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# Edge Connected Domination Polynomial of a Graph

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**Abstract** Let $G = (V, E)$ be a simple connected graph of order $n = |V|$ and size $m = |E|$. An edge connected dominating sets of $G$ is a set, say $F$, of edges of $G$ such that every edge in $G - F$ is adjacent to some edges in $F$ and the induced subgraph $< F >$ is connected. The edge connected domination number $\gamma_{ec}(G)$ is the minimum cardinality of an edge connected dominating sets of $G$. In this paper we introduce the edge connected domination polynomial of $G$. The edge connected domination polynomial of a connected graph $G$ of size $k$ is the polynomial $D_{ec}(G,x) = \sum_{k=\gamma_{ec}(G)}^{m} d_{ec}(G,k)x^k$, where $d_{ec}(G,k)$ is the number of edge connected dominating sets of $G$ of size $k$. In addition, we compute an edge connected domination polynomial and its roots for some special graphs with some of their basic properties.

## 1 Introduction

In this paper simple connected graphs will be considered. Let $G = (V, E)$, where $V$ is the set of vertices and $E$ is the set of edges and let $n = |V|$ be the order of $G$ and $m = |E|$ be the size of $G$. Two vertices $v_1, v_2$ of $G$, which are connected by an edge, are called adjacent vertices and two edges, having a vertex in common, are also called adjacent edges. An edge dominates its adjacent edges.[3]

An edge dominating sets $F$ of $G$ is called an edge connected dominating sets if the induced subgraph $< F >$ is connected. The minimum cardinality of an edge connected dominating sets of $G$ is called the edge connected domination number of $G$ and it is denoted by $\gamma_{ec}(G)$. An edge dominating sets with cardinality $\gamma_{ec}(G)$ is called $\gamma_{ec}$-set, we denote the family of edge dominating sets of a graph $G$ with cardinality $k$ by $D_{ec}(G,k)$. The roots of the edge connected domination polynomial are called the edge connected dominating roots of $G$, which is denoted by $R(D_{ec}(G,x))$.

The edge domination was introduced by Mitchell and Hedetniemi [4] and it was studied by Arumugam and Velammal [2]. For more information and motivation of domination polynomial and connected domination polynomial refer to [1,6].

## 2 Edge Connected Domination Polynomial of a Graph

**Definition 2.1.** Let $G = (V, E)$ be a simple connected graph of order $n = |V|$ and size $m = |E|$. The edge connected domination polynomial of a connected graph $G$ of size $k$ is the polynomial

$$D_{ec}(G,x) = \sum_{k=\gamma_{ec}(G)}^{m} d_{ec}(G,k)x^k,$$

where $d_{ec}(G,i)$ is the number of edge connected dominating sets of $G$ of size $i$ and $\gamma_{ec}(G)$ is the edge connected domination number of $G$.

**Example 2.2.** Let $G$ be the graph as shown in the figure 1 with $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Then the edge connected domination number is two and the edge connected dominating sets of size two are

$$\{e_2, e_3\}, \{e_2, e_4\}, \{e_2, e_5\}, \{e_3, e_4\}, \{e_3, e_6\},$$

the edge connected dominating sets of size three are 14, which are

$$\{e_1,e_2,e_3\},\{e_1,e_2,e_4\},\{e_1,e_2,e_5\},\{e_1,e_3,e_4\},\{e_1,e_3,e_6\},\{e_2,e_3,e_4\},\{e_2,e_3,e_5\},$$
$$\{e_2,e_3,e_6\},\{e_2,e_4,e_5\},\{e_2,e_4,e_6\},\{e_2,e_5,e_6\},\{e_3,e_4,e_5\},\{e_3,e_4,e_6\},\{e_3,e_5,e_6\},$$

the edge connected dominating sets of size four is 14, 6 edge connected dominating sets of size five and one edge connected dominating sets of size six.

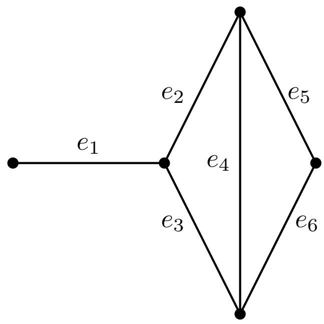


**Figure 1.** Special graph with labelled edges

Hence, $D_{ec}(G,x) = x^6 + 6x^5 + 14x^4 + 14x^3 + 5x^2$.

