+\frac{1}{2},2\beta-1;\nu+1;\frac{3}{2};\end{array}\frac{4y^2}{(z+h/2)^2},\frac{-\alpha^2}{(z+h/2)^2},\left(\frac{z-h/2}{z+h/2}\right)^2\right]\right\}. \tag{3.4}$$

**(v)** On setting $\beta=\frac{3}{2},\ \delta=3-\gamma$ in equation (2.1) and using a known result [7, p.595(10)], we get

$$\frac{\pi(\gamma-1)(\gamma-2)}{\sin\gamma\pi}\int_0^\infty t^{\sigma-1}e^{-zt}\ W_{k,m}(ht)J_\nu(\alpha t)[I_{1-\gamma}(yt)I_{\gamma-1}(yt)+I_{2-\gamma}(yt)I_{\gamma-2}(yt)]dt$$

$$=\frac{h^{m+\frac{1}{2}}\ \alpha^\nu\ \Gamma(C)\ \Gamma(C')}{2^\nu\Gamma(\nu+1)\Gamma(E)\left(z+h/2\right)^C}$$

$$.\left\{\quad F^{(3)}\left[\begin{array}{l}\frac{C}{2},\frac{C+1}{2}::\frac{C'}{2},\frac{C'+1}{2};-;-;\frac{3}{2};-;\frac{D}{2},\frac{D+1}{2};\\ \frac{E}{2},\frac{E+1}{2}::-;-;-;\gamma,\ 3\gamma-1;\nu+1;\frac{1}{2};\end{array}\frac{4y^2}{(z+h/2)^2},\frac{-\alpha^2}{(z+h/2)^2},\left(\frac{z-h/2}{z+h/2}\right)^2\right]\right.$$

$$+\frac{CD}{E}\left(\frac{z-h/2}{z+h/2}\right)$$

$$\left.F^{(3)}\left[\begin{array}{l}\frac{C+1}{2},\frac{C+2}{2}::\frac{C'}{2},\frac{C'+1}{2};-;-;\frac{3}{2};-;\frac{D+1}{2},\frac{D+2}{2};\\ \frac{E+1}{2},\frac{E+2}{2}::-;-;-;\gamma,3\gamma-1;\nu+1;\frac{3}{2}\quad;\end{array}\frac{4y^2}{(z+h/2)^2},\frac{-\alpha^2}{(z+h/2)^2},\left(\frac{z-h/2}{z+h/2}\right)^2\right]\right\}. \tag{3.5}$$

**(vi)** On setting $\beta=\frac{1}{4},\ \gamma=\frac{1}{2},\ \delta=\frac{5}{4}$ in equation (2.1) and using a known result [7, p.598(37)], we get

$$\frac{1}{4}\sqrt{\frac{\pi}{2y}}\int_0^\infty t^{\sigma-3/2}e^{-zt}\ W_{k,m}(ht)J_\nu(\alpha t)[\mathrm{erf}(\sqrt{2yt})+\mathrm{erfi}(\sqrt{2yt})]dt$$

$$=\frac{h^{m+\frac{1}{2}}\ \alpha^\nu\ \Gamma(C)\ \Gamma(C')}{2^\nu\Gamma(\nu+1)\Gamma(E)\left(z+h/2\right)^C}$$

$$.\left\{\quad F^{(3)}\left[\begin{array}{l}\frac{C}{2},\frac{C+1}{2}::\frac{C'}{2},\frac{C'+1}{2};-;-;\frac{1}{4};-;\frac{D}{2},\frac{D+1}{2};\\ \frac{E}{2},\frac{E+1}{2}::-;-;-;\frac{1}{2},\frac{5}{4};\nu+1;\frac{1}{2}\quad;\end{array}\frac{4y^2}{(z+h/2)^2},\frac{-\alpha^2}{(z+h/2)^2},\left(\frac{z-h/2}{z+h/2}\right)^2\right]\right.$$

$$+\frac{CD}{E}\left(\frac{z-h/2}{z+h/2}\right)$$

$$F^{(3)}\left[\begin{array}{l} \frac{C+1}{2},\frac{C+2}{2}::\frac{C'}{2},\frac{C'+1}{2};-;-;\frac{1}{4};-;\frac{D+1}{2},\frac{D+2}{2}\ ; \\ \frac{E+1}{2},\frac{E+2}{2}::-;-;-;\frac{1}{2},\frac{5}{4};\nu+1;\frac{3}{2}\ ; \end{array} \frac{4y^2}{(z+h/2)^2},\frac{-\alpha^2}{(z+h/2)^2},\left(\frac{z-h/2}{z+h/2}\right)^2\right]\Bigg\}. \tag{3.6}$$

