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An Efficient Three-Step Tenth–Order Method Without Second Order Derivative

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Abstract In this paper, we study and analyze new higher-order iterative methods free from second order derivative for solving nonlinear equations. The new proposed method is obtained by composing an iterative method obtained in Noor et al. [1] with Newton's method and approximating the first-appeared derivative in the last step by a combination of already evaluated function values. The convergence analysis of our methods is discussed. It is established that the new method have convergence order ten. Numerical tests show that the new method may be viewed as an alternative to the known methods.

1 Introduction

Finding iterative methods for solving the nonlinear equation $f(x) = 0$ is an important area of research in numerical analysis. It has interesting applications in several branches of pure and applied science. Due to their importance, several numerical methods have been suggested and analyzed under certain condition. These numerical methods have been constructed using different techniques such as Taylor series, decomposition method, quadrature formula, variational iteration method, and homotopy perturbation method and its variant forms. For more details, see [1-10]. In this paper, we revised and modify the method which given in Al-Subaihi et. al. [11], using predictor–corrector technique, and by replacing the second derivatives of the function by its suitable finite difference scheme. The error equation is refined theoretically to show that the proposed technique has tenth -order convergence. Commonly in the literature the efficiency of an iterative method is measured by the *efficiency index* defined as $I \approx p^{1/d}$ [12], where $p$is the order of convergence and $d$is the total number of functional evaluations per step. Therefore these methods have efficiency index $10^{1/5} \approx 1.585$ which are higher than $2^{1/2} \approx 1.4142$ of the Steffensen's method (SM) [13]. Several examples are given to illustrate the efficiency and performance of this method.

2 Iterative methods

Consider the nonlinear equation of the type

$$f(x) = 0 \tag{1}$$

For simplicity, we assume that *r* is a simple root of Eq. (1) and x_0 is an initial guess sufficiently close to *r*. Using the Taylor's series expansion of the function $f(x)$, we have

$$f(x_0) + (x - x_0)\, f'(x_0) + \frac{(x - x_0)^2}{2} f''(x_0) = 0 \tag{2}$$

First two terms of the equation (2) gives the first approximation, as

$$x = x_0 - \frac{f(x_0)}{f'(x_0)} \tag{3}$$

This allows us to suggest the following one-step iterative method for solving the nonlinear equation (1).
Algorithm 2.1. For a given x_0, find the approximate solution x_{n+1} by the iterative scheme

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

which is the Newton method. It is well known that algorithm 2.1 has a quadratic convergence.
Again from (2) we have

$$x = x_0 - \frac{f(x_0)}{f'(x_0) + \frac{1}{2}\,(x - x_0)\, f''(x_0)} \tag{4}$$

Substitution again from (3) into the right hand side of (4) gives the second approximation

$$x = x_0 - \frac{2f(x_0)f'(x_0)}{2f'^2(x_0) - f(x_0)f''(x_0)}.$$

This formula allows us to suggest the following iterative methods for solving the nonlinear Eq. (1).
Algorithm 2.2. For a given x_0, compute approximates solution x_{n+1} by the iterative scheme

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2[f'(x_n)]^2 - f(x_n)f''(x_n)}.$$

This is known as Halley's method and has cubic convergence [6]. Using Algorithm 2.1 as a predictor and Algorithm 2.2 as a corrector, Noor et. al. [14] suggest and analyze a two-step iterative method for solving the nonlinear equation.
Algorithm 2.3. For a given x_0, compute approximates solution x_{n+1} by the iterative schemes

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = y_n - \frac{2f(y_n)f'(y_n)}{2[f'(y_n)]^2 - f(y_n)f''(y_n)}.$$

Algorithm 2.3 is a two-step predictor–corrector Halley method and has sixth-order convergence [14]. Noor et al. in [1] approximate the second derivatives $f''(y_n)$by the function

$$f''(y_n) = \frac{2}{y_n - x_n}\left(2f'(y_n) + f'(x_n) - 3\frac{f(y_n) - f(x_n)}{y_n - x_n}\right) = P_f(x_n, y_n). \tag{5}$$