**Theorem 2.3.** *Let $G$ be a connected graph with size $m$. Then*

*i)* $d_{ec}(G,m) = 1$ *and* $d_{ec}(G,m-1) = m$.

*ii)* $d_{ec}(G,k) = 0$ *iff* $k < \gamma_{ec}(G)$ *or* $k > m$.

*iii)* $D_{ec}(G,x)$ *has no constant term.*

*iv)* $D_{ec}(G,x)$ *is a strictly increasing function in* $[0,\infty)$.

*v) Let $G$ be a connected graph and $H$ be any induced connected subgraph of $G$. Then,* $deg(D_{ec}(G,x)) \geq deg(D_{ec}(H,x))$.

*vi) Zero is a root of* $D_{ec}(G,x)$ *with multiplicity* $\gamma_{ec}(G)$.

*Proof.* i) Since $G$ has $m$ edges, there is only one way to choose all these edges which dominates all the edges and vertices. Therefore, $d_{ec}(G,m) = 1$. If we delete one edge, $e$, the remaining $m-1$ edges dominate all the edges and vertices of $G$, (This is done in $m$ ways). Therefore, $d_{ec}(G,m-1) = m$.

ii) Since $D_{ec}(G,k) = \phi$ if $k < \gamma_{ec}(G)$ or $D_{ec}(G,m+i) = \phi$, $i \geq 1$. Therefore, we have $d_{ec}(G,k) = 0$ if $k < \gamma_{ec}(G)$ or $k > m$. Conversely, if $k < \gamma_{ec}(G)$ or $k > m$, $d_{ec}(G,k) = 0$.

iii) Since $\gamma_{ec}(G) \geq 1$, the edge connected domination polynomial has no term of degree 0. Therefore, there is no constant term.

iv) The proof follows from the definition of edge connected domination polynomial.

v) We have $deg(D_{ec}(H,x)) =$ Number of edges in $H$ and $deg(D_{ec}(G,x)) =$ Number of edges in $G$. Since number of edges in $H \leq$ number of edge in $G$,
Thus we have, $deg(D_{ec}(H,x)) \leq deg(D_{ec}(G,x))$.

vi) The proof follows by part (iii) and Definition 2.1.

□

**Theorem 2.4.** *If $G$ is a connected graph consisting of two connected components $G_1$ and $G_2$, then*

$$D_{ec}(G,x) = D_{ec}(G_1,x)D_{ec}(G_2,x).$$

*Proof.* Let $G_1$ and $G_2$ be the connected components of a graph $G$ with former of size $m_1$ and the latter of size $m_2$. Let the edge connected domination number of $G_1$ and $G_2$ be $\gamma_{ec}(G_1)$ and $\gamma_{ec}(G_2)$.

For any $k \geq \gamma_{ec}(G)$, the edge connected dominating sets of $k$ edges, connected in $G$, arises by choosing an edge connected dominating sets of $j$ edges of $G_1$ and an edge connected dominating sets of $k-j$ edges of $G_2$. The number of edge connected dominating sets in $G_1 \cup G_2$ is equal to the coefficient of $x^k$ in $D_{ec}(G_1,x)D_{ec}(G_2,x)$. The number of edge connected dominating sets of $G$ is the coefficient of $x^k$ in $D_{ec}(G,x)$.

Hence the coefficient of $x^k$ in $D_{ec}(G,x)$ and $D_{ec}(G_1,x).D_{ec}(G_2,x)$ are equal.

