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A NOTE ON Ω-CONNECTEDNESS

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Abstract. Connectedness plays an important part in the study of topology. Several authors have generalized this notion by using generalized open and closed sets. In [9], the concept of Ω-open and Ω-closed sets have been introduced and studied. By using these set, we have introduced Ω-connectedness and investigated its properties.

1 Introduction

Let X be a topological space. In 1982, Hdeib [10] introduced the notion of ω-closeness. Using this concept, he introduced and studied ω-continuity. In 1968, the notions of θ-open subsets, θ-closed subsets and θ-closure were introduced by Veli$\check{c}$ko [20] for the purpose of studying the important class of H-closed spaces in terms of filter bases. He also showed that the collection of θ-open sets in a topological space X itself forms a topology $\Im_\theta$ on X. Dickman and Porter [4], [5], Joseph [13] extended the work of Veli$\check{c}$ko to study further properties of H-closed spaces. Noiri and Jafari [17], Caldas et al. [1] and [2], Steiner [18] and Cao et al [3] have also obtained several new and interesting results related to these sets. In [9], we have introduced the concept of $\Omega-$open, $\Omega-$ closed sets and studied their properties. Exploiting this concept, in this paper, we introduce and study the notion of Ω-connectedness. We start with the idea of Ω-separated sets which is keynote in introducing Ω-connectedness.

2 Preliminaries

Throughout this paper a space will always mean a topological space on which no separation axioms are assumed unless otherwise explicitly stated. Let (X,$\Im$) be a space and let A be a subset of X. The closure and interior of A are denoted as cl(A) and int(A) respectively. A point x $\in$ X is called a clocondensation point of A [9] if for each open set U containing x, the set cl(U) $\cap$ A is uncountable. A is called Ω-closed if it contains all its clocondensation points. The complement of an Ω-closed set is called Ω-open. A subset W of a space (X,$\Im$) is Ω-open if and only if for each x $\in$ W there exists an open set U containing x such that cl(U)-W is countable. The family of all Ω-open subsets of a space (X,$\Im$), denoted by $\Im_\Omega$, forms a topology on X . Let (X,$\Im$) be a space and A be a subset of X.The Ω-interior and Ω-closure of a subset A of a space (X,$\Im$) is denoted as Ω-cl(A) and Ω-int(A) in the space (X,$\Im_\Omega$). A function f : X$\rightarrow$Y is said to be Ω-continuous [9] if $\forall$ x $\in$ X and $\forall$ V open in Y containing f(x), $\exists$ an Ω-open subset containing x such that f(U) $\subset$ V.

3 $\Omega-$ Seprated Sets

Definition 3.1. Two nonempty subsets A and B of a topological space(X ,$\Im$) are said to be Ω-separated if A$\cap$(Ω-Cl(B)) = (Ω-Cl(A))$\cap$B = ϕ. Obviously, if A and B are two Ω separated sets, then A $\cap$ B = ϕ.Whenever X is expressed as a union of two Ω separated sets A and B, then we say that A and B form an Ω-separation of X.

Remark 3.2. Let (X ,$\Im$) be a topological space. If X can be written as union of two Ω-separated sets it does not necessarily mean that it can be written as union of two separated sets and vice versa as can be seen from the following examples :

Example 3.3. Let N be the set of natural numbers equipped with the topology $\Im$= $\{\phi$, X, N_m : m $\in N\}$ where N_m = $\{m, m+1, m+2, \ldots..\}$ then it is clear that $\Im_\Omega$ is discrete topology on N as N is countable and therefore X has Ω-separation since every Ω-open set is Ω-closed. However, X has no separation with respect to $\Im$.

Example 3.4. Let R be the real line having the topology $\Im$= $\{\phi$, X, Q$\}$. Evidently, $\Im_\Omega$ is the co-countable topology. Here X is neither separated nor Ω - separated. Note that Q is open in $\Im$ while closed in $\Im_\Omega$.

Example 3.5. Let R be the real line equipped with discrete topology then $\Im_\Omega$ is discrete topology and R is separated with respect to both the topologies.

Example 3.6. Let R be the real line with point exclusion topology then $\Im_\Omega$ is cocountable topology therefore R is separated but not Ω-separated.

Example 3.7. The two Ω-separated sets are always disjoint, since A∩B $\subset$ A∩(Ω-Cl(B)) =ϕ.

Theorem 3.8. *For any non – empty subsets A and B of a topological space(X ,$\Im$), the following are equivalent:*

(i) A and B are Ω-separated.