Now using the technique of updating the solution, therefore, using Algorithm 2.3 as a predictor and Algorithm 2.1 as a corrector, we suggest and analyze a new three-step iterative methods for solving the nonlinear equation (1).
Algorithm 2.4. For a given x_0, compute approximates solution x_{n+1} by the iterative schemes

$$\begin{aligned} y_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n &= y_n - \frac{2f(y_n)f'(y_n)}{2[f'(y_n)]^2 - f(y_n)P_f(x_n, y_n)}. \\ x_{n+1} &= z_n - \frac{f(z_n)}{f'(z_n)} \end{aligned}$$

Algorithm 2.4 has twelfth-order convergence. Per iteration this method requires three evaluations of the function, and three evaluations of first derivative. We have that the efficiency index of the method 2.4 is $12^{1/6} \approx 1.513$.
To improve the efficiency index, we approximate the first-appeared derivative in the last step $f'(z_n)$ by a combination of already evaluated function values using divided differences. This procedure was used by A. Cordero et al. [15].
To explain the idea, consider the Taylor polynomial of degree 2 for the function $f(z_n)$

$$f(z_n) = f(y_n) + (z_n - y_n)\,f'(y_n) + \frac{(z_n - y_n)^2}{2} f''(y_n) \tag{6}$$

This implies that

$$f'(y_n) = \frac{f(z_n) - f(y_n)}{(z_n - y_n)} - \frac{(z_n - y_n)}{2} f''(y_n) = f[z_n, y_n] - \frac{(z_n - y_n)}{2} f''(y_n) \tag{7}$$

where

$$f[z_n, y_n] = \frac{f(z_n) - f(y_n)}{(z_n - y_n)}$$

then from (7)

$$f''(y_n) = \frac{2\{f[z_n, y_n] - f'(y_n)\}}{(z_n - y_n)} \tag{8}$$

again from (6)

$$f'(z_n) = f'(y_n) + (z_n - y_n)f''(y_n) \tag{9}$$

Substitute the estimation of $f'(y_n)$ and $f''(y_n)$ into the last expression, to get

$$f'(z_n) = f[z_n, y_n] + (z_n - y_n)f[z_n, y_n, y_n] \tag{10}$$

where

$$f[z_n, y_n, y_n] = \frac{f[z_n, y_n] - f'(y_n)}{(z_n - y_n)}$$

from (5) and (7) in (9) we can have another approximation formula to the function $f'(z_n)$as

$$f'(z_n) = f[z_n, y_n] + \frac{1}{2}(z_n - y_n)P_f(x_n, y_n) \tag{11}$$

Now by substituting (10) into (5), we obtain the following new proposed three-step iterative method for solving equation (1) which are the main motivation of this paper.
Algorithm 2.5. For a given x_0, compute approximates solution x_{n+1} by the iterative schemes

$$\begin{aligned} y_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n &= y_n - \frac{2f(y_n)f'(y_n)}{2[f'(y_n)]^2 - f(y_n)P_f(x_n, y_n)}. \\ x_{n+1} &= z_n - \frac{f(z_n)}{f[z_n, y_n] + (z_n - y_n)f[z_n, y_n, y_n]} \end{aligned}$$

Algorithm 2.5 is called the predictor-corrector Modified Halley method (MHS1) and has tenth-order convergence. Per iteration of the iterative method 2.5 requires thee evaluations of the function and two evaluations of first derivative. We have that the efficiency index of the method 2.5 is $10^{1/5} \approx 1.585$ which is better than $12^{1/6} \approx 1.513$ of the method 2.4 and is better than $12^{1/7} \approx 1.426$ of the method 2.3.
Again by substituting (11) into (5) , we obtain the following new proposed three-step iterative method for solving equation (1):
Algorithm 2.6. For a given x_0, compute approximates solution x_{n+1} by the iterative schemes

$$\begin{aligned} y_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n &= y_n - \frac{2f(y_n)f'(y_n)}{2[f'(y_n)]^2 - f(y_n)P_f(x_n, y_n)}. \\ x_{n+1} &= z_n - \frac{2f(z_n)}{2f[z_n, y_n] + (z_n - y_n)P_f(x_n, y_n)} \end{aligned}$$