Therefore, $D_{ec}(G,x) = D_{ec}(G_1,x).D_{ec}(G_2,x)$. □

**Theorem 2.5.** *For any simple connected graph $G$ with $n$ components, say $G_1, G_2, \cdots, G_n$, then*

$$D_{ec}(G,x) = D_{ec}(G_1,x)D_{ec}(G_2,x)\cdots D_{ec}(G_n,x).$$

*Proof.* The proof follows from the Theorem 2.4. □

**Theorem 2.6.** *For any path $P_n$ with $n \geq 4$ (with $m \geq 3$ edges), then*

$$D_{ec}(P_n,x) = x^m + 2x^{m-1} + x^{m-2}, \text{ where } m = n-1.$$

*Proof.* Let $G$ be path $P_n$ with $m \geq 3$ and let $P_n = v_1e_1v_2e_2\cdots v_{n-1}e_mv_n$. The edge connected domination number of $P_n$ is $m-2$ and there is only one edge connected domination sets of size $m-2$. That means, $d_{ec}(P_n, m-2) = 1$. Moreover, there are only two edge connected dominating sets of size $m-1$ namely $\{e_2, \cdots, e_m\}$ and $\{e_1, e_2, \cdots, e_{m-1}\}$.

Therefore, $d_{ec}(P_n, m-1) = 2$ and clearly there is only one edge connected dominating sets of size $m$. Hence, $D_{ec}(P_n,x) = x^m + 2x^{m-1} + x^{m-2}$ and it is clear that the roots of $D_{ec}(P_n,x)$ are 0 with multiplicity $m-2$ and $-1$ with multiplicity 2. □

**Theorem 2.7.** *For any cycle graph $C_n$ with $n$ vertices and $m$ edges, then*

$$D_{ec}(C_n,x) = x^m + mx^{m-1} + mx^{m-2}, \text{ where } m = n.$$

*Proof.* Let $G$ be a cycle, $C_n$, with $n$ vertices and let $C_n = v_1e_1v_2e_2\cdots v_ne_nv_1$. In a cycle graph the order is equal to the its size, that is $n = m$. The edge connected domination number of $C_n$ is $m-2$ and there are $m$ possibilities for the edge connected dominating sets of size $(m-1)$ and $(m-2)$. That means, $d_{ec}(C_n, m-1) = d_{ec}(C_n, m-2) = m$.

Furthermore, there are only one edge connected dominating sets of size $m$.

Hence, $D_{ec}(C_n,x) = x^m + mx^{m-1} + mx^{m-2}$ and $R(D_{ec}(C_n,x))$ are 0 with multiplicity $(m-2)$, $\dfrac{-m+\sqrt{m^2-4m}}{2}$ and $\dfrac{-m-\sqrt{m^2-4m}}{2}$. □

**Theorem 2.8.** *For any star graph $S_{1,n}$ with $n+1$ vertices and $m$ edges, where $n \geq 2$, then*

$$d_{ec}(S_{1,n},k) = \binom{m}{k}, \text{ where } m = n.$$

*Proof.* Let $S_{1,n}$ be the star graph with $n+1$ vertices and $m$ edges, and the edge connected domination number of $S_{1,n}$ is one, $\gamma_{ec}(S_{1,n}) = 1$. Let $d_{ec}(S_{1,n},k)$ is a dominating set of $S_{1,n}$ of size $k$ then there are $\binom{m}{k}$ possibilities of a connected edge subsets of $S_{1,n}$ with cardinality $k$.

Therefore $d_{ec}(S_{1,n},k) = \binom{m}{k}$, where $m = n$. □

**Theorem 2.9.** *For any star graph $S_{1,n}$ with $n+1$ vertices and $m$ edges, where $n \geq 2$,*

$$D_{ec}(S_{1,n},x) = (x+1)^m - 1.$$

*Proof.* By Definition 2.1, we have:
$D_{ec}(S_{1,n},x) = \sum_{k=\gamma_{ec}(S_{1,n})}^{m} d_{ec}(S_{1,n},k)x^k$ and by Theorem 2.4, we have:

$$
\begin{aligned}
D_{ec}(S_{1,n},x) &= \sum_{k=1}^{m} \binom{m}{k} x^k \\
&= \binom{m}{1} x + \binom{m}{2} x^2 + \cdots + \binom{m}{m} x^m \\
&= [\binom{m}{1} x + \binom{m}{2} x^2 + \cdots + \binom{m}{m} x^m + 1] - 1 \\
&= \sum_{k=0}^{m} \binom{m}{k} x^k - 1 \\
&= (x+1)^m - 1.
\end{aligned}
$$

