Palestine Journal of Mathematics

Vol. 4(2) (2015) , 446–454

A New Similarity Measure of Generalized Fuzzy numbers Based on Left and Right Apex Angles (II)

Salim Rezvani and M. Mousavi

Communicated by Ayman Badawi

MSC 2010 Classifications: Primary 47S40; Secondary 08A72, 03E72.

Keywords and phrases: Generalized Trapezoidal Fuzzy Numbers, Similarity Measures, Left and Right Apex Angles.

Abstract In part (I)[12], calculated two fuzzy numbers on left and right apex angles in measure of generalized fuzzy numbers. As a continuation of the part I, we considered fuzzy numbers do separately on left and right apex angles. Means, in part (I) for example we have $a = \frac{1}{n}\sum_1^n b_i - \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i - a_i)]$, but this paper we calculated $a = b - \tan(\tan^{-1}(b-a))$.

1 Introduction

The similarity measure of fuzzy numbers is very important in many research fields such as pattern recognition [[5],[6]] and risk analysis in fuzzy environment [[1],[3],[15]]. Some methods have been presented to calculate the degree of similarity between fuzzy numbers [[1]-[4],[8],[15]]. In [16], Wen presented A modified similarity measure of generalized fuzzy numbers. Pandey et al.,[7] proposed a new aggregation operator for trapezoidal fuzzy numbers based on the arithmetic means of the left and right apex angles. Also, Rezvani[8] proposed a new similarity measure of generalized fuzzy numbers based on left and right apex angles I.
In part I[8], calculated two fuzzy numbers on left and right apex angles in measure of generalized fuzzy numbers. As a continuation of the part I, we considered fuzzy numbers do separately on left and right apex angles. Means, in part I for example we have $a = \frac{1}{n}\sum_1^n b_i - \tan[\frac{1}{n}\sum_1^n \tan^{-1}(b_i - a_i)]$, but this paper we calculated $a = b - \tan(\tan^{-1}(b-a))$.

2 Preliminaries

Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions,

(i) μ_A is a continuous mapping from R to the closed interval [0,1],

(ii) $\mu_A(x) = 0, -\infty < x \leq a$,

(iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a, b]$,

(iv) $\mu_A(x) = w, b \leq x \leq c$,

(v) $\mu_A(x) = R(x)$ is strictly decreasing on $[c, d]$,

(vi) $\mu_A(x) = 0, d \leq x < \infty$

Where $0 < w \leq 1$ and a, b, c, and d are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by

$$A = (a, b, c, d; w) . \tag{2.1}$$

A $A = (a, b, c, d; w)$ is a fuzzy set of the real line R whose membership function $\mu_A(x)$ is defined as

$$\mu_A(x) = \begin{cases} w\frac{x-a}{b-a} & if \quad a \leq x \leq b \\ w & if \quad b \leq x \leq c \\ w\frac{d-x}{d-c} & if \quad c \leq x \leq d \\ 0 & Otherwise \end{cases} \tag{2.2}$$

3 Proposed Approach

In this section some important results, that are useful for the proposed approach, are proved.

The concept of the method to calculate the degree of similarity between generalized fuzzy numbers, the horizontal center-of-gravity, the perimeter, the height and the area of the two fuzzy numbers are considered.

Suppose that $A_1 = (a_1, b_1, c_1, d_1; w_1)$ and $A_2 = (a_2, b_2, c_2, d_2; w_2)$ be the generalized trapezoidal fuzzy numbers. where $0 \leq a_1 \leq b_1 \leq c_1 \leq d_1 \leq 1$ and $0 \leq a_2 \leq b_2 \leq c_2 \leq d_2 \leq 1$. Then the degree of similarity $S(A_1, A_2)$ between the generalized trapezoidal fuzzy numbers A_1 and A_2 is calculated as follows:

$$S(A_1, A_2) = [1 - \mid x^*_{A_1} - x^*_{A_2} \mid] \times [1 - \mid w_{A_1} - w_{A_2} \mid] \times \frac{min(P(A_1), P(A_2)) + min(A(A_1), A(A_2))}{max(P(A_1), P(A_2)) + max(A(A_1), A(A_2))} \tag{3.1}$$