(ii) There exist Ω-closed sets F and G satisfying A $\subset$ F$\subset$ (X $\sim$ B) and B $\subset$ G$\subset$ (X $\sim$ A).

(iii) There exist Ω-open sets U and V satisfying A $\subset$ U$\subset$ (X $\sim$ B) and B $\subset$ V$\subset$ (X $\sim$ A).

Proof. The Proof is straightforward and hence omitted.

Theorem 3.9. *Let A and B be subsets of a topological space(X, $\Im$). If A and B are Ω-separated, $\phi\neq$ C $\subset$ A and $\phi\neq$ D $\subset$ B, then C and D are Ω-separated.*

Proof. Since A and B are Ω-separated sets, A ∩ (Ω-Cl(B))= ϕ and (Ω-Cl(A))∩B = ϕ. By hypothesis C$\subset$A, we have (Ω-cl(C)) ∩ D = ϕ.Similarly, we have C ∩ (Ω-Cl(D)) = ϕ. Therefore, C and D are Ω-separated sets.

Theorem 3.10. *Let C be a Ω-closed subset of a topological space (X, $\Im$) and let A and B be Ω-separated sets such that C = A∪B, then A and B are Ω-closed sets.*

Proof. Let C = A∪B, where (Ω-Cl(A)) ∩ B = ϕ=A ∩ (Ω-Cl(B)). Now, C ∩ (Ω-Cl(A)) = (A∪B) ∩ (Ω-Cl(A)) = A. Since the intersection of two Ω-closed sets is Ω-closed, therefore A is Ω-closed. Similarly, it can be shown that B is Ω-closed.

Theorem 3.11. *Let A and B be non-empty subsets in a topological space (X, $\Im$). Then the following statements hold:*
(1) If A and B are Ω -separated and P $\subset$ A, Q $\subset$ B, then P and Q are also Ω -separated.
(2) If A∩B = ϕ such that A and B are Ω -closed (Ω -open), then A and B are Ω -separated.
(3) If A and B are Ω -closed (Ω -open) and H = A∩(X∼B) and G= B∩(X∼A), then H and G are Ω -separated.

Proof. (1) Since P $\subset$ A, (Ω-cl(P)) $\subset$ (Ω-cl(A)). Then B ∩ (Ω-cl(A)) = ϕ implies Q ∩ (Ω-cl(A)) =ϕ and Q ∩ (Ω-cl(P)) =ϕ. Similarly P ∩ (Ω-cl(Q))=ϕ. Hence P and Q are Ω-separated.
(2) Since A= (Ω-cl(A)), B = (Ω-cl(B)) and A∩B = ϕ, (Ω-cl(A)) ∩ B = ϕ and (Ω-cl(B)) ∩ A = ϕ. Hence A and B are Ω-separated sets. If A and B are Ω-open, then their complements are Ω-closed.
(3) If A and B are Ω-open, then X-A and X-B are Ω-closed. Since H $\subset$ X∼ B, (Ω-cl(H)) $\subset$ (Ω-cl(X∼B)) = X $\sim$ B and so (Ω-cl(H)) ∩ B = ϕ. Thus G ∩ Ω-Cl(H)= ϕ. Similarly, H ∩ (Ω-cl(G)) =ϕ. Hence H and G are Ω-separated sets.

Theorem 3.12. *Two sets A and B of a topological space (X, $\Im$) are Ω-separated if and only if there exist Ω-open sets U and V such that A ⊂ U, B ⊂ V, A∩V=ϕ and B ∩ U = ϕ.*

Proof. Let A and B be Ω-separated sets. Let V = X∼(Ω-cl(A)) and U = X∼(Ω-cl(B)). Then U, V are Ω-open sets such that A⊂ U, B ⊂ V, A∩V = ϕ and B ∩ V = ϕ. Conversely, let U,V be two Ω-open subsets of X satisfying A ⊂ U, B ⊂ V, A∩V=ϕ and B∩U=ϕ. Since X∼V and X∼ U are Ω-closed sets, (Ω-cl(A))⊂ X∼V ⊂ X∼B and (Ω-cl(B))⊂ X∼U ⊂ X∼A. Thus (Ω-cl(A)) ∩ B=ϕ and (Ω-cl(B)) ∩ A = ϕ.

