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Exponential growth of solutions for a coupled nonlinear wave equations with nonlinear damping and source terms

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Abstract In this paper, we study initial-boundary conditions for a coupled nonlinear wave equations with weak damping terms. The exponential growth for sufficiently large initial data is proved.

1 Introduction

In this paper, we study the following initial boundary value problem

$$\begin{cases} u_{tt}+u_t+|u_t|^{p-1}u_t=div\left(\rho\left(|\nabla u|^2\right)\nabla u\right)+f_1(u,v), & (x,t)\in\Omega\times(0,T),\\ v_{tt}+v_t+|v_t|^{q-1}v_t=div\left(\rho\left(|\nabla v|^2\right)\nabla v\right)+f_2(u,v), & (x,t)\in\Omega\times(0,T),\\ u(x,t)=v(x,t)=0, & (x,t)\in\partial\Omega\times(0,T),\\ u(x,0)=u_0(x),\ u_t(x,0)=u_1(x), & x\in\Omega,\\ v(x,0)=v_0(x),\ v_t(x,0)=v_1(x), & x\in\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain of R^n $(n=1,2,3)$ with smooth boundary $\partial\Omega$ in $p,q\geq 1;$ $f_i:R^2\longrightarrow R$ are given functions to be specified later.

We define ρ by

$$\rho(s)=b_1+b_2s^m,\ q\geq 0, \tag{1.2}$$

where b_1,b_2 are nonnegative constants, and $b_1+b_2>0$.

Wu and Li [6] obtained the blow up of the solution of problem (1.1), for negative initial energy. In the absence of the linear weak damping $(u_t$ and $v_t)$ terms, the problem (1.1) reduced to the following system

$$\begin{cases} u_{tt}+|u_t|^{p-1}u_t=div\left(\rho\left(|\nabla u|^2\right)\nabla u\right)+f_1(u,v),\\ v_{tt}+|v_t|^{q-1}v_t=div\left(\rho\left(|\nabla v|^2\right)\nabla v\right)+f_2(u,v). \end{cases} \tag{1.3}$$

Wu et al. [7] obtained the global existence and blow up of the solution of problem (1.3) under some suitable conditions. Fei and Hongjun [2] considered problem (1.3) and improved the blow up result obtained in [7], for a large class of initial data in positive initial energy, using the some techniques as in Payne and Sattinger [4] and some estimates used firstly by Vitillaro [8]. Recently, Pişkin and Polat [5] studied the local and global existence, energy decay and blow up of the solution of problem (1.3). Also, for more information about (1.1) and (1.3), see references [2, 5, 6].

Our purpose in this paper is to give the blow up property in infinity time, i.e. exponential growth.

This paper is organized as follows. In section 2, we give the local existence result. In section 3, we study the energy will grow up as an exponential as time goes to infinity, provided that the initial data are large enough or $E(0)<E_1$, where $E(0)$ and E_1 are defined in (2.5) and (3.2).

2 Preliminaries

Throughout this paper, we denote $\|.\|$ and $\|.\|_p$ denote the usual $L^2(\Omega)$ norm and $L^p(\Omega)$ norm, respectively.

Concerning the functions $f_1(u,v)$ and $f_2(u,v)$, we take

$$\begin{aligned} f_1(u,v) &= a|u+v|^{2(r+1)}(u+v) + b|u|^r u|v|^{r+2}, \\ f_2(u,v) &= a|u+v|^{2(r+1)}(u+v) + b|v|^r v|u|^{r+2}, \end{aligned}$$

where $a, b > 0$ are constants and r satisfies

$$\begin{cases} -1 < r \text{ if } n \le 2, \\ -1 < r \le 1 \text{ if } n = 3. \end{cases} \tag{2.1}$$

One can easily verify that

$$u f_1(u,v) + v f_2(u,v) = 2(r+2) F(u,v), \ \forall (u,v) \in R^2, \tag{2.2}$$

where

$$F(u,v) = \frac{1}{2(r+2)} \left[a|u+v|^{2(r+2)} + 2b|uv|^{r+2} \right]. \tag{2.3}$$

We have the following result.