Where $x^*_{A_1}$ and $x^*_{A_2}$ are the horizontal center-of-gravity of the generalized trapezoidal fuzzy numbers A_1 and A_2 is calculated as follows:

$$x^*_{A_1} = \frac{y^*_{A_1}(c_1 + b_1) + (d_1 + a_1)(w_{A_1} - y^*_{A_1})}{2w_{A_1}} \tag{3.2}$$

$$y^*_{A_1} = \begin{cases} \frac{w_{A_1}(\frac{c_1 - b_1}{d_1 - a_1} + 2)}{6} & if \quad a_1 \neq d_1 \ and \ 0 < w_{A_1} \leq 1 \\ \frac{w_{A_1}}{2} & if \quad a_1 = d_1 \ and \ 0 < w_{A_1} \leq 1 \end{cases} \tag{3.3}$$

$P(A_1)$ and $P(A_2)$ are the perimeters of two generalized trapezoidal fuzzy numbers which are calculated as follows:

$$P(A_1) = \sqrt{(a_1 - b_1)^2 + w^2_{A_1}} + \sqrt{(c_1 - d_1)^2 + w^2_{A_1}} + (c_1 - b_1) + (d_1 - a_1) \tag{3.4}$$

$$P(A_2) = \sqrt{(a_2 - b_2)^2 + w^2_{A_2}} + \sqrt{(c_2 - d_2)^2 + w^2_{A_2}} + (c_2 - b_2) + (d_2 - a_2) \tag{3.5}$$

$A(A_1)$ and $A(A_2)$ are the areas of two generalized trapezoidal fuzzy numbers which are calculated as follows:

$$A(A_1) = \frac{1}{2} w_{A_1}(c_1 - b_1 + d_1 - a_1) \tag{3.6}$$

$$A(A_2) = \frac{1}{2} w_{A_2}(c_2 - b_2 + d_2 - a_2) \tag{3.7}$$

The larger the value of $S(A_1, A_2)$, the more the similarity measure between two generalized trapezoidal fuzzy numbers A_1 and A_2.

Aggregation operations on fuzzy numbers are operations by which several fuzzy numbers are combined to produce a single fuzzy number.

i) Arithmetic Mean

The arithmetic mean aggregation operator defined on n trapezoidal fuzzy numbers (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2),..., (a_n, b_n, c_n, d_n) produces the result (a, b, c, d) where

$a = \frac{1}{n} \sum_1^n a_i$, $b = \frac{1}{n} \sum_1^n b_i$, $c = \frac{1}{n} \sum_1^n c_i$ and $d = \frac{1}{n} \sum_1^n d_i$.

ii) Geometric Mean

The arithmetic mean aggregation operator defined on n trapezoidal fuzzy numbers (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2),..., (a_n, b_n, c_n, d_n) produces the result (a, b, c, d) where

$a = (\prod_1^n a_i)^{\frac{1}{n}}$, $b = (\prod_1^n b_i)^{\frac{1}{n}}$, $c = (\prod_1^n c_i)^{\frac{1}{n}}$ and $d = (\prod_1^n d_i)^{\frac{1}{n}}$.

An Aggregation Operators trapezoidal fuzzy numbers are given in [7]

Consider the trapezoidal fuzzy number shown in Figure 1. If the value of this trapezoidal fuzzy number is $v \in [b, c]$ the corresponding possibility $\mu = 1$. The left side apex angle of this trapezoidal fuzzy number is $\mathcal{L}apb$. The right side apex angle of this trapezoidal fuzzy number is $\mathcal{L}drc$. The left and right side apex angles of the trapezoid refer to the apex angles subtended to the left and the right of the interval [b,c] respectively. But