Algorithm 2.6 is called the predictor-corrector Modified Halley's method (MHS2) and has tenth-order convergence. Per iteration of the iterative method 2.6 requires thee evaluations of the function and two evaluations of first derivative. We have that the efficiency index of the method 2.6 is $10^{1/5} \approx 1.585$ which is better than $12^{1/6} \approx 1.513$ of the method 2.4 and is better than $12^{1/7} \approx 1.426$ of the method 2.3.

To be more precise, we now approximate $f'(y_n)$, to reduce the number of evaluations per iteration by a combination of already known data in the past steps. Toward this end, an estimation of the function $P_2(t)$ is taken into consideration as follows

$$P_2(t) = a + b(t - x_n) + c(t - x_n)^2, \qquad P_2'(t) = b + 2c(t - x_n)$$

By substituting in the known values

$$\begin{aligned} P_g(y_n) &= f(y_n) = a + b(y_n - x_n) + c(y_n - x_n)^2 \\ P_g'(y_n) &= f'(y_n) = b + 2c(y_n - x_n), \\ P_g(x_n) &= f(x_n) = a, \qquad P_g'(x_n) = f'(x_n) = b \end{aligned}$$

we could easily obtain the unknown parameters. Thus we have

$$f'(y_n) = 2f[x_n, y_n] - f'(x_n) = P_g(x_n, y_n) \tag{12}$$

then algorithm 2.6 can be written in the form of the following algorithm.
Algorithm 2.7. For a given x_0, compute approximates solution x_{n+1} by the iterative schemes

$$\begin{aligned} y_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n &= y_n - \frac{2f(y_n)P_g}{2[P_g]^2 - f(y_n)P_f}. \\ x_{n+1} &= z_n - \frac{2f(z_n)}{2f[z_n, y_n] + (z_n - y_n)P_f} \end{aligned}$$

Algorithm 2.7 is called the predictor-corrector Modified Halley's method (MHS3) and has seventh-order convergence. Per iteration of the iterative method 2.7 requires thee evaluations of the function and one evaluations of first derivative. We have that the efficiency index of the method 2.7 is $7^{1/4} \approx 1.626$ which is better than $10^{1/5} \approx 1.585$ of the method 2.6 and this is the main motivation of our paper.

3 Convergence analysis

Let us now discuss the convergence analysis of the above mentioned methods Algorithm 2.5 and Algorithm 2.7. In a similar way, we can discuss the convergence of other algorithms.

Theorem 3.1. *Let r be a sample zero of sufficient differentiable function $f :\subseteq R \to R$ for an open interval I. If $x_0$is sufficiently close to r, then the two step method defined by our algorithm 2.6 has convergence is at least of order ten.*

Proof.

$$\begin{aligned} y_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n &= y_n - \frac{2f(y_n)f'(y_n)}{2[f'(y_n)]^2 - f(y_n)P_f} \\ x_{n+1} &= z_n - \frac{f(z_n)}{f[z_n, y_n] + (z_n - y_n)f[z_n, y_n, y_n]} \end{aligned} \tag{13}$$

Let $r$be a simple zero of f. Since $f$is sufficiently differentiable, by expanding $f(x_n)$ and $f'(x_n)$ about r, we get

$$f(x_n) = f(r) + (x_n - r)f'(r) + \frac{(x_n - r)^2}{2!}f^{(2)}(r) + \frac{(x_n - r)^3}{3!}f^{(3)}(r) + \frac{(x_n - r)^4}{4!}f^{(4)}(r) + \cdots,$$

then

$$f(x_n) = f'(r)[e_n + c_2e_n^2 + c_3e_n^3 + c_4e_n^4 + \cdots], \tag{14}$$

and

$$f'(x_n) = f'(r)[1 + 2c_2e_n + 3c_3e_n^2 + 4c_4e_n^3 + 5c_5e_n^4 + \cdots], \tag{15}$$

where $c_k = \frac{1}{k!}\frac{f^{(k)}(r)}{f'(r)}$, $k = 1, 2, 3, \ldots$ and $e_n = x_n - r$.
Now from (14) and (15), we have

$$\frac{f(x_n)}{f'(x_n)} = e_n - c_2e_n^2 + 2(c_2^2 - c_3)e_n^3 + (7c_2c_3 - 4c_2^3 - 3c_4)e_n^4 + \cdots,$$