Lemma 2.1 [3]. There exist two positive constants c_0 and c_1 such that

$$c_0 \left(|u|^{2(r+2)} + |v|^{2(r+2)} \right) \le 2(r+2) F(u,v) \le c_1 \left(|u|^{2(r+2)} + |v|^{2(r+2)} \right) \tag{2.4}$$

is satisfied.

We define

$$\begin{aligned} E(t) &= \frac{1}{2}\left(\|u_t\|^2 + \|v_t\|^2 \right) + \frac{1}{2}\int_\Omega \left(P\left(|\nabla u|^2\right) + P\left(|\nabla v|^2\right) \right) dx \\ &\quad - \int_\Omega F(u,v)\, dx. \end{aligned} \tag{2.5}$$

where $P(s) = \int_0^s \rho(\xi)\, d\xi$, $s \ge 0$.

Lemma 2.2 [5]. $E(t)$ is a nonincreasing function for $t \ge 0$ and

$$E'(t) = -\left(\|u_t\|^2 + \|v_t\|^2 + \|u_t\|_{p+1}^{p+1} + \|v_t\|_{q+1}^{q+1} \right) \le 0. \tag{2.6}$$

Lemma 2.3 (Sobolev-Poincare inequality) [1]. Let p be a number with $2 \le p < \infty$ $(n = 1, 2)$ or $2 \le p \le 2n/(n-2)$ $(n \ge 3)$, then there is a constant $C_* = C_*(\Omega,\ p)$ such that

$$\|u\|_p \le C_* \|\nabla u\| \text{ for } u \in H_0^1(\Omega).$$

Next, we state the local existence theorem [5, 6].

Theorem 2.1 (Local existence). Suppose that (2.1) holds. Then there exist p, q satisfying

$$\begin{cases} 1 \le p, q \quad \text{if } n \le 2, \\ 1 \le p, q \ \le 5 \quad \text{if } n = 3, \end{cases}$$

and further $(u_0, v_0) \in \left(W_0^{1,2(m+1)}(\Omega) \cap L^{2(r+2)}(\Omega) \right) \times \left(W_0^{1,2(m+1)}(\Omega) \cap L^{2(r+2)}(\Omega) \right)$, $(u_1, v_1) \in L^2(\Omega) \times L^2(\Omega)$. Then problem (1.1) has a unique local solution

$$u, v \in C\left([0,T); W_0^{1,2(m+1)}(\Omega) \cap L^{2(r+2)}(\Omega) \right),$$

$$\begin{aligned} u_t &\in C\left([0,T); L^2(\Omega)\right) \cap L^{p+1}(\Omega \times [0,T)) \text{ and} \\ v_t &\in C\left([0,T); L^2(\Omega)\right) \cap L^{q+1}(\Omega \times [0,T)). \end{aligned}$$

3 Exponential Growth

In this section, we will prove that the energy is unbounded when the initial data are large enough in some sense.

Lemma 3.1 [2]. Suppose that (2.1) holds. Then there exists $\eta > 0$ such that for any $(u, v) \in \left(W_0^{1,2(m+1)}(\Omega) \cap L^{2(r+2)}(\Omega)\right) \times \left(W_0^{1,2(m+1)}(\Omega) \cap L^{2(r+2)}(\Omega)\right)$ the inequality

$$\|u+v\|_{2(r+2)}^{2(r+2)} + 2\|uv\|_{r+2}^{r+2} \leq \eta \left(\int_\Omega \left(P\left(|\nabla u|^2\right) + P\left(|\nabla v|^2\right)\right) dx \right)^{r+2} \tag{3.1}$$

holds.