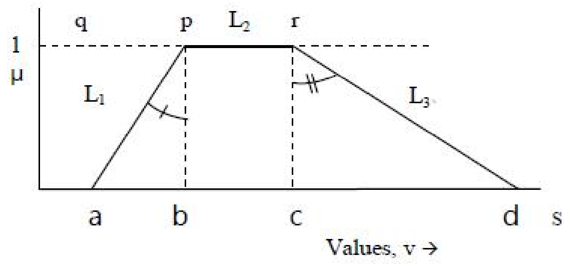

Figure 1. Trapezoidal Fuzzy Number

$$\mathcal{L}apb = \frac{\pi}{2} - \mathcal{L}bap \tag{3.8}$$

and

$$\mathcal{L}drc = \mathcal{L}sdr - \frac{\pi}{2} \tag{3.9}$$

Considering the left side and averaging over n trapezoidal fuzzy numbers we have

$$\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i = \frac{1}{n}\sum_{1}^{n}(\mathcal{L}(\frac{\pi}{2} - bap)_i) \tag{3.10}$$

$$\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i = \frac{\pi}{2} - \frac{1}{n}\sum_{1}^{n}(\mathcal{L}bap)_i \tag{3.11}$$

The left side of the above equation represents the contributions of the left lines(aggregate apex angle). It can be seen that

$$\tan(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i) = \frac{1}{\tan(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}bap)_i)} \tag{3.12}$$

It can be shown that

$$\tan(\frac{1}{n}\sum_{1}^{n}(\mathcal{L}apb)_i) = [tan(\frac{1}{n}\sum_{1}^{n}tan^{-1}(b_i - a_i))]^{-1} \tag{3.13}$$

In this paper, we considered fuzzy numbers do separately on left and right apex angles. Means

$$a = b - \tan(\tan^{-1}(b - a)) \tag{3.14}$$

and

$$d = c + \tan[\tan^{-1}(d - c)] \tag{3.15}$$

So, Left and Right Apex Angles combines the concepts of the center of gravity, the area, the perimeter and the height of generalized fuzzy numbers for new approach. Means, Put the values (16,17) in (4-9). The proposed method is now presented as follows:

$*$ Step 1

Find y_A^* and y_B^*

$$y_A^* = \begin{cases} w_A \times [(c_A - b_A/c_A + \tan^{-1}(d_A - c_A)] - b_A + \tan(\tan^{-1}(b_A - a_A)) + 2)/6 \\ \quad if \quad b_A - \tan(\tan^{-1}(b_A - a_A)) \neq c_A + \tan[\tan^{-1}(d_A - c_A)] \quad and \quad 0 < w_A \leq 1 \\ w_A/2 \\ \quad if \quad b_A - \tan(\tan^{-1}(b_A - a_A)) = c_A + \tan[\tan^{-1}(d_A - c_A)] \quad and \quad 0 < w_A \leq 1 \end{cases} \tag{3.16}$$

and

$$y_B^* = \begin{cases} w_B \times [(c_B - b_B/c_B + \tan^{-1}(d_B - c_B)] - b_B + \tan(\tan^{-1}(b_B - a_B)) + 2)/6 \\ \quad if \quad b_B - \tan(\tan^{-1}(b_B - a_B)) \neq c_B + \tan[\tan^{-1}(d_B - c_B)] \quad and \quad 0 < w_B \leq 1 \\ w_A/2 \\ \quad if \quad b_B - \tan(\tan^{-1}(b_B - a_B)) = c_B + \tan[\tan^{-1}(d_B - c_B)] \quad and \quad 0 < w_B \leq 1 \end{cases} \tag{3.17}$$

∗ Step 2

Find x_A^* and x_B^*

$$x_A^* = [y_A^*(c_A + b_A) + (c_A + \tan[\tan^{-1}(d_A - c_A)] + b_A - \tan(\tan^{-1}(b_A - a_A)))(w_A - y_A^*)/2w_A \tag{3.18}$$

and

$$x_B^* = [y_B^*(c_B + b_B) + (c_B + \tan[\tan^{-1}(d_B - c_B)] + b_B - \tan(\tan^{-1}(b_B - a_B)))(w_B - y_B^*)/2w_B \tag{3.19}$$