From (13), we get

$$y_n = r + c_2e_n^2 + 2(c_3 - c_2^2)e_n^3 + (-7c_2c_3 + 4c_2^3 + 3c_4)e_n^4 + \cdots, \tag{16}$$

From (16), we get,

$$f(y_n) = f'(r)[(y_n - r) + c_2(y_n - r)^2 + c_3(y_n - r)^3 + c_4(y_n - r)^4 + \cdots],$$

$$f'(y_n) = f'(r)[1 + 2c_2^2e_n^2 + 4(c_2c_3 - c_2^3)e_n^3 + (-11c_2^2c_3 + 8c_2^4 + 6c_2c_4)e_n^4 + \cdots].$$

then

$$\begin{aligned} \tfrac{f(y_n)}{f'(y_n)} &= c_2e_n^2 - 2(c_2^2 - c_3) - (7c_2c_3 - 3c_2^3 - 3c_4)e_n^4 + 2(8c_2^2c_3 - 2c_2^4 - 3c_3^2 - 5c_2c_4 + 2c_5)e_n^5 \\ &\quad - (13c_2c_5 - 22c_4c_2^2 - 5c_5 - 6c_2^5 - 32c_3c_2^3 + 17c_4c_3 - 29c_2c_3^2)e_n^6 + \cdots \end{aligned}$$

$$P_f(x_n, y_n) = \frac{2}{y_n - x_n}\left(2f'(y_n) + f'(x_n) - 3\frac{f(y_n) - f(x_n)}{y_n - x_n}\right)$$

$$\begin{aligned} P_f(x_n, y_n) = f'(r)[&2c_2 + (6\,c_2\,c_3 - 2c_4)\,e_n^2 - 4(3\,c_3(c_2^2 -\ c_3) - c_2c_4\ +\ c_5)\,e_n^3 \\ &+ 2\,(12\,c_2^3\,c_3 -\ 21\,c_2\,c_3^2 +\ c_2^2c_4 +\ 13\,c_3\,c_4 + (c_2 - 3)\,c_5)\,e_n^4 + \cdots] \end{aligned} \tag{17}$$

Substituting into (13), to get

$$z_n = y_n - \frac{2f(y_n)f'(y_n)}{2[f'(y_n)]^2 - f(y_n)P_f} = r + c_2^2(2\ c_2^3\ -\ c_2\ c_3\ +\ c_4)\ e_n^6 + O(e_n^7) \quad (18)$$

Now, expand $f(z_n)$ about *r* to get

$$\begin{aligned} f(z_n) &= f'(r)[c_2^2\ (2\ c_2^3 - c_2\ c_3 + c_4)\ e_n^6 + 2\ c_2\ (-6\ c_2^5 + 9\ c_2^3\ c_3 - 3\ c_2^2\ c_4 \\ &\quad + 2\ c_3\ c_4 +\ c_2\ (-3\ c_3^2 \end{aligned}$$

$$f[z_n, y_n] + (z_n - y_n)f[z_n, y_n, y_n] = f'(r)[1 - c_2^2\ c_3\ e_n^4 + 4\ c_2c_3(c_2^2 - c_3)\ e_n^5 + \cdots]$$

Substituting into (13), to get

$$x_{n+1} = z_n - \frac{f(z_n)}{f[z_n, y_n] + (z_n - y_n)f[z_n, y_n, y_n]} \quad (19)$$

or

$$x_{n+1} = r - c_2^4c_3(2c_2^3\ -\ c_2\ c_3\ +\ c_4)\ e_n^{10} + O(e_n^{11}) \quad (18)$$

