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Mean cordiality of some snake graphs

R. Ponraj and S. Sathish Narayanan

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Abstract Let f be a function from the vertex set $V(G)$ to $\{0,1,2\}$. For each edge uv assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called a mean cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i,j \in \{0,1,2\}$, where $v_f(x)$ and $e_f(x)$ respectively are denote the number of vertices and edges labeled with x $(x = 0,1,2)$. A graph with a mean cordial labeling is called a mean cordial graph. In this paper we investigate mean cordial labeling behavior of double triangular snake, alternate triangular snake, double alternate triangular snake.

1 Introduction

All graphs in this paper are finite, undirected and simple. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. Let p, q denotes the number of vertices and edges in G. Ponraj et al. defined the mean cordial labeling of a graph in [3]. Mean cordial labeling behavior of path, cycle, star, complete graph, wheel, comb, mG, $P_m \cup P_n$, P_n^2, triangular snake etc have been investigated in [3, 6]. Also, Albert william et al. [1] have studied about the mean cordial labeling behaviour of certain graphs like subdivision of a bistar $S(B_{m,n})$, particular type of caterpillar, Banana tree and path banana tree. Here we investigate the mean cordial labeling behavior of double triangular snake, alternate triangular snake, double alternate triangular snake. The symbol $\lceil x \rceil$ stands for smallest integer greater than or equal to x. Terms and definitions not defined here are used in the sense of Harary [4].

2 Preliminary Results

In this section we write some basic definitions and results which are needed for the next section.

Definition 2.1. Let f be a function from $V(G)$ to $\{0,1,2\}$. For each edge uv of G assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called a mean cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i,j \in \{0,1,2\}$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x $(x = 0,1,2)$ respectively. A graph with a mean cordial labeling is called a mean cordial graph.

Definition 2.2. The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.

Definition 2.3. An alternate triangular snake $A(T_n)$ is obtained from a path $u_1u_2\ldots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is every alternate edge of a path is replaced by C_3.

Definition 2.4. A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is, a double alternate triangular snake is obtained from a path $u_1u_2\ldots u_n$ by joining u_i and u_{i+1} (alternatively) to two new vertices v_i and w_i.

Definition 2.5. A double triangular snake $D(T_n)$ consists of two triangular snakes that have a common path.

Theorem 2.6. *[5]* The triangular snake T_n $(n > 1)$ is mean cordial iff $n \equiv 0 \pmod 3$.

3 Main Results

Theorem 3.1. Mean cordial labeling behaviour of Alternate triangular snake $A(T_n)$ is given below:

a. Mean cordial if the triangle starts from u_2 and ends with u_{n-1}.

b. Mean cordial if the triangle starts from u_1, ends with u_n and $n \equiv 0 \pmod 3$.

c. Not Mean cordial if the triangle starts from u_1, ends with u_n and $n \equiv 1, 2 \pmod 3$.

d. Mean cordial if the triangle starts from u_2, ends with u_n.

Proof. **Case a.** The triangle starts from u_2 and ends with u_{n-1}.
In this case $p = \frac{3n-2}{2}$, $q = 2n - 3$.
Sub case 1. $n \equiv 1 \pmod 3$.
Let $n = 3t + 1,\ t > 1$. Assign the label '0' to $t + 1$ path vertices $u_1, u_2, \ldots, u_{t+1}$. Then assign '2' to the next t path vertices; assign '1' to the remaining path vertices. Then we move to the vertices of degree 2. Label the vertices $v_1, v_2, \ldots, v_{t-2}$ by '0'. Then assign the label '2' to the vertices $v_{t-1}, v_t, \ldots, v_{2t-3}$. Finally assign the label '1' to the vertices $v_{2t-2}, v_{2t-1}, \ldots, v_{3t-5}$. The above vertex labeling f, satisfies the mean cordial condition by table 1.

i	0	1	2
$v_f(i)$	$\frac{n}{2}$	$\frac{n-2}{2}$	$\frac{n}{2}$
$e_f(i)$	$\frac{2n-5}{3}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$

Table 1.

When $t = 1$, the corresponding mean cordial labeling of $A(T_4)$ is given in figure 3.1.