∗ Step 3

Find $P(A)$ and $P(B)$

$$\begin{aligned} P(A) &= \sqrt{(b_A - \tan(\tan^{-1}(b_A - a_A)) - b_A)^2 + w_A^2} + \sqrt{(c_A - c_A - \tan[\tan^{-1}(d_A - c_A)])^2 + w_A^2} + \\ &\quad (c_A - b_A) + (c_A + \tan[\tan^{-1}(d_A - c_A)] - b_A + \tan(\tan^{-1}(b_A - a_A))) \\ &= \sqrt{(-\tan(\tan^{-1}(b_A - a_A)))^2 + w_A^2} + \sqrt{(-\tan[\tan^{-1}(d_A - c_A)])^2 + w_A^2} + \\ &\quad (c_A - b_A) + (c_A + \tan[\tan^{-1}(d_A - c_A)] - b_A + \tan(\tan^{-1}(b_A - a_A))) \end{aligned} \tag{3.20}$$

and

$$\begin{aligned} P(B) &= \sqrt{(b_B - \tan(\tan^{-1}(b_B - a_B)) - b_B)^2 + w_B^2} + \sqrt{(c_B - c_B - \tan[\tan^{-1}(d_B - c_B)])^2 + w_B^2} + \\ &\quad (c_B - b_B) + (c_B + \tan[\tan^{-1}(d_B - c_B)] - b_B + \tan(\tan^{-1}(b_B - a_B))) \\ &= \sqrt{(-\tan(\tan^{-1}(b_B - a_B)))^2 + w_B^2} + \sqrt{(-\tan[\tan^{-1}(d_B - c_B)])^2 + w_B^2} + \\ &\quad (c_B - b_B) + (c_B + \tan[\tan^{-1}(d_B - c_B)] - b_B + \tan(\tan^{-1}(b_B - a_B))) \end{aligned} \tag{3.21}$$

∗ Step 4

Find $A(A)$ and $A(B)$

$$A(A) = \frac{1}{2} w_A (c_A - b_A + c_A + \tan[\tan^{-1}(d_A - c_A)] - b_A + \tan(\tan^{-1}(b_A - a_A))) \tag{3.22}$$

and

$$A(B) = \frac{1}{2} w_B (c_B - b_B + c_B + \tan[\tan^{-1}(d_B - c_B)] - b_B + \tan(\tan^{-1}(b_B - a_B))) \tag{3.23}$$

∗ Step 5

Calculating $S(A, B)$ with eq.(3) .

4 A comparison of the similarity measures

In this section, we extend 15 sets of fuzzy numbers presented in Wei and Chen[15] into 18 sets of fuzzy numbers, shown in Fig. 2, and compare the calculation results of the proposed method with the results of the existing similarity measures, shown in Table 1.

Example 4.1. Let $A = (0.1, 0.2, 0.3, 0.4; 1)$ and $B = (0.1, 0.25, 0.25, 0.4; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.25 - 0.25 \mid] \times [1- \mid 1-1 \mid] \times \frac{min(2.41, 2.32) + min(0.15, 0.2)}{max(2.41, 2.32) + max(0.15, 0.2)}$$

$$= 1 \times 1 \times \frac{2.32 + 0.15}{2.41 + 0.2} = 0.9464$$

Example 4.2. Let $A = (0.1, 0.2, 0.3, 0.4; 1)$ and $B = (0.1, 0.2, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.25 - 0.25 \mid] \times [1- \mid 1-1 \mid] \times \frac{min(2.41, 2.41) + min(0.15, 0.15)}{max(2.41, 2.41) + max(0.15, 0.15)} = 1$$