**(vii)** On setting $\gamma = \frac{3}{2}$, in equation (2.1) and using a known result [7, p.595(11)] and integral transform [3, p.216(16)], we get

$$\frac{\sqrt{\pi}}{2}\Gamma(\delta)y^{1/2-\delta}\int_0^\infty t^{\sigma+1/2-\delta-1}e^{-zt}\,W_{k,m}(ht)J_\nu(\alpha t)[L_{\delta-3/2}(2yt)]dt$$

$$=\frac{h^{m+\frac{1}{2}}\,\alpha^\nu\,\Gamma(C)\,\Gamma(C')}{2^\nu\Gamma(\nu+1)\Gamma(E)\left(z+\frac{h}{2}\right)^C}$$

$$\cdot\Bigg\{F^{(3)}\left[\begin{array}{l} \frac{C}{2},\frac{C+1}{2}::\frac{C'}{2},\frac{C'+1}{2};-;-;\beta;-;\frac{D}{2},\frac{D+1}{2}\ ; \\ \frac{E}{2},\frac{E+1}{2}::-;-;-;\frac{3}{2},\delta;\nu+1;\frac{1}{2}\ ; \end{array} \frac{4y^2}{(z+\frac{h}{2})^2},\frac{-\alpha^2}{(z+\frac{h}{2})^2},\left(\frac{z-\frac{h}{2}}{z+\frac{h}{2}}\right)^2\right]$$

$$+\frac{CD}{E}\left(\frac{z-h/2}{z+h/2}\right)$$

$$F^{(3)}\left[\begin{array}{l} \frac{C+1}{2},\frac{C+2}{2}::\frac{C'}{2},\frac{C'+1}{2};-;-;\beta;-;\frac{D+1}{2},\frac{D+2}{2}\ ; \\ \frac{E+1}{2},\frac{E+2}{2}::-;-;-;\frac{3}{2},\delta;\nu+1;\frac{3}{2}\ ; \end{array} \frac{4y^2}{(z+h/2)^2},\frac{-\alpha^2}{(z+h/2)^2},\left(\frac{z-\frac{h}{2}}{z+\frac{h}{2}}\right)^2\right]\Bigg\}. \tag{3.7}$$

# 4 Conclusion

In the present investigation, we have established on infinite series of three variables involving Whittaker, Bessel and generalized hypergeometric functions into a series of Srivastava's triple hypergeometric function $F^{(3)}$. A number of known and new transformations for Kamp$e'$ de F$e'$riet function $F^{(2)}$ and Legendre function are obtained as special cases. The result appears in the paper may be found useful in some areas of mathematical physics and engineering.

# References


[1] J. L Burchnall and T.W Chaundy, *Expansions of Appell's double hypergeometric functions-II,* Quart. J.Math.Oxford Ser. **12** (1941), 112-128.

[2] A. Erdelyi, *Higher transcendental functions*, Vol.I, McGraw-Hill, New York (1953).

[3] A. Erdelyi, *Table of integral transforms*, Vol.I, McGraw-Hill, New York (1954).

[4] A. Erdelyi, *Table of integral transforms*, Vol.II, McGraw-Hill, New York (1954).

[5] B. C. Carlson, *Some extensions of Lardner's relations between ${}_0F_3$ and Bessel functions*, SIAM J. Math. Anal. No. **2**, 232-242 (1970).

[6] B. Khan and M. A. Pathan, *Some transformations of Appell's function $F_4$* Tamkang J. Math. Vol.**13**(1), 25-35 (1982).

[7] A. P. Prudnikov, *Integrals and Series*, Vol.3, More special functions, Gorden and Breach Science Publishers, New York (1990).

[8] M. A. Pathan, *On a transformation of a general hypergeometric series of four variables*, Nederl , Akad. Wetensch. Proc Ser. A. 82=Indag Math 41, New York , 171-175 (1979).

[9] E.D. Rainville, *Special functions,* The Macmillan Company, New York (1960).

[10] H.M. Srivastava and H.L. Manocha, *A tretise on generating functions,* Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and Sons, New York (1984).

[11] R. K. Saxena, *Integrals involving the products of Bessel functions* Proc.Glasgow Math. Assoc.**6**, 130-132 (1984).

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