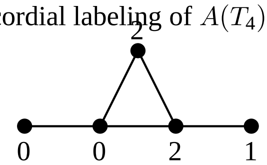

Figure 3.1

Sub case 2. $n \equiv 2 \pmod 3$.
Let $n = 3t+2$. Assign the label '0' to the vertices of the first $\frac{t}{2}$ triangles and then '1' to the next $\frac{t}{2}$ triangles. Then assign the label '2' to the vertices of the remaining $\frac{t}{2}$ triangles. Finally assign the label $'0'$, $'2'$ to the pendent vertices u_1 and u_n respectively. In this case the vertex and edge condition is given in table 2.

i	0	1	2
$v_f(i)$	$\frac{n}{2}$	$\frac{n-2}{2}$	$\frac{n}{2}$
$e_f(i)$	$\frac{2n-4}{3}$	$\frac{2n-4}{3}$	$\frac{2n-1}{3}$

Table 2.

Sub case 3. $n \equiv 0 \pmod 3$.
Let $n = 3t$. Assign the labels to the vertices as in sub case 2 then relabel the vertices u_1 by 2 and the vertex u_{2t+1}, a vertex of t^{th} triangle by 2. The table 3 shows that the above vertex labeling f is a mean cordial labeling.

i	0	1	2
$v_f(i)$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n-2}{2}$
$e_f(i)$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$

Table 3.

For the cases b & c, $p = \frac{3n}{2}$ and $q = 2n - 1$.
Case b. The triangle starts from u_1, ends with u_n and $n \equiv 0 \pmod 3$.
Let $n = 3t$. Assign the label $'0'$ to the vertices of the first $\frac{t}{2}$ triangles. Then $'1'$ to the vertices of the next $\frac{t}{2}$ triangles. Finally assign the label $'2'$ to the vertices of the remaining $\frac{t}{2}$ triangles. The table 4 establishes that the above vertex labeling f, satisfies the mean cordiality condition.
Case c. The triangle starts from u_1, ends with u_n and $n \equiv 1, 2 \pmod 3$.

i	0	1	2
$v_f(i)$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$
$e_f(i)$	$\frac{2n-3}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$

Table 4.

Sub case 1. $n \equiv 1 \pmod 3$.

Suppose f is a mean cordial labeling, then $v_f(0) = v_f(1) = v_f(2) = \frac{n}{2}$. This forces the maximum value of $e_f(0)$ is $\lfloor \frac{2n-1}{3} \rfloor - 1$. That is $e_f(0) \leq \lfloor \frac{2n-1}{3} \rfloor - 1$. Since the size of $A(T_n)$ is $2n-1$, f can not be satisfies the edge condition of the mean cordial labeling.

Sub case 2. $n \equiv 2 \pmod 3$.

Suppose f is a mean cordial labeling, then $v_f(0) = v_f(1) = v_f(2) = \frac{n}{2}$. In this case $e_f(0) \leq \frac{2n-4}{3}$, a contradiction.

Case d. The triangle starts from u_2, ends with u_n.

Here $p = \frac{3n-1}{2}$, $q = 2n-2$.

Sub case 1. $n \equiv 0 \pmod 3$.

Let $n = 3t,\ t > 1$. Assign the label $'0'$ to the first $t+1$ vertces $u_1, u_2, \ldots, u_{t+1}$ of the path. Then label the next $t-1$ vertices $u_{t+2}, u_{t+3}, \ldots, u_{2t}$ by $'1'$ and assign the label $'2'$ to the remaining vertices of the path. Now we move to the vertices with degree 2. Assign the label $'0'$ to the first $\frac{t-1}{2}$ vertices and then the next $\frac{t+1}{2}$ vertices receives the label $'1'$. Finally assign the label $'2'$ to the remaining $\frac{t-1}{2}$ vertices. From the table 5, we can conclude that the above vertex labeling, say f, is a mean cordial labeling.

i	0	1	2
$v_f(i)$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n-1}{2}$
$e_f(i)$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$	$\frac{2n}{3}$

Table 5.

When $t = 1$, the mean cordial labeling of $A(T_3)$ is given in figure 3.2.

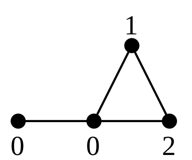

Figure 3.2

Sub case 2. $n \equiv 1 \pmod 3$.

Let $n = 3t+1$. Assign the label $'0'$ to the first $\frac{t}{2}$ triangles and then label the vertices of the next $\frac{t}{2}$ triangles by $'1'$. Then assign the label $'2'$ to the vertices of the remaining $\frac{t}{2}$ triangles. Finally assign the label $'0'$ to the vertex u_1. The vertex and edge conditions of the above labeling f is given in table 6.

i	0	1	2
$v_f(i)$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n-1}{2}$
$e_f(i)$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$

Table 6.