Example 4.3. Let $A = (0.1, 0.2, 0.3, 0.4; 1)$ and $B = (0.4, 0.55, 0.55, 0.7; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.55 - 0.25 \mid] \times [1- \mid 1-1 \mid] \times \frac{min(2.41, 2.32) + min(0.2, 0.15)}{max(2.41, 2.32) + max(0.2, 0.15)}$$

$$= 0.7 \times 1 \times \frac{2.32 + 0.15}{2.41 + 0.2} = 0.6625$$

Example 4.4. Let $A = (0.1, 0.2, 0.3, 0.4; 1)$ and $B = (0.4, 0.5, 0.6, 0.7; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.55 - 0.25 \mid] \times [1- \mid 1-1 \mid] \times \frac{min(2.41, 2.41) + min(0.2, 0.2)}{max(2.41, 2.41) + max(0.2, 0.2)}$$

$$= 0.7 \times 1 \times 1 = 0.7$$

Example 4.5. Let $A = (0.1, 0.2, 0.3, 0.4; 1)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.8)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.2 - 0.25 \mid] \times [1- \mid 1-0.8 \mid] \times \frac{min(2.41, 2.02) + min(0.2, 0.16)}{max(2.41, 2.02) + max(0.2, 0.16)}$$

$$= 0.95 \times 0.8 \times 0.83 = 0.6308$$

Example 4.6. Let $A = (0.3, 0.3, 0.3, 0.3; 1)$ and $B = (0.3, 0.3, 0.3, 0.3; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.3 - 0.3 \mid] \times [1- \mid 1-1 \mid] \times \frac{min(2,2) + min(0,0)}{max(2,2) + max(0,0)} = 1$$

Example 4.7. Let $A = (0.2, 0.2, 0.2, 0.2; 1)$ and $B = (0.3, 0.3, 0.3, 0.3; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.3 - 0.2 \mid] \times [1- \mid 1-1 \mid] \times \frac{min(2,2) + min(0,0)}{max(2,2) + max(0,0)}$$

$$= 0.9 \times 1 \times 1 = 0.9$$

Example 4.8. Let $A = (0.1, 0.2, 0.2, 0.3; 1)$ and $B = (0.3, 0.3, 0.3, 0.3; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.3 - 0.2 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.21, 2) + min(0.1, 0)}{max(2.21, 2) + max(0.1, 0)}$$

$$= 0.9 \times 1 \times 0.866 = 0.7794$$

Example 4.9. Let $A = (0.1, 0.2, 0.2, 0.3; 1)$ and $B = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.3 - 0.2 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.21, 2.21) + min(0.1, 0.1)}{max(2.21, 2.21) + max(0.1, 0.1)}$$

$$= 0.9 \times 1 \times 1 = 0.9$$

Example 4.10. Let $A = (0.1, 0.4, 0.4, 0.7; 1)$ and $B = (0.3, 0.4, 0.4, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.4 - 0.4 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.68, 2.21) + min(0.3, 0.1)}{max(2.68, 2.21) + max(0.3, 0.1)}$$

$$= 1 \times 1 \times 0.7752 = 0.7752$$

Example 4.11. Let $A = (0.2, 0.3, 0.5, 0.6; 1)$ and $B = (0.3, 0.4, 0.4, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.4 - 0.4 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.61, 2.21) + min(0.3, 0.1)}{max(2.61, 2.21) + max(0.3, 0.1)}$$

$$= 1 \times 1 \times 0.7938 = 0.7938$$

Example 4.12. Let $A = (0.4, 0.4, 0.4, 0.8; 1)$ and $B = (0.3, 0.4, 0.4, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.4 - 0.53 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(1.48, 2.21) + min(0.2, 0.1)}{max(1.48, 2.21) + max(0.2, 0.1)}$$

$$= 0.87 \times 1 \times 0.656 = 0.5707$$

Example 4.13. Let $A = (0.2, 0.3, 0.4, 0.5; 1)$ and $B = (0.3, 0.4, 0.5, 0.6; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.43 - 0.35 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.41, 2.41) + min(0.2, 0.2)}{max(2.41, 2.41) + max(0.2, 0.2)}$$