Hence, $D_{ec}(S_{1,n},x) = (x+1)^m - 1$. □

**Theorem 2.10.** *The bi-star graph $B_{n_1,n_2}$ with $n_1+n_2$ vertices and $m_1+m_2+1$ edges, where $n_1,n_2 \geq 2$, then*

$$d_{ec}(B_{n_1,n_2},k) = \binom{m_1+m_2}{k-1}.$$

*Where $n_1-1=m_1$, $n_2-1=m_2$ and $\gamma_{ec}(B_{n_1,n_2})=1$.*

*Proof.* Let $B_{n_1,n_2}$ be the bi-star graph of order $n_1+n_2$ and size $n_1+n_2+1$. The edge connected domination number of $B_{n_1,n_2}$ is one as there is one edge between $n_1$ and $n_2$ which dominated all the edges of $B_{n_1,n_2}$. The number of edge connected dominating sets of size two is $\binom{m_1+m_2}{1}$ and of size three is $\binom{m_1+m_2}{2}$ and so on.

In general, we have $\binom{m_1+m_2}{k-1}$ edge connected dominating sets of size $k$. □

**Theorem 2.11.** *Let $G$ be a bi-star graph $B_{n_1,n_2}$, then*

$$D_{ec}(B_{n_1,n_2},x) = x(1+x)^{m_1+m_2}.$$

*Proof.* By Definition 2.1, we have:
$D_ec(B_{n_1,n_2},x) = \sum_{k=\gamma_{ec}(B_{n_1,n_2})}^{m} d_{ec}(B_{n_1,n_2},k)x^k$ and by Theorem 2.10, we have:

$$
\begin{aligned}
D_ec(B_{n_1,n_2},x) &= \sum_{k=1}^{m=m_1+m_2+1} \binom{m_1+m_2}{k-1} x^k \\
&= \binom{m_1+m_2}{0} x + \binom{m_1+m_2}{1} x^2 + \binom{m_1+m_2}{2} x^3 + \cdots \\
&+ \binom{m_1+m_2}{m_1+m_2} x^{m_1+m_2+1} \\
&= x[1 + \binom{m_1+m_2}{1} x + \binom{m_1+m_2}{2} x^2 + \cdots + \binom{m_1+m_2}{m_1+m_2} x^{m_1+m_2}] \\
&= x[\sum_{k=0}^{m_1+m_2} \binom{m_1+m_2}{k} x^k] \\
&= x(x+1)^{m_1+m_2}.
\end{aligned}
$$

Hence, $D_{ec}(B_{n_1,n_2}, x) = x(1+x)^{m_1+m_2}$ and $R(D_{ec}(B_{n_1,n_2}, x))$ are 0 with multiplicity 1 and $-1$ with multiplicity $m_1 + m_2$. □

**Definition 2.12.** Let $Y_t$ be a graph obtained from $Y_1 = K_{1,3}$ by identifying each end vertex of $Y_{t-1}$ with the central vertex of $K_{1,2}$. There exist $3(2^{t-1})$ end vertices which forms $3(2^{t-2})$ pairs for $t \geq 2$. The order of $Y_t$ is $n(Y_t) = 3(2^t) - 2$ and size $m(Y_t) = 3(2^t) - 3$. [5]

The radius of $Y_t$ is $t$, while the diameter of $Y_t$ is $2t$, moreover, it is a unicentral tree.

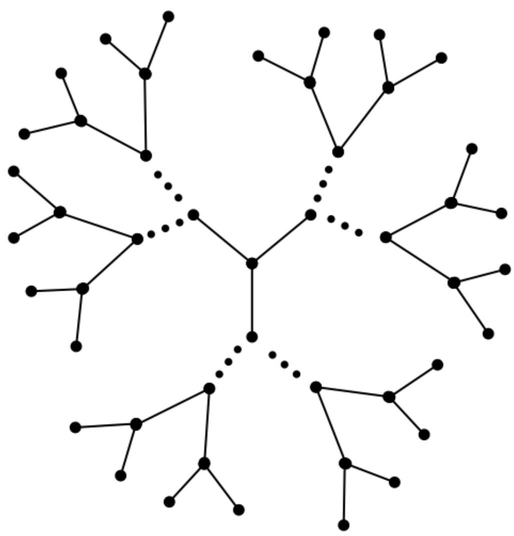

**Figure 2.** The graph of $Y_t$

**Theorem 2.13.** *The edge connected dominating sets and the edge connected domination number of $Y_t$ is given by* $d_{ec}(Y_t, k) = \binom{3(2^{t-1})}{k + 3 - 3(2^{t-1})}$ *and* $\gamma_{ec}(Y_t) = 3(2^t) - 3$*, for all* $t \geq 2$.