For the sake of simplicity and to prove our result, we take $a = b = 1$ and introduce

$$B = \eta^{\frac{1}{2(r+2)}},\ \alpha_1 = B^{-\frac{r+2}{r+1}},\ E_1 = \left(\frac{1}{2} - \frac{1}{2(r+2)}\right)\alpha_1^2, \tag{3.2}$$

where η is the optimal constant in (3.1). Next, we will state a lemma which is similar to the one introduced firstly by Vitillaro in [8] to study a class of a single wave equation.

Lemma 3.2 [2]. Suppose that (2.1) holds. Let (u, v) be the solution of problem (1.1). Assume further that $E(0) < E_1$ and

$$\left(\int_\Omega \left(P\left(|\nabla u_0|^2\right) + P\left(|\nabla v_0|^2\right)\right) dx \right)^{\frac{1}{2}} > \alpha_1. \tag{3.3}$$

Then there exists a constant $\alpha_2 > \alpha_1$ such that

$$\left(\int_\Omega \left(P\left(|\nabla u|^2\right) + P\left(|\nabla v|^2\right)\right) dx \right)^{\frac{1}{2}} \geq \alpha_2, \tag{3.4}$$

and

$$\left(\|u+v\|_{2(r+2)}^{2(r+2)} + 2\|uv\|_{r+2}^{r+2} \right)^{\frac{1}{2(r+2)}} \geq B\alpha_2, \tag{3.5}$$

for all $t \in [0, T)$.

Theorem 3.1. Assume that (2.1) and $2(r+2) > \max\{p+1, q+1\}$ hold. Then any solution of problem (1.1) with initial data satisfying

$$\left(\int_\Omega \left(P\left(|\nabla u_0|^2\right) + P\left(|\nabla v_0|^2\right)\right) dx \right)^{\frac{1}{2}} > \alpha_1 \text{ and } E(0) < E_1,$$

grows exponentially.

Proof. We set

$$H(t) = E_1 - E(t), \quad t \geq 0. \tag{3.6}$$

By the definition of $H(t)$ and (2.6)

$$H'(t) = -E'(t) = \|u_t\|_{p+1}^{p+1} + \|v_t\|_{q+1}^{q+1} \geq 0,$$

hence we have $H(t) \geq H(0) = E_1 - E(0) > 0$.

Let us define the functional

$$L(t) = H(t) + \varepsilon \int_\Omega (uu_t + vv_t)\, dx + \frac{\varepsilon}{2}\left(\|u\|^2 + \|v\|^2\right) \tag{3.7}$$

for ε small to be chosen later.

Our goal is to show that $L(t)$ satisfies a differential inequality of the form

$$L'(t) \geq CL(t) \quad \text{for all } t \geq 0.$$

This, of course, will lead to exponential growth.

By taking the time derivative of (3.7) and using equations (1.1), we have

$$\begin{aligned}
L'(t) &= H'(t) + \varepsilon\left(\|u_t\|^2 + \|v_t\|^2\right) + \varepsilon \int_\Omega (uu_{tt} + vv_{tt})\,dx \\
&\quad + \varepsilon \int_\Omega (uu_t + vv_t)\,dx \\
&= \left(\|u_t\|_{p+1}^{p+1} + \|v_t\|_{q+1}^{q+1}\right) + \varepsilon\left(\|u_t\|^2 + \|v_t\|^2\right) \\
&\quad - \varepsilon b_1\left(\|\nabla u\|^2 + \|\nabla v\|^2\right) \\
&\quad - \varepsilon b_2\left(\|\nabla u\|_{2(m+1)}^{2(m+1)} + \|\nabla v\|_{2(m+1)}^{2(m+1)}\right) \\
&\quad + \varepsilon \int_\Omega (u\, f_1(u,v) + v f_2(u,v))\,dx \\
&\quad - \varepsilon \int_\Omega \left(u\,|u_t|^{p-1}\,u_t + v\,|v_t|^{q-1}\,v_t\right) dx. \qquad (3.8)
\end{aligned}$$