Theorem 3.13. *Let A and B be non-empty disjoint subsets of a topological space(X, $\Im$) and let E be a subset of X such that E = A∪B. Then A and B are Ω-separated in X if and only if each of A and B are Ω-closed (Ω-open) in E.*

Proof. Let A and B be Ω -separated sets in X. Then A ∩ (Ω-cl(B)) =ϕ which implies that (Ω-cl(B))⊂ (X∼A) i.e. B contains all Ω -limit points of B which are in A∪B = E. Hence B is Ω-closed in E. Similarly A is also Ω-closed in E.

Theorem 3.14. *Let (X, $\Im$) be a topological space. If A and B are Ω-separations of X itself, then A and B are Ω -closed sets of (X, $\Im$) .*

Proof. Since A and B are Ω-separated, A ∩ (Ω-cl(B)) = (Ω-cl(A)) ∩ B=ϕ. Then A ∩ (Ω-cl(B)) =ϕ if and only if B is Ω -closed in A∪B = X. Similarly, we can show that A is Ω-closed in X.

Theorem 3.15. *X has Ω separation if and only if X has a subset which is both Ω-open and Ω-closed.*

Proof. Let A be such subset of X then X∼A is both Ω-open and Ω-closed such that A ∩ (X∼A) = ϕ while A ∪ (X∼A) = X. Conversely, let X = A ∪ B such that both A and B are disjoint, nonempty and Ω-open. Then A is Ω-closed also.

4 Properties Of Ω – Connected Spaces

In this section, we introduce and study Ω -connected spaces and also investigate some of their basic properties.

Definition 4.1. A subset A of a topological space (X, $\Im$) is said to be Ω -connected if it cannot be expressed as the union of two Ω -separated sets. Otherwise, the set A is called Ω-disconnected.

Example 4.2. Let N be the set of natural numbers equipped with the topology $\Im$ = {ϕ, X, N_m : m ∈ N} where N_m ={m,m+1,m+2,. . . ..} then $\Im_\Omega$ = discrete topology as N is countable and therefore disconnected as every Ω-open set is Ω-closed whereas X is $\Im$ -connected.

Example 4.3. Let R be the real line having the topology $\Im$ = {ϕ, X, Q} then $\Im_\Omega$ is cocountable topology. Here both $\Im$ and $\Im_\Omega$ are connected. Note that Q is open in $\Im$ while closed in $\Im_\Omega$.

Example 4.4. Let R be the real line equipped with real topology then $\Im_\Omega$ is also discrete topology and both the topologies are disconnected.

Example 4.5. Let R be the real line with point exclusion topology then $\Im_\Omega$ is cocountable topology therefore R is disconnected but Ω-connected.

Theorem 4.6. *Let A ⊂ B ∪ C such that A is a nonempty Ω-connected set in a topological space (X, $\Im$) and B , C be Ω-separated sets. Then only one of the following conditions holds:*
(a) A ⊂ B and A∩C = ϕ.
(b) A⊂ C and A∩B = ϕ.

Proof. If A ∩ C = ϕ then A ⊂ B. Similarly, if A∩B = ϕ, then A ⊂ C. Since A ⊂ B ∩ C, then both A ∩ B = ϕ and A ∩ C = ϕ cannot hold simultaneously. Conversely, suppose that A∩B $\neq \phi$ and A ∩ C $\neq\phi$, then, A∩B and A∩C are Ω-separated sets such that A = (A∩B) ∪(A∩C) which contradicts with the Ω-connectedness of A. Hence only one of the conditions (a) and (b) must hold.

Theorem 4.7. *If A and B are Ω -separated sets in a topological space (X, ℑ) such that X=A ∪ B and if an Ω-connected set S is contained in A∪B, then either S ⊂A or S ⊂ B.*

Proof. We are given that X = A ∪ B. Now, S = X ∩ S = (A ∪ B)∩S = (S∩A) ∪ (S∩B). Since (S∩A)⊂ A and(S∩B)⊂ B therefore they form separation of S. Since S is Ω-connected, therefore, either (S∩A) is empty or(S∩ B) is empty that is either S ⊂ B or S⊂ A.

Theorem 4.8. *Let B be a subset of a topological space(X , ℑ) such that there exists a Ω-connected set A satisfying A ⊂B ⊂ (Ω-cl(A)) then B is also Ω -connected.*

Proof. Let B= P∪ Q, where P and Q are Ω-separated sets. Then either A⊂P or A⊂ Q and hence either B ⊂ (Ω-cl(A)) ⊂ (Ω-cl(P)) ⊂ (X∼ Q) or B⊂ (X∼ P). Therefore either P = ϕ or Q = ϕ.