*Proof.* Let $t = 1$, we have 3 edges connected dominating sets of size one, 3 edge connected dominating sets of size two and one edge connected dominating sets of size 3.

In general, we have $d_{ec}(Y_1) = \binom{3}{k}$, for $k = 1, 2$ and 3.

Let $t = 2$, we have one edge connected dominating sets of size 3, 6 edge connected dominating sets of size four and so on. Therefore, we have $d_{ec}(Y_2) = \binom{6}{k-3}$, for $k = 3, 4, \cdots, 9$, with $\gamma_{ec}(Y_2) = 3$.

For $t \geq 3$, the $Y_t$ graph has $3(2^t - 1)$ edges and $3(2^t) - 3$ end edges. Thus by calculating, we have one edge connected dominating set of size $3(2^{t-1})$, in general $\gamma_{ec}(Y_t) = 3(2^{t-1} - 1)$.

Hence, we have:

$$\begin{aligned}
d_{ec}(Y_t, k) &= \binom{3(2^t - 1) - (3(2^{t-1} - 1))}{k - (3(2^{t-1} - 1))} \\
&= \binom{3(2^t) - 3(2^{t-1})}{k - 3(2^{t-1} - 1)} \\
&= \binom{3(2^{t-1})(2 - 1)}{k - 3(2^{t-1} - 1)} \\
d_{ec}(Y_t, k) &= \binom{3(2^{t-1})}{k - 3(2^{t-1} - 1)} \text{ and } \gamma_{ec}(Y_t) = 3(2^{t-1} - 1).
\end{aligned}$$

□

**Theorem 2.14.** *The edge connected domination polynomial of $Y_t$ is given by*

$$D_{ec}(Y_t,x) = x^{3(2^{t-1}-1)}(x+1)^{3(2^{t-1})}.$$

*Proof.* By Defintion 2.1, we have $D_{ec}(Y_t,x) = \sum_{k=\gamma_{ec}(Y_t)}^{m} d_{ec}(Y_t,k)x^k$, and by Theorem 2.13, we have:

$$\begin{aligned}
D_{ec}(Y_t,x) &= \sum_{k=3(2^{t-1}-1)}^{3(2^t-1)} \binom{3(2^{t-1})}{k+3(1-2^{t-1})} x^k \\
&= \binom{3(2^{t-1})}{0} x^{3(2^{t-1}-1)} + \binom{3(2^{t-1})}{1} x^{3(2^{t-1}-1)+1} + \binom{3(2^{t-1})}{2} x^{3(2^{t-1}-1)+2} + \cdots \\
&+ \binom{3(2^{t-1})}{3(2^{t-1})} x^{3(2^{t-1})} \\
&= x^{3(2^{t-1})}[1 + \binom{3(2^{t-1})}{1} x + \binom{3(2^{t-1})}{2} x^2 + \cdots + \binom{3(2^{t-1})}{3(2^{t-1})} x^{3(2^{t-1})}] \\
&= x^{3(2^{t-1}-1)}[\sum_{k=0}^{3(2^{t-1})} \binom{3(2^{t-1})}{k} x^k].
\end{aligned}$$

Hence, $D_{ec}(Y_t,x) = x^{3(2^{t-1}-1)}(x+1)^{3(2^{t-1})}$ and $R(D_{ec}(Y_t,x))$ are 0 and $-1$ with multiplicity $3(2^{t-1}-1)$ and $3(2^{t-1})$, respectively. □