Sub case 3. $n \equiv 2 \pmod 3$.

Let $n = 3t+2$. Consider the path vertices $u_1, u_2, \ldots, u_n$. Assign the label $'0'$ to the vertices $u_1, u_2, \ldots, u_{t+1}$ and label the next $t+1$ vertices $u_{t+2}, u_{t+3}, \ldots, u_{2t+2}$ by $'1'$. Then assign the label $'2'$ to the vertices $u_{2t+3}, u_{2t+4}, \ldots, u_{3t+2}$. Then we move to the vertices of degree 2. These are labeled in the following pattern.

w_1	w_2	$\ldots\ldots$	w_{t-1}	w_t	w_{t+1}	$\ldots\ldots$	w_{2t-3}	w_{2t-2}	w_{2t-1}	$\ldots\ldots$	w_{3t-4}
0	0	$\ldots\ldots$	0	1	1	$\ldots\ldots$	1	2	2	$\ldots\ldots$	2

Finally assign the label $'0'$ to the pendent vertex. In this case the following table 7 shows that the above vertex labeling f, is a mean cordial labeling.

i	0	1	2
$v_f(i)$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n-1}{2}$
$e_f(i)$	$\frac{2n-4}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$

Table 7.

□

A mean cordial labeling of $A(T_{12})$ with the condition that the triagle starts from u_2, ends with u_{n-1} is given in figure 3.3

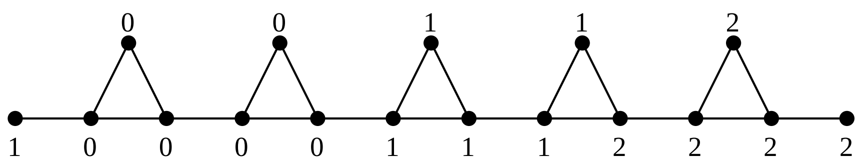

Figure 3.3

Theorem 3.2. Mean cordial labeling behaviour of double alternate triangular snake $DA(T_n)$ is given below:

a. $DA(T_n)$ is mean cordial if the triangle starts from u_2 and ends with u_{n-1} and $n \equiv 0, 2 \pmod 3$.

b. Not mean cordial if the triangle starts from u_2, ends with u_{n-1} and $n \equiv 1 \pmod 3$.

c. Mean cordial if the triangle starts from u_1, ends with u_n, $n > 2$. In this case $DA(T_2)$ is not mean cordial.

d. Mean cordial if the triangle starts from u_2, ends with u_n and $n \equiv 1 \pmod 3$.

e. Not mean cordial if the triangle starts from u_2, ends with u_n and $n \equiv 0, 2 \pmod 3$.

Proof. For the cases a & b, $p = 2n - 2$ and $q = 3n - 5$.

Case a. The triangles starts from u_2 and ends with u_{n-1} and $n \equiv 0, 2 \pmod 3$.

Sub case 1. $n \equiv 0 \pmod 3$.

noindent Assign the label to the vertices of the first $\frac{t}{2}$ double triangles by $'0'$, next $\frac{t}{2}$ double triangles by $'1'$ and the last $\frac{t-2}{2}$ double triangles by $'2'$. Then replace the label of the vertex u_{2t+1} by $'2'$. Finally assign the label $'2'$ to the pendent vertices. The labeling f given in above is mean cordial from table 8.

i	0	1	2
$v_f(i)$	$\frac{2n}{3}$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$
$e_f(i)$	$n-1$	$n-2$	$n-2$

Table 8.

Sub case 2. $n \equiv 2 \pmod 3$.

Label the vertices of $DA(T_n)$ as in subcase 1 and assign the label 2 to the vertices of the last double triangles. Then replace the label of the vertex u_{t+1} by $'0'$. The vertex condition and edge condition of the labelings f is shown in table 9.

i	0	1	2
$v_f(i)$	$\frac{2n-1}{3}$	$\frac{2n-4}{3}$	$\frac{2n-1}{3}$
$e_f(i)$	$n-2$	$n-1$	$n-2$

Table 9.

Case b. The triangles starts from u_2, ends with u_{n-1} and $n \equiv 1 \pmod 3$.

Suppose f is a mean cordial labeling. Then $v_f(0) = v_f(1) = v_f(2) = \frac{2n-2}{3}$. But $e_f(0) \leq \left\lfloor \frac{3n-5}{3} \right\rfloor - 1$. This is a contradiction.