From (19), $e_{n+1} = x_{n+1} - r$ then we will have

$$e_{n+1} = -c_2^4c_3(2\ c_2^3\ -\ c_2\ c_3\ +\ c_4)\ e_n^{10} + O(e_n^{11}) \quad (20)$$

which shows that Algorithm 2.5 is at least a tenth order convergent method, the required result. □

Theorem 3.2. *Let r be a sample zero of sufficient differentiable function $f\ :\subseteq\ R \to R$ for an open interval I. If x_0 is sufficiently close to r, then the two step method defined by our algorithm 2.8 has convergence is at least of order seven.*

Proof. Consider to

$$\begin{aligned} z_n &= y_n - \frac{2f(y_n)P_g}{2[P_g]^2 - f(y_n)P_f} \\ x_{n+1} &= z_n - \frac{2f(z_n)}{2f[z_n, y_n] + (z_n - y_n)P_f} \end{aligned} \quad (21)$$

Again by using Taylor's expansion we can get

$$\begin{aligned} P_g = 2f[x_n, y_n] - f'(x_n) &= f'(r)[1 + (2\ c_2^2 - c_3)\ e_n^2 - 2\ (2\ c_2^3 - 3\ c_2\ c_3 + c_4)\ e_n^3 \\ &+\ (8\ c_2^4 -\ 16\ c_2^2c_3\ +\ 4\ c_3^2 + 8\ c_2\ c_4\ -\ 3\ c_5)\ e_n^4 + \cdots] \end{aligned} \quad (22)$$

from (16), (17) and (22) in (21) we get

$$z_n = y_n - \frac{2f(y_n)P_g}{2[P_g]^2 - f(y_n)P_f} = r - c_2c_3e_n^4 +\ (2c_3(c_2^2 -\ c_3) - 2c_2c_4)e_n^5 + \cdots \quad (22)$$

$$f(z_n) = f'(r)[-c_2\ c_3\ e_n^4 +\ (2c_3(c_2^2 -\ c_3) - 2c_2c_4)e_n^5 + \cdots], \quad (23)$$

$$2f[z_n, y_n] + (z_n - y_n)P_f = f'(r)[1 - 2c_2c_3\ e_n^3 +\ (2c_3(c_2^2 -\ 2c_3) - 3c_2c_4)\ e_n^4 \cdots] \quad (24)$$

Substituting from (23) and (24) into (21), to get

$$x_{n+1} = z_n - \frac{2f(z_n)}{2f[z_n, y_n] + (z_n - y_n)P_f} = r + 2c_2^2c_3^2e_n^7\ + O(e_n^8)$$

or, in the final form

$$e_{n+1} = 2c_2^2c_3^2e_n^7\ + O(e_n^8) \quad (25)$$

which shows that Algorithm 2.7 has seventh- order of convergence. □

4 Numerical examples

For comparisons, we have used the ninth-order Noor et al. [1] (NRM) defined respectively by

$$\begin{aligned} y_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\ z_n &= y_n - \frac{2f(y_n)f'(y_n)}{2[f'(y_n)]^2 - f(y_n)P_f}. \\ x_{n+1} &= z_n - \frac{f'(x_n) + f'(y_n)}{3f'(y_n) - f'(x_n)} \times \frac{f(z_n)}{f'(x_n)} \end{aligned}$$

In this study, we present some numerical examples to illustrate the efficiency and the accuracy of the new developed iterative methods (Table 1). We compare our new methods namely (MS1) to (MS4), with Noor et al. [1] (NRM). Our examples are tested with precision $\varepsilon = 10^{-200}$and the following stopping criteria is used for computer programs: $|x_{n+1} - x_n| + |f(x_{n+1})| < \varepsilon$.