From (2.5) and (3.6), it follows that

$$\begin{aligned}
& -b_2\left(\|\nabla u\|_{2(m+1)}^{2(m+1)} + \|\nabla v\|_{2(m+1)}^{2(m+1)}\right) \\
=\ & 2(m+1)H(t) - 2(m+1)E_1 \\
& + (m+1)\left(\|u_t\|^2 + \|v_t\|^2\right) \\
& + b_1(m+1)\left(\|\nabla u\|^2 + \|\nabla v\|^2\right) \\
& - 2(m+1)\int_\Omega F(u,v)\,dx. \qquad (3.9)
\end{aligned}$$

Inserting (3.9) into (3.8), we get

$$\begin{aligned}
L'(t) &= \left(\|u_t\|_{p+1}^{p+1} + \|v_t\|_{q+1}^{q+1}\right) + \varepsilon(m+2)\left(\|u_t\|^2 + \|v_t\|^2\right) \\
&\quad + \varepsilon b_1 m\left(\|\nabla u\|^2 + \|\nabla v\|^2\right) \\
&\quad + \varepsilon\left(1 - \frac{m+1}{r+2}\right)\left(\|u+v\|_{2(r+2)}^{2(r+2)} + 2\,\|uv\|_{r+2}^{r+2}\right) \\
&\quad + 2\varepsilon(m+1)H(t) - 2\varepsilon(m+1)E_1 \\
&\quad - \varepsilon \int_\Omega \left(u\,|u_t|^{p-1}\,u_t + v\,|v_t|^{q-1}\,v_t\right) dx.
\end{aligned}$$

Then using (3.5), we obtain

$$\begin{aligned}
L'(t) &\geq \left(\|u_t\|_{p+1}^{p+1} + \|v_t\|_{q+1}^{q+1}\right) + \varepsilon(m+2)\left(\|u_t\|^2 + \|v_t\|^2\right) \\
&\quad + \varepsilon b_1 m\left(\|\nabla u\|^2 + \|\nabla v\|^2\right) + 2\varepsilon(m+1)H(t) \\
&\quad + \varepsilon c'\left(\|u+v\|_{2(r+2)}^{2(r+2)} + 2\,\|uv\|_{r+2}^{r+2}\right) \\
&\quad - \varepsilon \int_\Omega \left(u\,|u_t|^{p-1}\,u_t + v\,|v_t|^{q-1}\,v_t\right) dx, \qquad (3.10)
\end{aligned}$$

where $c' = 1 - \frac{m+1}{r+2} - 2(m+1)E_1(B\alpha_2)^{-2(r+2)}$. It is clear that $c' > 0$, since $\alpha_2 > B^{-\frac{r+2}{r+1}}$. In order to estimate the last two terms in (3.10), we use the following Young's inequality

$$XY \leq \frac{\delta^k X^k}{k} + \frac{\delta^{-l} Y^l}{l},$$

where $X, Y \geq 0,\ \delta > 0,\ k, l \in R^+$ such that $\frac{1}{k} + \frac{1}{l} = 1$. Consequently, applying the above inequality we have

$$\int_\Omega u u_t |u_t|^{p-1} dx \leq \frac{\delta_1^{p+1}}{p+1} \|u\|_{p+1}^{p+1} + \frac{p\delta_1^{-\frac{p+1}{p}}}{p+1} \|u_t\|_{p+1}^{p+1} \tag{3.11}$$

and

$$\int_\Omega v v_t |v_t|^{q-1} dx \leq \frac{\delta_2^{q+1}}{q+1} \|v\|_{q+1}^{q+1} + \frac{q\delta_2^{-\frac{q+1}{q}}}{q+1} \|v_t\|_{q+1}^{q+1}. \tag{3.12}$$