**Definition 2.15.** Let $Y_t^*$ be a graph obtained from $Y_1 = K_{1,3}$ by identifying each end vertex of $Y_{t-1}^*$ with an end vertex of $Y_1 = K_{1,3}$. The order of $Y_t^*$ is $n(Y_t^*) = 9(2^{t-1}) - 5$ and its size is $m(Y_t^*) = 9(2^{t-1} - 6)$. The number of end vertices in $Y_t^*$ is $3(2^{t-1})$. [5]

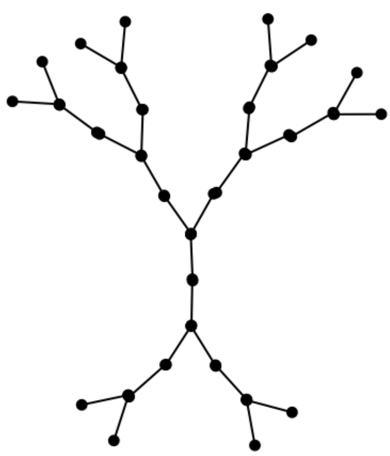

**Figure 3.** For example, when t = 3, this is the graph of $Y_3^*$

Moreover, the radius of $Y_t^*$ is $1+2^{t-1}$ and diameter $2+2^t$ for $t \geq 2$, which is also a unicentral tree.

**Theorem 2.16.** *The edge connected dominating sets and the edge connected domination number of $Y_t^*$ is given by* $d_{ec}(Y_t^*,k) = \binom{3(2^{t-1})}{k+6-3(2^t)}$, *and* $\gamma_{ec}(Y_t^*) = 3(2^t) - 6$, *for all* $t \geq 2$.

*Proof.* For $n = 1$, the proof is similar to the same case of $d_{ec}(Y_t,k)$.

Let $n = 2$, we have one edge connected dominating sets of size 6, 15 edge connected dominating sets of size 8 and so on. In general, we have $d_{ec}(Y_2^*, k) = \binom{6}{k-6}$ for $k = 6, 7, \cdots, 9$ and $\gamma_{ec}(Y_2^*) = 6$.

By calculating, we have $d_{ec}(Y_3^*, k) = \binom{12}{k-18}$ for $k = 18, 19, \cdots, 30$

$d_{ec}(Y_4^*, k) = \binom{24}{k-42}$ for $k = 42, 43, \cdots, 66$

$\vdots$

The $Y_t^*$ graph have $9(2^{t-1}) - 6$ edges and the edge connected dominating number of $Y_t^*$ is $3(2^t) - 6$.

Thus, we have:

$$\begin{aligned} d_{ec}(Y_t^*, k) &= \binom{9(2^{t-1}) - 6 - (3(2^t) - 6)}{k - (3(2^t) - 6)} \\ &= \binom{9(2^{t-1}) - 3(2^t)}{k - 3(2^t) + 6} \end{aligned}$$

Hence, $d_{ec}(Y_t^*, k) = \binom{3(2^{t-1})(3-2)}{k - 3(2^t) + 6} = \binom{3(2^{t-1})}{k - 3(2^t) + 6}$ and $\gamma_{ec}(Y_t^*) = 3(2^t) - 6$.

□

**Theorem 2.17.** *The edge connected domination polynomial of $Y_t^*$ is given by*

$$D_{ec}(Y_t^*, x) = x^{3(2^{t-1})-6}(x+1)^{3(2^{t-1})}.$$

*Proof.* By Definition 2.1, we have:

$D_{ec}(Y_t^*, x) = \sum_{k=\gamma_{ec}(Y_t^*)}^{9(2^{t-1})-6} d_{ec}(Y_t^*, k)x^k$ and by Theorem 2.16, we have:

$$\begin{aligned} D_{ec}(Y_t^*, x) &= \sum_{k=3(2^{t-1}-1)}^{9(2^{t-1})-6} \binom{3(2^{t-1})}{k+6-3(2^t)} x^k \\ &= \binom{3(2^{t-1})}{0} x^{3(2^t)-6} + \binom{3(2^{t-1})}{1} x^{3(2^t)-6+1} + \binom{3(2^{t-1})}{2} x^{3(2^t)-6+2} + \cdots \\ &+ \binom{3(2^{t-1})}{3(2^{t-1})} x^{9(2^t)-6} \\ &= x^{3(2^{t-1})-6}[1 + \binom{3(2^{t-1})}{1} x + \binom{3(2^{t-1})}{2} x^2 + \cdots + \binom{3(2^{t-1})}{3(2^{t-1})} x^{3(2^{t-1})}] \\ &= x^{3(2^{t-1})-6}[\sum_{k=0}^{3(2^{t-1})} \binom{3(2^{t-1})}{k} x^k] \end{aligned}$$