Theorem 4.9. *If A is Ω-connected set of a topological space(X, ℑ), then so is (Ω-cl(A)) .*

Proof. Follows from **Theorem 4.8**.

Theorem 4.10. *If $\{C_\alpha$: $\alpha \in\Delta\}$ is a family of Ω-connected sets in a topological space (X, ℑ) satisfying the property that any two of them are not Ω-separated, then C = $\bigcup_{\alpha\in\Delta} C_\alpha$ is Ω - connected.*

Proof. Let C = A∪B , where A and B are Ω-separated sets. Then for each $\alpha\in\Delta$ either C_{α}⊂A or C_{α}⊂B. Since no two members of the family $\{C_{\alpha} : \alpha \in\Delta\}$ are Ω-separated, either C_{α}⊂ A for each $\alpha\in\Delta$ or C_{α}⊂B for each $\alpha\in\Delta$. So either B=ϕ or A=ϕ .

Theorem 4.11. *If C = $\bigcup_{\alpha\in\Delta} C_{\alpha}$, where each C_α is Ω-connected set in a topological space(X, ℑ) and also $C_{\alpha}\cap C_{\alpha}$'$\neq\phi$ for $\alpha, \alpha'\in\Delta$, then C is Ω-connected.*

Proof. Obvious and hence omitted.

Theorem 4.12. *If C=$\bigcup_{\alpha\in\Delta} C_{\alpha}$, where each C_{α} is Ω-connected in a topological space (X, ℑ) and $\bigcap_{\alpha\in\Delta} C_{\alpha}\neq\phi$ for each $\alpha\in\Delta$,then C is Ω-connected.*

Proof. Suppose that C is not Ω -connected. Let C = A∪B, where A and B are Ω-separated sets. Then for each $\alpha\in\Delta$ either C_{α}⊂ A or C_{α}⊂ B. Since $\bigcap_{\alpha\in\Delta} C_{\alpha}\neq\phi$, we have a point x $\in\bigcap_{\alpha\in\Delta} C_{\alpha}$. Then either x ∈A or x∈ B. Let x ∈ A. Since x ∈ C_{α} for each $\alpha\in\Delta$, then C_{α} ∈ A for each $\alpha\in\Delta$ which means that B contains no element of C as A and B are disjoint. Hence B is empty. Similarly if x ∈ B then due to same reason A will be empty. Thus C is Ω-connected.

Theorem 4.13. *For a topological space(X, ℑ) , the following statements are equivalent:*
(1) X is Ω -connected.
(2) X cannot be expressed as the union of two nonempty disjoint Ω -open sets.
(3) X contains no nonempty proper subset which is both Ω -open and Ω -closed.

Proof. (1) ⇒ (2): Suppose that X is Ω-connected and if X can be expressed as the union of two nonempty disjoint sets A and B such that A and B are Ω-open sets. Consequently A⊂ X∼ B. Then (Ω-cl(A)) ⊂ Ω-cl(X∼ B) = X∼B. Therefore, (Ω-cl(A)) ∩ B = ϕ. Similarly we can prove A ∩ (Ω-cl(B)) =ϕ. This is a contradiction to the fact that X is Ω-connected. Therefore, X cannot be expressed as the union of two nonempty disjoint Ω-open sets.
(2) ⇒ (3): Suppose that X cannot be expressed as the union of two nonempty disjoint sets A and B such that both A and B are Ω-open sets. If X contains a nonempty proper subset A which is both Ω-open and Ω-closed. Then X = A∪(X∼ A). Hence A and X∼ A are disjoint Ω -open sets whose union is X. This is the contradiction to our assumption. Hence, X contains no nonempty proper subset which is both Ω-open and Ω-closed.
(3) ⇒ (1): Suppose that X contains no nonempty proper subset which is both Ω -open and Ω -closed and X is not Ω -connected. Then X can be expressed as the union of two nonempty disjoint sets A and B such that (A∩ (Ω-cl(B)) ∪(((Ω-cl(A))∩B) =ϕ. Since A∩B = ϕ, A = X∼ B and B = X∼ A. Since (Ω-cl(A))∩B=ϕ, (Ω-cl(A))⊂ X∼ B. Hence (Ω-cl(A))⊂ A. Therefore, A is Ω-closed. Similarly, B is Ω-closed. Since A = X∼ B, A is Ω-open. Therefore, there exists a non-empty proper subset A which is both Ω-open and Ω-closed. This is a contradiction to our assumption. Therefore, X is Ω-connected.