Case c. The triangles starts from u_1, ends with u_n.

In this case $p = 2n$ and $q = 3n - 1$.
Consider the graph $DA(T_2)$. Suppose f is a mean cordial labeling. Then we have two cases. $v_f(0) = 2$ or $v_f(0) = 1$. If $v_f(0) = 1$ then $e_f(0) = 0$, a contradiction. Suppose $v_f(0) = 2$. Note that the label $'0'$ should be assigned to the adjacent vertices (otherwise $e_f(0) = 0$). Then $e_f(0) = e_f(2) = 1$, $e_f(1) = 3$ or $e_f(0) = 1$, $e_f(1) = 4$, $e_f(2) = 0$, a contradiction. Therefore $DA(T_2)$ is not mean cordial.

Sub case 1. $n \equiv 0 \pmod 3$.

Assign the label to the vertices of the first $\frac{t}{2}$ double triangles by $'0'$. Put the label $'1'$ to the vertices of the next $\frac{t}{2}$ triangles. Finally assign the label $'2'$ to the vertices of the last $\frac{t}{2}$ double triangles. The table 10 shows that f is a mean cordial labeling.

i	0	1	2
$v_f(i)$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$e_f(i)$	$n-1$	n	n

Table 10.

Sub case 2. $n \equiv 1 \pmod 3$.

Assign the label $'0'$ to the first $t+1$ path vertices then assign the label $'1'$ to the next t vertices of the path. The remaining t vertices of the path are labeled by $'2'$. The vertices v_i and w_i are labeled as given below.

v_1	v_2	......	v_t	v_{t+1}	v_{t+2}	......	v_{2t}	v_{2t+1}	v_{2t+2}	......	v_{3t-1}
0	0	......	0	1	1	......	1	2	2	......	2

w_1	w_2	......	w_{t-1}	w_t	w_{t+1}	......	w_{2t-1}	w_{2t}	w_{2t+1}	......	w_{3t-1}
0	0	......	0	1	1	......	1	2	2	......	2

The values of $v_f(i)$ and $e_f(i)$ are given in table 11

i	0	1	2
$v_f(i)$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$
$e_f(i)$	$n-1$	n	n

Table 11.

Sub case 3. $n \equiv 2 \pmod 3$.

Assign the label $'0'$ to the vertices of first $\frac{t+2}{2}$ double triangles then assign the label to the next $\frac{t}{2}$ vertices of the double triangles by $'1'$ and the last $\frac{t}{2}$ triangles by 2. Finally replace the labels of the vertices u_{t+2}, $w_{\frac{t+2}{2}}$, u_{2t+2}, w_{t+1} by 1, 1, 1, 2 respectively. The following table 12 shows that the above vertex labeling f is a mean cordial labeling.

i	0	1	2
$v_f(i)$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$
$e_f(i)$	$n-1$	n	n

Table 12.

For the cases d & e, $p = 2n - 1$ and $q = 3n - 3$.

Case d. The triangle starts from u_2, ends with u_n and $n \equiv 1 \pmod 3$.
Assign the label $'0'$ to the vertices of the first $\frac{t}{2}$ double triangles, $'1'$ to the vertices of the next $\frac{t}{2}$ double triangles and $'2'$ to the vertices of the last $\frac{t}{2}$ double triangles. Put the label $'0'$ to the pendent vertex u_1. Table 13 establish that the labeling f given above is a mean cordial labeling.

Case e. The triangle starts from u_2, ends with u_n.

Sub case 1. $n \equiv 0 \pmod 3$.

i	0	1	2
$v_f(i)$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$
$e_f(i)$	$n-1$	$n-1$	$n-1$

Table 13.

Suppose f is a mean cordial labeling. In this case, either $v_f(0) = \frac{2n}{3}$ or $\frac{2n-3}{3}$. In both cases $e_f(0) \le n-2$, a contradiction.

Sub case 2. $n \equiv 2 \pmod 3$.

Here $v_f(0) = \frac{2n-3}{3}$. But $e_f(0) \le n-2$. This contradiction proves that there does not exists a mean cordial labeling. □

A mean cordial labeling of $DA(T_{10})$ with the condition that the triagles starts from u_1, ends with u_n is given in figure 3.4

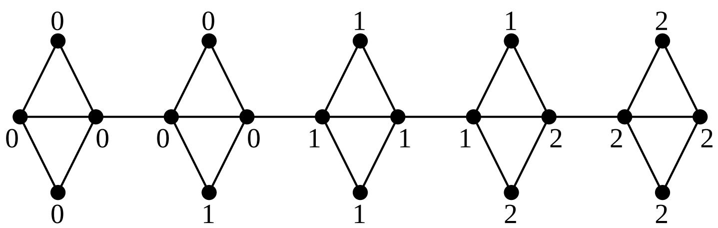

Figure 3.4

Theorem 3.3. *The double triangular snake $D(T_n)$ is mean cordial iff $n > 3$.*

Proof. **Case 1.** $n = 2$.