$$= 0.92 \times 1 \times 1 = 0.92$$

Example 4.14. Let $A = (0.1, 0.2, 0.2, 0.3; 1)$ and $B = (0.1, 0.2, 0.2, 0.3; 0.7)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.14 - 0.2 \mid] \times [1- \mid 1 - 0.7 \mid] \times \frac{min(2.21, 1.62) + min(0.1, 0.07)}{max(2.21, 1.62) + max(0.1, 0.07)}$$

$$= 0.94 \times 0.7 \times 0.732 = 0.4817$$

Example 4.15. Let $A = (0.1, 0.2, 0.2, 0.3; 1)$ and $B = (0.2, 0.2, 0.2, 0.2; 0.7)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.12 - 0.2 \mid] \times [1- \mid 1 - 0.7 \mid] \times \frac{min(2.21, 1.4) + min(0.1, 0)}{max(2.21, 1.4) + max(0.1, 0)}$$

$$= 0.92 \times 0.7 \times 0.61 = 0.3928$$

Example 4.16. Let $A = (0.1, 0.4, 0.4, 0.7; 0.825)$ and $B = (0.3, 0.4, 0.4, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.4 - 0.33 \mid] \times [1- \mid 0.825 - 1 \mid] \times \frac{min(2.36, 2.21) + min(0.25, 0.1)}{max(2.36, 2.21) + max(0.25, 0.1)}$$

$$= 0.93 \times 0.825 \times 0.88 = 0.6752$$

Example 4.17. Let $A = (0.2, 0.3, 0.5, 0.6; 0.79205)$ and $B = (0.3, 0.4, 0.4, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.4 - 0.32 \mid] \times [1- \mid 0.79205 - 1 \mid] \times \frac{min(1.4, 2.21) + min(0.24, 0.1)}{max(1.4, 2.21) + max(0.24, 0.1)}$$

$$= 0.92 \times 0.79205 \times 0.61 = 0.4445$$

Example 4.18. Let $A = (0.2, 0.3, 0.5, 0.6; 1)$ and $B = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy number, then

$$S(A,B) = [1- \mid 0.3 - 0.4 \mid] \times [1- \mid 1 - 1 \mid] \times \frac{min(2.61, 2.21) + min(0.3, 0.1)}{max(2.61, 2.21) + max(0.3, 0.1)}$$

$$= 0.9 \times 1 \times 0.79 = 0.711$$

Table (1): A comparison of the ranking results for different approaches

sets	Chen[2]	Lee[4]	Chen[1]	Wei[15]	Hejazi[3]	Wen[16]	Rezvani[12]	proposed method
set 1	0.975	0.9617	0.8357	0.95	0.9004	0.9473	1	0.9464
set 2	1	1	1	1	1	1	1	1
set 3	0.7	0.5	0.42	0.682	0.6465	0.6631	1	0.6625
set 4	0.7	0.5	0.49	0.7	0.7	0.7	1	0.7
set 5	1	1	0.8	0.8248	0.6681	0.6659	0.6474	0.6308
set 6	1	$*$	1	1	1	1	1	1
set 7	0.9	0	0.81	0.9	0.9	0.9	1	0.9
set 8	0.9	0.5	0.54	0.8411	0.37	0.3896	1	0.7794
set 9	0.9	0.6667	0.81	0.9	0.9	0.9	1	0.9
set 10	0.9	0.8333	0.9	0.7833	0.6261	0.7731	1	0.7752
set 11	0.9	0.75	0.72	0.8003	0.6448	0.7938	1	0.7938
set 12	0.9	0.8	0.8325	0.8289	0.7361	0.7478	1	0.5707
set 13	0.9	0.75	0.81	0.9	0.9	0.9	1	0.92
set 14	1	1	0.7	0.7209	0.5113	0.5104	0.467	0.4817
set 15	0.95	0.75	0.9048	0.6215	0.383	0.4242	0.47	0.3928
set 16	0.9	0.8333	0.7425	0.814	0.6261	0.7321	0.66	0.6752
set 17	0.9	0.75	0.8911	0.838	0.6448	0.7432	0.603	0.4445
set 18	0.9	0.75	0.6976	0.8003	0.6448	0.7144	1	0.711