Theorem 4.14. *A topological space(X, $\Im$) is Ω-connected if and only if for every pair of points x , y in X , there is a Ω -connected subset of X which contains both x and y.*

Proof. The necessity is immediate since the Ω-connected space itself contains these two points. For the sufficiency, suppose that for any two points x and y, there is a Ω-connected subset $C_{x,y}$ of X such that x , y $\in C_{x,y}$. Let a $\in$ X be a fixed point and consider the family $\{C_{a,x}: x \in X\}$ of all Ω-connected subsets of X which contain the points a and x. Then $X=\bigcup_{x\in X} C_{a,x}$ and $\bigcap_{x\in X} C_{a,x} \neq \phi$. Therefore X is Ω-connected.

Theorem 4.15. *For a topological space (X,$\Im$) the following are equivalent:*
(1). (X,$\Im$) is Ω-connected
(2). The only subsets of (X,$\Im$) which are both Ω-open and Ω-closed are the empty set ϕ and X.
(3). Each Ω-continuous map of (X,$\Im$) into a discrete space (Y,σ) with at least two points is a constant map.

Proof. (1) $\Rightarrow$ (2) : Let G be an Ω-open and Ω-closed subset of (X,$\Im$). Then X$\sim$ G is also both Ω-open and Ω-closed. Then X = G$\cup$ (X$\sim$ G) a disjoint union of two non-empty Ω-open sets which contradicts the fact that (X,$\Im$) is Ω -connected. Hence G = ϕ or X.
(2) $\Rightarrow$ (1): Suppose that X = A $\cup$ B where A and B are disjoint non-empty Ω -open subsets of (X,$\Im$). Since A = X -B, then A is both Ω-open and Ω-closed. By assumption A = ϕ or X, which is a contradiction. Hence (X,$\Im$) is Ω-connected.
(2) $\Rightarrow$ (3): Let f : (X,$\Im$) $\rightarrow$ (Y, σ) be a Ω-continuous map, where (Y, σ) is discrete space with at least two points. Then $f^{-1}\{y\}$ is Ω - closed and Ω -open for each y $\in$Y. By assumption, $f^{-1}\{y\}$ = ϕ or X for each y $\in$Y .If $f^{-1}\{y\}$ = ϕ for each y $\in$Y , then f fails to be a map. Therefore there exists a point say z$\in$Y such that $f^{-1}\{z\}$ = X. This shows that f is a constant map.
(3) $\Rightarrow$ (2): Let G be both Ω-open and Ω-closed in (X,$\Im$). Suppose G $\neq \phi$. Let f : (X,$\Im$) $\rightarrow$ (Y, σ) be a Ω -continuous map defined by f(G) = a and f(X -G) = b where a $\neq$ b and a; b $\in$Y . By assumption, f is constant so G = X.

Theorem 4.16. *Every Ω-connected space is connected .*

Proof. Let X be Ω-connected and if possible let X be disconnected then there is a proper subset A of X which is both open and closed. But such a set is also both Ω-open and Ω-closed which is a contradiction thus Ω-connected is connected.

Theorem 4.17. *Let f : (X,$\Im$)$\rightarrow$ (Y, σ) be an Ω-continuous surjection and (X,$\Im$) be Ω-connected, then (Y, σ) is connected.*

Proof. Suppose that (Y, σ) is not connected. Let Y = A $\cup$B where A and B are disjoint non-empty open subsets in (Y, σ)). Since f is Ω-continuous, X = f^{-1}(A) $\cup f^{-1}$(B), where f^{-1}(A) and f^{-1}(B) are disjoint non-empty Ω-open subsets in (X,$\Im$). This contradicts the fact that (X,$\Im$) is Ω-connected. Hence (Y, σ) is connected.

Definition 4.18. A function f : (X,$\Im$) $\rightarrow$(Y, σ) is said to be strongly Ω-continuous function if inverse image of every Ω-closed set is closed.

Definition 4.19. A function f : (X,$\Im$) $\rightarrow$(Y, σ) is said to be Ω-irresolute function if inverse image of every Ω-closed set is Ω-closed.

The Proofs of the following theorem are straightforward and hence omitted.