Inserting the estimates (3.11) and (3.12) into (3.10), we have

$$\begin{aligned} L'(t) \geq\ & \varepsilon(m+2)\left(\|u_t\|^2 + \|v_t\|^2\right) + \varepsilon b_1 m\left(\|\nabla u\|^2 + \|\nabla v\|^2\right) \\ & + 2\varepsilon(m+1)H(t) + \varepsilon c_2\left(\|u\|_{2(r+2)}^{2(r+2)} + \|v\|_{2(r+2)}^{2(r+2)}\right) \\ & - \varepsilon\frac{\delta_1^{p+1}}{p+1}\|u\|_{p+1}^{p+1} + \left(1 - \varepsilon\frac{p\delta_1^{-\frac{p+1}{p}}}{p+1}\right)\|u_t\|_{p+1}^{p+1} \\ & - \varepsilon\frac{\delta_2^{q+1}}{q+1}\|v\|_{q+1}^{q+1} + \left(1 - \varepsilon\frac{q\delta_2^{-\frac{q+1}{q}}}{q+1}\right)\|v_t\|_{q+1}^{q+1}, \end{aligned} \tag{3.13}$$

where $c_2 = c' c_0$.

Since $2(r+2) > \max\{p+1, q+1\}$, from the embedding $L^{2(r+2)}(\Omega) \hookrightarrow L^{p+1}(\Omega)$ and $L^{2(r+2)}(\Omega) \hookrightarrow L^{p+1}(\Omega)$, we have

$$\|u\|_{p+1}^{p+1} \leq c_3 \|u\|_{2(r+2)}^{p+1}$$

and

$$\|v\|_{q+1}^{q+1} \leq c_4 \|v\|_{2(r+2)}^{q+1}$$

for some positive constants c_3 and c_4. Using the algebraic inequality

$$z^v \leq z + 1 \leq \left(1 + \frac{1}{a}\right)(z + a),\ \forall z \geq 0,\ 0 < v \leq 1,\ a \geq 0, \tag{3.14}$$

and since $H(t) \geq H(0)$, we get

$$\begin{aligned} \|u\|_{2(r+2)}^{p+1} & \leq d\left(\|u\|_{2(r+2)}^{2(r+2)} + H(0)\right) \\ & \leq d\left(\|u\|_{2(r+2)}^{2(r+2)} + H(t)\right), \end{aligned} \tag{3.15}$$

where $d = 1 + \frac{1}{H(0)}$. Similarly

$$\|v\|_{2(r+2)}^{q+1} \leq d\left(\|v\|_{2(r+2)}^{2(r+2)} + H(t)\right). \tag{3.16}$$

Inserting (3.15) and (3.16) into (3.13), we have

$$\begin{aligned} L'(t) &\geq \varepsilon(m+2)\left(\|u_t\|^2+\|v_t\|^2\right)+\varepsilon b_1 m\left(\|\nabla u\|^2+\|\nabla v\|^2\right)\\ &+\varepsilon\left(2(m+1)-d\frac{\delta_1^{p+1}}{p+1}-d\frac{\delta_2^{q+1}}{q+1}\right)H(t)\\ &+\varepsilon\left(c_2-d\frac{\delta_1^{p+1}}{p+1}-d\frac{\delta_2^{q+1}}{q+1}\right)\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right)\\ &+\left(1-\varepsilon\frac{p\delta_1^{-\frac{p+1}{p}}}{p+1}\right)\|u_t\|_{p+1}^{p+1}+\left(1-\varepsilon\frac{q\delta_2^{-\frac{q+1}{q}}}{q+1}\right)\|v_t\|_{q+1}^{q+1}. \end{aligned} \tag{3.17}$$

Now, once δ_1 and δ_2 are fixed, we can choose ε small enough such that

$$1-\varepsilon\frac{p\delta_1^{-\frac{p+1}{p}}}{p+1}>0,\ 1-\varepsilon\frac{q\delta_2^{-\frac{q+1}{q}}}{q+1}>0,$$