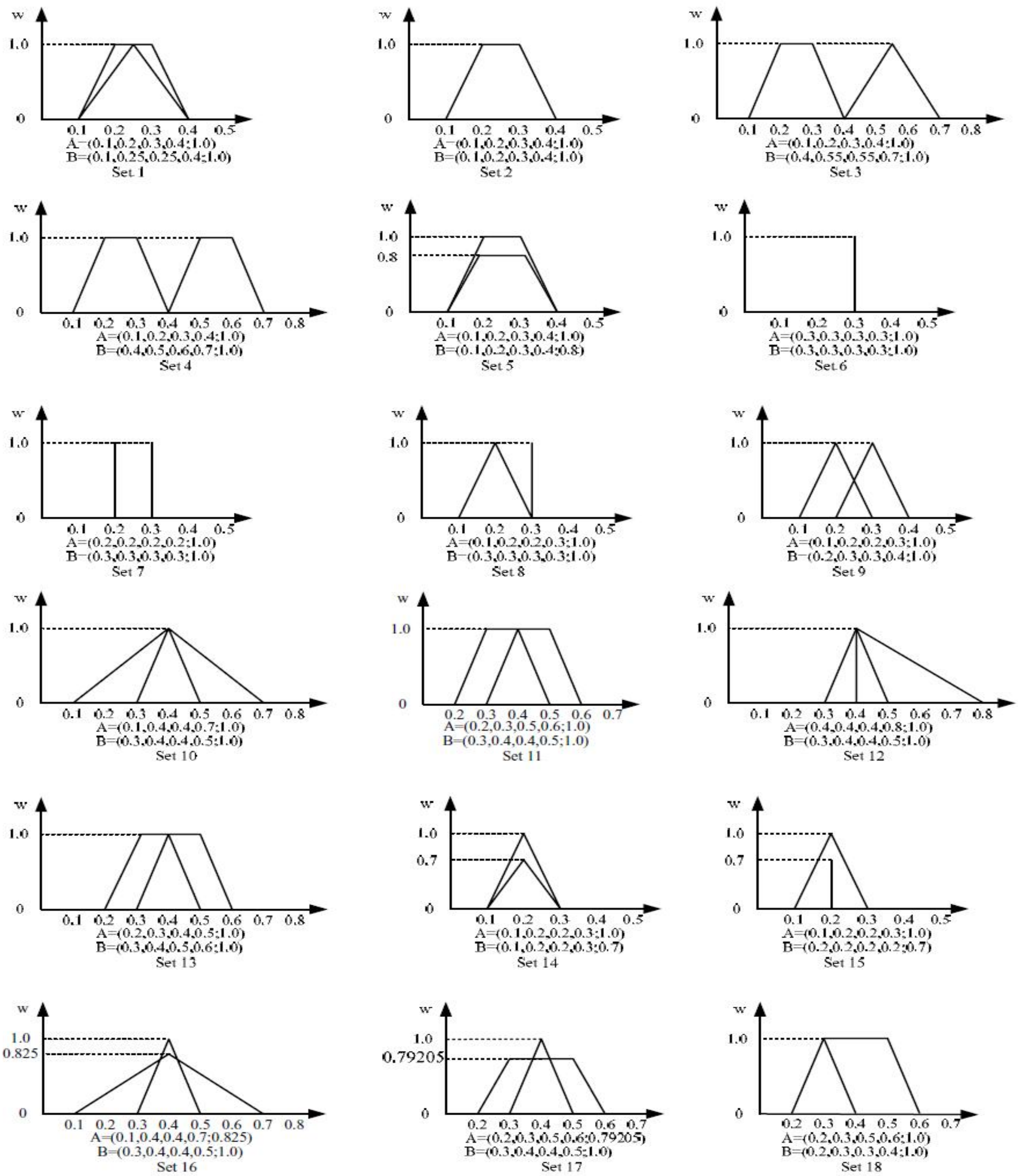

Figure 2. Eighteen sets of generalized fuzzy numbers

5 Conclusions

In this paper, left and right apex angles combines the concepts of the center of gravity, the area, the perimeter and the height of generalized fuzzy numbers for calculating the degree of similarity between generalized fuzzy numbers. The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

References

[1] Chen S. J., Chen S. M., Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers, IEEE Trans. Fuzzy Syst., vol. 11, no. 1, p. 45-56 (2003).

[2] Chen S. M., New methods for subjective mental workload assessment and fuzzy risk analysis, Cybern. Syst.: Int. J, vol. 27, no. 5, p. 449-472 (1996).

[3] Hejazi S.R., Doostparast A., Hosseini S.M., An improved fuzzy risk analysis based on a new similarity measures of generalized fuzzy numbers, Expert Systems with Applications Volume 38, Issue 8, Pages 9179-9185 (August 2011).

[4] Lee H. S., Optimal consensus of fuzzy opinions under group decision making environment, Fuzzy sets Syst., vol. 132, no. 3, p. 303-315 (2002).

[5] Li, D.F., Cheng, C.T., New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions.Pattern Recognition Lett. 23, p. 221Ű225 (2002).

[6] Liang, Z.Z., Shi, P.F., Similarity measures on intuitionistic fuzzy numbers. *Pattern Recognition Lett.*, p. 2687Ű2693 (2003).

[7] Manju Pandey et al., New Aggregation Operator for Trapezoidal Fuzzy Numbers based on the Arithmetic Means of the Left and Right Apex Angles, *International Journal of Advanced Research in Computer Science and Software Engineering*, Volume 2, Issue 3, p. 430-433 (March 2012).