Theorem 4.20. *Let f : (X,$\Im$) $\rightarrow$(Y, σ) be a Ω-irresolute surjection and (X,$\Im$) is Ω-connected, then (Y, σ) is Ω-connected.*

Theorem 4.21. *The image of a connected space under strongly Ω-continuous map is Ω-connected.*

Theorem 4.22. *If (X,$\Im$) is Ω-disconnected and $\Im$' is finer than $\Im$ then (X,$\Im$') is Ω-disconnected.*

Theorem 4.23. *If (X,$\Im$) is Ω-connected and $\Im$' is coarser than $\Im$ then (X,$\Im$') is Ω-connected.*

Theorem 4.24. *If every two points of E$\subset$ X are contained in some Ω-connected space of E then E is Ω-connected subset of X.*

Proof. Let E be not Ω-connected then for some A,B ⊂ X , E = A ∪ B such that Ω-cl(A) ∩ B =ϕ = A ∩(Ω-cl(B)). Since both A and B are nonempty let a ∈ A and b ∈ B for some a,b ∈ E then there exists a Ω-connected subset F of E such that a,b ∈ F. Since F ⊂ A ∪ B either F ⊂ A or F ⊂ B.Without loss of generality let we assume that F ⊂ A then a,b ∈ A which means that A ∩ B≠ which is a contradiction. Hence E has to be Ω-connected.

5 Locally Ω-Connected

Definition 5.1. A subset A of a topological space (X, ℑ) is said to be locally Ω-connected at x ∈ X if for every Ω-open set U containing x there exists a Ω-open and Ω-connected set V containing x and contained in U. If X is locally Ω-connected at each of its points then X is said to be locally Ω-connected.

Example 5.2. Let R be the set of real numbers with usual topology then R is Ω-connected and locally Ω-connected as well.

Example 5.3. The subspace (1,2) ∪ (2,3) of the real line is Ω disconnected but locally Ω-connected.

Example 5.4. In discrete topology R is Ω-disconnected but locally Ω-connected.

Definition 5.5. A function f : (X,ℑ)→ (Y; σ) is said to be oΩ-open function if image of every open set in X is Ω-open in Y.

Definition 5.6. A function f : (X,ℑ) → (Y; σ) is said to be Ωo-open function if image of every Ω-open set in X is open in Y.

Definition 5.7. A function f : (X,ℑ) → (Y; σ) is said to be ΩΩ-open function if image of every Ω-open set in X is Ω-open in Y.

Theorem 5.8. *The image of a locally Ω-connected space under Ωo-continuous, Ω-open map is locally connected.*

Proof. Let f : (X,ℑ) → (Y; σ) be a Ω-continuous, Ωo-open map of X into Y. Let y ∈ Y then there exists x ∈ X such that f(x) = y. LetV_y be an open set containing y then $f^{-1}(V_y)$ is a Ω-open set containing x. Since X is locally Ω-connected it contains a Ωopen-set U_y which is Ω-connected. This implies that y∈ f(U_y) such that f(U_y) is open (as f is Ωo-open) and connected (as f is Ω -continuous) and f(U_y) is contained in V_y . Hence Y is locally connected.

Theorem 5.9. *The image of a locally Ω-connected space under Ω-irresolute, ΩΩ -open map is locally* $\Omega-$*connected.*

Proof. Let f : (X,ℑ) → (Y; σ) be an ΩΩ-irresolute, Ω-open map of X into Y. Let y ∈Y then there exists x ∈ X such that f(x) = y. Let V_y be an Ω-open set containing y then $f^{-1}(V_y)$ is an Ω-open set containing x. Since X is locally Ω-connected it contains a Ω-open set U_y which is Ω-connected and implies that y∈ f(U_y) such that f(U_y) is Ω-open (as f is Ω-open) and Ω-connected (as f is Ω-irresolute) and f(U_y) is contained in V_y . Hence Y is locally Ω-connected .

Theorem 5.10. *The image of a locally connected space under strongly Ω -continuous, oΩ-open map is locally Ω-connected.*

Proof. Let f : (X,ℑ) → (Y; σ) be a strongly Ω-continuous,oΩ-open map of X into Y. Let y ∈ Y then there exists x∈X such that f(x) = y. Let V_y be a Ω-open set containing y then $f^{-1}(V_y)$ is an open set containing x. Since X is locally connected it contains an open set U_y which is connected. This implies that y∈ f(U_y) such that f(U_y) is Ω-open (as f is oΩ-open) and Ω-connected (as f is strongly Ω-continuous) and f(U_y) is contained in V_y . Hence Y is locally Ω-connected .

Theorem 5.11. *A Ω-open subset of a locally Ω-connected space is locally Ω-connected.*

Theorem 5.12. *A space (X,ℑ) is locally Ω-connected if and only if it has a basis consisting of Ω-connected Ω-open sets.*

Proof. Follows from the definition of locally $\Omega-$ connectedness.

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