Follows from case c of theorem 3.2.

Case 2. $n = 3$.

In this case, $v_f(0) = 2$ or 3. If $v_f(0) = 2$ then $e_f(0) \le 1$ which is not possible. If $v_f(0) = 3$ then $'0'$ should be labeled to the vertices of any triangle. Otherwise the value of $e_f(0)$ lower than 3. In the case of $'0'$ labeled in the vertices of the triangle, the value $e_f(2)$ is not greater than 2. This is a contradiction to the size. Hence $D(T_3)$ is not mean cordial.

Case 3. $n > 3$.

Sub case 1. $n \equiv 1 \pmod 3$.

Let $n = 3t+1$. Assign the label $'0'$ to v_i $(1 \le i \le t)$, $'1'$ to v_{t+i} $(1 \le i \le t)$ and $'2'$ to v_{2t+i} $(1 \le i \le t+1)$. Label the vertices u_i $(1 \le i \le t+1)$ by $'0'$, u_{t+1+i} $(1 \le i \le t)$ by 1 and u_{2t+1+i} by 2. Then we move to the vertex w_i. Assign the labels to w_i as in v_i. The following table 14 shows that the above labeling f is a mean cordial labeling.

i	0	1	2
$v_f(i)$	n	$n-1$	$n-1$
$e_f(i)$	$\frac{5n-5}{3}$	$\frac{5n-5}{3}$	$\frac{5n-5}{3}$

Table 14.

Sub case 2. $n \equiv 0 \pmod 3$.

Let $n = 3t,\ t > 1$. Assign the label $'0'$ to the vertices u_i $(1 \le i \le t+1)$, v_i $(1 \le i \le t)$ and w_i $(1 \le i \le t-1)$. Put the label $'1'$ to the vertices u_j $(t+2 \le j \le 2t)$, v_j $(t+1 \le j \le 2t)$ and w_j $(t \le j \le 2t-1)$. Finally assign the label $'2'$ to the vertices u_r $(2t+1 \le r \le 3t)$, v_r $(2t+1 \le r \le 3t-1)$ and w_r $(2t \le r \le 3t-1)$. The table 15 given below shows that the above labeling f is a mean cordial labeling.

i	0	1	2
$v_f(i)$	n	$n-1$	$n-1$
$e_f(i)$	$\frac{5n-6}{3}$	$\frac{5n-6}{3}$	$\frac{5n-3}{3}$

Table 15.

Sub case 3. $n \equiv 2 \pmod 3$.

Let $n = 3t + 2$. First we consider the path vertices. Assign the label $'0'$ to u_i $(1 \leq i \leq t + 1)$, $'1'$ to u_j $(t + 2 \leq j \leq 2t + 2)$ and 2 to u_r $(2t + 3 \leq r \leq 3t)$. Now we move to the vertices v_i $(1 \leq i \leq n - 1)$. The first $t + 1$ vertices are labeled by $'0'$ and the vertices v_i $(t + 2 \leq i \leq 2t + 1)$ are labeled by 1 then the last t vertices of v_i are labeled by $'2'$. The vertex labeling of w_i $(1 \leq i \leq n - 1)$ is given below.

w_1	w_2	......	w_t	w_{t+1}	w_{t+2}	......	w_{2t}	w_{2t+1}	w_{2t+2}	......	w_{3t-1}
0	0	......	0	1	1	......	1	2	2	......	2

Let f be the labeling defined above. The values $e_f(i)$ and $v_f(i)$ where $i = 0, 1, 2$ given in table 16 proves that f is a mean cordial labeling.

i	0	1	2
$v_f(i)$	n	$n-1$	$n-1$
$e_f(i)$	$\frac{5n-7}{3}$	$\frac{5n-4}{3}$	$\frac{5n-4}{3}$

Table 16.

□

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Author information

R. Ponraj and S. Sathish Narayanan, Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627 412, Tamil Nadu, India.
E-mail: `ponrajmaths@gmail.com; sathishrvss@gmail.com`