Hence, $D_{ec}(Y_t^*, x) = x^{3(2^{t-1})-6}(x+1)^{3(2^{t-1})}$.

In addition, $R(D_{ec}(Y_t^*, x))$ are 0 and $-1$ with multiplicity $3(2^{t-1}) - 6$ and $3(2^{t-1})$, respectively. □

## 3 Edge Connected Domination Polynomial of Spider Graph

In this section, we determine the edge connected dominating sets and edge connected domination polynomial of the spider and bispider graphs.

**Definition 3.1.** The spider graph is a graph obtained from a star graph by introducing each end vertex by one vertex, in other word, a tree with at most one vertex of degree more than two is called a spider graph and denoted by $S_p$, for all $p \geq 2$ of size $m = 2p$.

**Example 3.2.** Here are some examples of spider graphs:

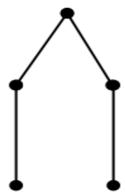

**Figure 4.** $S_2$

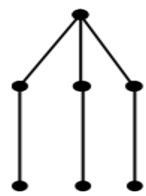

**Figure 5.** $S_3$

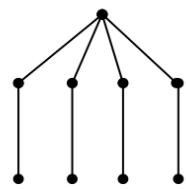

**Figure 6.** $S_4$

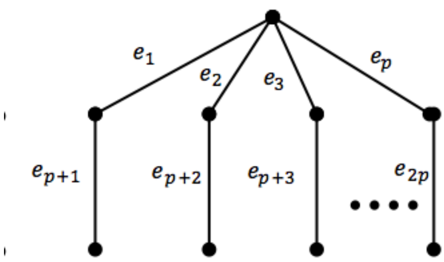


**Figure 7.** $S_p$

**Theorem 3.3.** *The edge connected dominating sets of size $k$ for spider graph is $\binom{p}{k-p}$ and $\gamma_{ec}(S_p) = p$.*

*Proof.* Let $E_1 = \{e_1, e_2, e_3, \cdots, e_p\}$ and $E_2 = \{e_{p+1}, e_{p+2}, e_{p+3}, \cdots, e_{2p}\}$.

There is one edge connected dominating sets of size $p$, $E_1$, which is the minimum ones, i.e., $d_{ec}(S_p, p) = 1$ and $\gamma_{ec}(S_p) = p$.

There are $\binom{p}{1}$ ways to extend the edge connected dominating sets of size $p+1$, i.e., $d_{ec}(S_p, p+1) = \binom{p}{1}$ and there are $\binom{p}{2}$ edge connected dominating sets of size $p+2$, that is $d_{ec}(S_p, p+2) = \binom{p}{2}$, and so on.

In general, we have $d_{ec}(S_p, k) = \binom{p}{k-p}$ where $p \leq k \leq 2p$. $\square$

**Theorem 3.4.** *The edge connected dominating polynomial of $S_p$ is*

$$D_{ec}(S_p, x) = \sum_{k=p}^{2p} \binom{p}{k-p} x^k.$$

*Proof.* By Definition 2.1 and Theorem 3.3, we have:

$$
\begin{aligned}
D_{ec}(S_p, x) &= \sum_{k=p}^{2p} \binom{p}{k-p} x^k \\
&= \binom{p}{0-p} x^p + \binom{p}{p+1-p} x^{p+1} + \cdots + \binom{p}{2p-p} x^{2p} \\
&= \binom{p}{0} x^p + \binom{p}{1} x^{p+1} + \cdots + \binom{p}{p} x^{2p} \\
&= x^p[1 + \binom{p}{1} x + \binom{p}{2} x^2 + \cdots + \binom{p}{p} x^p] \\
&= x^p[\sum_{k=0}^{p} \binom{p}{k} x^p] \\
&= x^p(x+1)^p.
\end{aligned}
$$

$R(D_{ec}(S_p, x))$ are 0 and $-1$ with multiplicity $p$. □

**Definition 3.5.** The bispider graph is a graph obtained by edge introducing between two star graphs and the introducing is the rooted vertices, which is denoted by $S_{p_1,p_2}$ of order $2p_1+2p_2+2$ and size $2p_1 + 2p_2 + 1$.