[8] S. Rezvani, Graded Mean Representation Method with Triangular Fuzzy Number, World Applied Sciences Journal 11 (7): 871-876 (2010).

[9] S. Rezvani, Multiplication Operation on Trapezoidal Fuzzy Numbers, Journal of Physical Sciences, Vol. 15, 17-26 (2011).

[10] S. Rezvani, Ranking Method of Trapezoidal Intuitionistic Fuzzy Numbers, Annals of Fuzzy Mathematics and Informatics, Volume 5, No. 3, pp. 515-523 (2013).

[11] S. Rezvani, A New Method for Ranking Fuzzy Numbers with Using TRD Distance Based on mean and standard deviation, International Journal of Mechatronics, Electrical and Computer Technology, Vol. 4(12), pp. 840-856 (2014).

[12] S. Rezvani, A New Similarity Measure of Generalized Fuzzy numbers Based on Left and Right Apex Angles (I), Palestine Journal of Mathematics, Vol. 4(1), 117Ű126 (2015).

[13] Wei, S. H., Chen, S. M. A new approach for fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. Expert Systems with Applications, 36, p. 589Ű598 (2009).

[14] Jiang Wen et al., A modified similarity measure of generalized fuzzy numbers, Procedia Engineering 15, 2773-2777 (2011).

[15] Wei, S. H., Chen, S. M. A new approach for fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. *Expert Systems with Applications*, 36, p. 589Ű598 (2009).

[16] Jiang Wen et al., A modified similarity measure of generalized fuzzy numbers, *Procedia Engineering* 15, p. 2773-2777 (2011).

Author information

Salim Rezvani, Department of Mathematics Marlik Higher Education Institute of Nowshahr, Nowshahr,, Iran.
E-mail: salim_rezvani@yahoo.com

M. Mousavi, Department of chemistry, Azad University of Shahroud, Shahroud, Iran.
E-mail:

Received: July 18, 2014.

Accepted: November 4, 2014.