and

$$L(0)=H(0)+\varepsilon\int_\Omega(u_0u_1+v_0v_1)\,dx>0.$$

Consequently (3.17) takes the form

$$\begin{aligned} L'(t) &\geq \varepsilon(m+2)\left(\|u_t\|^2+\|v_t\|^2\right)+\varepsilon b_1 m\left(\|\nabla u\|^2+\|\nabla v\|^2\right)\\ &+\varepsilon\left(2(m+1)-d\frac{\delta_1^{p+1}}{p+1}-d\frac{\delta_2^{q+1}}{q+1}\right)H(t)\\ &+\varepsilon\left(c_2-d\frac{\delta_1^{p+1}}{p+1}-d\frac{\delta_2^{q+1}}{q+1}\right)\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right)\\ &\geq \theta\left(\|u_t\|^2+\|v_t\|^2+H(t)+\|\nabla u\|^2+\|\nabla v\|^2+\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right), \end{aligned} \tag{3.18}$$

where $\theta=\min\left\{\varepsilon(m+2),\ \varepsilon b_1 m,\ \left(2(m+1)-d\frac{\delta_1^{p+1}}{p+1}-d\frac{\delta_2^{q+1}}{q+1}\right),\ \varepsilon\left(c_2-d\frac{\delta_1^{p+1}}{p+1}-d\frac{\delta_2^{q+1}}{q+1}\right)\right\}$.

Then we have

$$L(t)\geq L(0)>0,\ \forall t\geq 0.$$

On the other hand, applying Hölder inequality, we obtain

$$\begin{aligned} \left|\int_\Omega uu_t dx+\int_\Omega vv_t dx\right| &\leq \|u\|\|u_t\|+\|v\|\|v_t\|\\ &\leq C\left(\|u\|_{2(r+2)}\|u_t\|+\|v\|_{2(r+2)}\|v_t\|\right). \end{aligned} \tag{3.19}$$

Young inequality gives

$$\begin{aligned} \left|\int_\Omega uu_t dx+\int_\Omega vv_t dx\right| &\leq \frac{C}{2}\left(\|u\|_{2(r+2)}^2+\|u_t\|^2+\|v\|_{2(r+2)}^2+\|v_t\|^2\right)\\ &\leq \frac{C}{2}\left(\left(\|u\|_{2(r+2)}^{2(r+2)}\right)^{\frac{1}{r+2}}+\|u_t\|^2+\left(\|v\|_{2(r+2)}^{2(r+2)}\right)^{\frac{1}{r+2}}+\|v_t\|^2\right) \end{aligned} \tag{3.20}$$

Since $r>-1$, the algebraic inequality (3.14) yields

$$\begin{aligned} \left|\int_\Omega uu_t dx+\int_\Omega vv_t dx\right| &\leq \frac{C}{2}\left(\left(1+\frac{1}{H(0)}\right)\left(\|u\|_{2(r+2)}^{2(r+2)}+H(t)\right)+\|u_t\|^2\right.\\ &\left.+\left(1+\frac{1}{H(0)}\right)\left(\|v\|_{2(r+2)}^{2(r+2)}+H(t)\right)+\|v_t\|^2\right). \end{aligned}$$

Note that

$$\begin{aligned} L(t) &\leq H(t) + \varepsilon \int_{\Omega} (uu_t + vv_t)\, dx \\ &\leq H(t) + \left| \int_{\Omega} uu_t dx + \int_{\Omega} vv_t dx \right| \\ &\leq C' \Big(\|u_t\|^2 + \|v_t\|^2 + H(t) + \|\nabla u\|^2 + \|\nabla v\|^2 \\ &\quad + \|u\|_{2(r+2)}^{2(r+2)} + \|v\|_{2(r+2)}^{2(r+2)} \Big). \end{aligned} \tag{3.21}$$

Combination of (3.18) and (3.21), gives

$$L'(t) \geq CL(t) \quad \text{for all } t \geq 0. \tag{3.22}$$

An integration of (3.22) from 0 to t gives to

$$L(t) \geq L(0)\, e^{Ct}.$$

This completes the proof.

Remark 3.1. When $E(0) < 0$, by setting $H(t) = -E(t)$, the similar result is obtained by applying the same arguments in proof of Theorem 3.1.

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