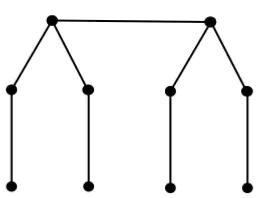

**Figure 8.** $S_{2,2}$

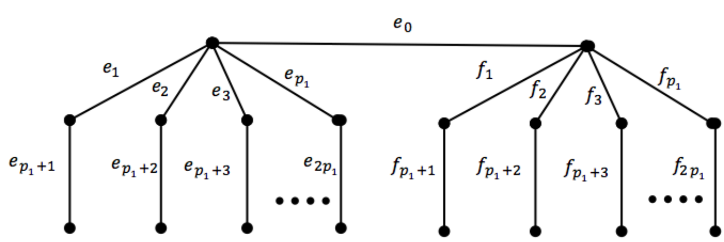


**Figure 9.** The graph of $S_{p_1,p_2}$

**Theorem 3.6.** *The edge connected dominating sets of bispider graph is*

$$\binom{p_1+p_2}{k-(p_1+p_2+1)} \text{ and } \gamma_{ec}(S_{p_1,p_2}) = p_1+p_2+1.$$

*Proof.* The proof is similar to Theorem 3.3. □

**Theorem 3.7.** *The edge connected dominating polynomial of $S_{p_1,p_2}$ is*

$$D_{ec}(S_{p_1,p_2}, x) = \sum_{k=p_1,p_2+1}^{2p_1+2p_2+1} \binom{p_1+p_2}{k-(p_1+p_2+1)} x^k.$$

*Proof.* By Definition 2.1 and Theorem 3.4, we have:

$$\begin{aligned}
D_{ec}(S_{p_1,p_2},x) &= \sum_{k=p_1,p_2+1}^{2p_1+2p_2+1} \binom{p_1+p_2}{k-(p_1+p_2+1)} x^k \\
&= \binom{p_1+p_2}{0} x^{p_1+p_2+1} + \binom{p_1+p_2}{1} x^{p_1+p_2+2} + \cdots \\
&+ \binom{p_1+p_2}{2p_1+2p_2+1-p_1-p_2-1} x^{2p_1+2p_2+1} \\
&= x^{p_1+p_2+1}[1 + \binom{p_1+p_2}{1} x + \binom{p_1+p_2}{2} x^2 + \cdots + \binom{p_1+p_2}{p_1+p_2} x^{p_1+p_2}] \\
&= x^{p_1+p_2+1}(x+1)^{p_1+p_2}.
\end{aligned}$$

$R(D_{ec}(S_{p_1,p_2},x))$ are 0 and $-1$ with multiplicity $p_1+p_2+1$ and $p_1+p_2$, respectively. □

## References


[1] Saeid Alikhani and Yee-hock Peng, Introduction to domination polynomial of a graph *arXiv preprint arXiv:0905.2251*, (2009).

[2] S. Arumugam and S. Velammal. Edge domination in graphs, *Taiwanese journal of Mathematics* **51**, 173–179 (1998).

[3] Frank Harary, Graph theory, *Addison-Wesley, Reading, MA,* **Vol. 9**, (1969).

[4] S. T. Hedetniemi and S. Mitchell, Edge domination in trees, *Proc. 8th SE Conf. Combin., Graph Theory and Computing, Congr. Numer.* **Vol. 19**, (1977).

[5] N. B. Ibrahim, On the Nullity of Some Sequential Element Identified, Element Introduced Graphs, MSc. Thesis, (2013)

[6] E. Sampathkumar and H. B. Walikar, The connected domination number of a graph, *J. Math. Phys*, (1979).


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