Displayed in Table 1 are the number of iterations (IT), such that the stopping criteria satisfied, the absolute values of the function $f(x_n)$ after the required iterations. Moreover, displayed is the distance of two consecutive approximations $\delta = |x_n - x_{n-1}|$, the time pier second and the computational order of convergence (*COC*). Where the computational order of convergence (*COC*) can be approximated using the formula,

$$COC \approx \frac{\ln|\,(x_{n+1} - x_n)/(x_n - x_{n-1})|}{\ln|\,(x_n - x_{n-1})/(x_{n-1} - x_{n-2})|}$$

All the computations are performed using Maple 15 with 10000 significant digits. The following examples are used for numerical testing:

$f_1(x) = x^3 + 4x^2 - 10,\quad x_0 = 1\ ,\quad f_2(x) = \sin^2 x - x^2 + 1,\quad x_0 = 1.3\,.$
$f_3(x) = \cos x - x,\quad x_0 = 1.7,\quad f_4(x) = (x-1)^3 - 1,\quad x_0 = 2.5\,.$
$f_5(x) = x^3 - 10,\quad x_0 = 2\ ,\quad f_6(x) = e^{-x} + \cos x,\quad x_0 = 2\ ,$
$f_7(x) = \sin x - x/2,\quad x_0 = 2\,.$

Results are summarized in Table 1, as it shows, new algorithms are comparable with Noor method [1] and in most cases gives better or equal results.

5 Conclusions

In this paper, we revised and modify the method presented recently by Al-Subaihi et. al. [11], and used it for solving of nonlinear equations. These method based on a Halley iterative method and using predictor–corrector technique. The error equations are given theoretically to show that the proposed technique have seventh - and tenth -order convergence. The new methods attain efficiency indices of 1.626 and 1.585, which makes them competitive. In addition, the proposed methods have been tested on a series of examples published in the literature and show good results when compared them with the previous literature.

Table 1. Comparison of different methods

Method	*NRM*	*MHS1*	*MHS2*	*MHS3*
f_1 , x_0=1				
IT	4	4	4	4
$\|f(x_n)\|$	7.51e-4458	2.19E-7691	3.40E-7327	4.34E-2099
δ	7.91E-496	1.46E-769	2.83E-733	2.84E-300
Time/s	0.141	0.249	0.141	0.141
COC	9	10	10	7
f_2, x_0=1.3				
IT	4	4	4	4
$\|f(x_n)\|$	6.72E-6932	1E-9999	1E-9999	5.28E-3070
δ	8.30E-771	4.19E-1177	1.74E-1111	5.82E-439
Time/s	4.118	4.244	4.071	4.360
COC	9	10	10	7
f_3, x_0=1.7				
IT	4	4	4	4
$\|f(x_n)\|$	1.56E-5892	7.95E-9767	2.38E-9827	5.06E-1777
δ	7.20E-655	9.16E-977	8.35E-983	4.91E-254
Time/s	4.087	4.212	4.181	4.165
COC	9	10	10	7
f_4, x_0=2.5				
IT	4	4	4	5
$\|f(x_n)\|$	328E-2868	7.37E-5099	1.23E-4696	4.28E-8040
δ	2.20E-318	1.60E-510	2.49E-470	3.50E-1149
Time/s	0.109	0.203	0.125	0.156
COC	9	10	10	7
f_5, x_0=2				
IT	4	4	4	4
$\|f(x_n)\|$	1.71E-7415	5.00E-9999	1.00E-9998	1.93E-2986
δ	2.05E-824	1.11E-1196	5.09E-1163	4.84E-427
Time/s	0.125	0.203	0.110	0.125
COC	9	10	10	7
f_6, x_0=2				
IT	4	4	4	4
$\|f(x_n)\|$	6.07E-7232	0	0	1.42E-2593
δ	1.11E-803	6.60E-1164	4.39E-1134	1.10E-370
Time/s	5.008	5.179	4.836	4.883
COC	9	10	10	7
f_7, x_0=2				
IT	4	4	4	4
$\|f(x_n)\|$	4.73E-7904	0.10E-9999	0.10E-9999	2.74E-3372
δ	1.08E-878	1.88E-1329	1.78E-1346	5.31E-482
Time/s	4.290	4.258	4.196	4.353
COC	9	10	10	7

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