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TOTAL VERTEX IRREGULARITY STRENGTH OF SOME GRAPHS

P.Jeyanthi and A.sudha

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Abstract. A vertex irregular total k-labeling of a graph G with vertex set V and edge set E is an assignment of positive integer labels $\{1, 2, ..., k\}$ to both vertices and edges so that the weights calculated at vertices are distinct. The total vertex irregularity strength of G, denoted by $tvs(G)$ is the minimum value of the largest label k over all such irregular assignment. In this paper, we study the total vertex irregularity strength of cycle quadrilateral snake, sunflower, double wheel, fungus, triangular book and quadrilateral book.

1 Introduction

As a standard notation, assume that $G = (V, E)$ is a finite, simple and undirected graph with p vertices and q edges. A labeling of a graph is any mapping that sends some set of graph elements to a set of numbers (usually positive integers). If the domain is the vertex -set (or) the edge- set, the labeling are called respectively vertex labeling (or) edge labeling. If the domain is $V \cup E$ then we call the labeling a total labeling. Chartrand et al. [6] introduced labelings of the edges of a graph G with positive integers such that the sum of the labels of edges incident with a vertex is different for all the vertices. Such labelings were called *irregular assignments* and *the irregularity strength s(G)* of a graph G is known as the minimum k for which G has an irregular assignment using labels at most k. The irregularity strength $s(G)$ can be interpreted as the smallest integer k for which G can be turned into a multigraph G' by replacing each edge by a set of at most k parallel edges, such that the degrees of the vertices in G' are all different. Karonski et al. [8] conjectured that the edges of every connected graph of order at least 3 can be assigned labels from $\{1, 2, 3\}$ such that for all pairs of adjacent vertices the sums of the labels of the incident edges are different. Motivated by irregular assignments Bača et al. [5] defined a labeling $f : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ to be a vertex irregular total k-labeling if for every two different vertices x and y the vertex-weights $wt_f(x) \neq wt_f(y)$ where the vertex-weight $wt_f(x) = f(x) + \sum_{xy \in E} f(xy)$. A minimum k for which G has a vertex irregular total k-labeling is defined as the total vertex irregularity strength of G and denoted by $tvs(G)$. It is easy to see that irregularity strength $s(G)$ of a graph G is defined only for graphs containing at most one isolated vertex and no connected component of order 2. On the other hand, the total vertex irregularity strength $tvs(G)$ is defined for every graph G. If an edge labeling $f : E \to \{1, 2, \ldots, \delta(G)\}$ provides the irregularity strength $s(G)$, then we extend this labeling total labeling ϕ in such a way

$$\phi(xy) = f(xy) \qquad \text{for every } xy \in E(G),$$

$$\phi(x) = 1 \qquad \text{for every } x \in V(G).$$

Thus, the total labeling ϕ is a vertex irregular total labeling and for graphs with no component of order ≤ 2 has $tvs(G) \leq s(G)$. Nierhoff [9] proved that for all (p, q)-graphs G with no component of order at most 2 and $G \neq K_3$the irregularity strength $s(G) \leq p - 1$. From this result it follows that

$$tvs(G) \leq p - 1. \tag{1.1}$$

Bača et al. [5] proved that if a tree T with n pendant vertices and no vertices of degree 2, then $\left\lceil \frac{n+1}{2} \right\rceil \leq tvs(T) \leq n$. Additionally, they gave a lower bound and an upper bound on total vertex irregular strength for any graph G with v vertices and e edges,minimum degree δ and maximum degree Δ, $\left\lceil \frac{|V|+\delta}{\Delta+1} \right\rceil \leq tvs(G) \leq |V|+\Delta-2\delta+1$. In the same paper, they gave the total vertex irregular strengths of cycles, stars, and complete graphs, that is, $tvs(C_n) = \left\lceil \frac{n+2}{3} \right\rceil$, $tvs(K_{1,n}) = \left\lceil \frac{n+1}{2} \right\rceil$ and $tvs(K_n) = 2$. Ahmad et al. [1, 3] determined an exact value of the total vertex irregularity strength for wheel related graphs and cubic graphs. Wijaya et al. [16] determined an exact value of the total vertex irregularity strength for complete bipartite graphs. Wijaya and Slamin [15] found the exact values of *tvs* for wheels, fans, suns and friendship graphs. Nurdin et al. [11] proved the following lower bound of *tvs* for any graph G.

Theorem 1.1. *Let G be a connected graph having n_i vertices of degree i ($i = \delta, \delta+1, \delta+2, \ldots, \Delta$) where δ and Δ are the minimum and the maximum degree of G, respectively. Then*

$$tvs(G) \geq max\left\{\left\lceil \frac{\delta + n_\delta}{\delta+1} \right\rceil, \left\lceil \frac{\delta + n_\delta + n_{\delta+1}}{\delta+2} \right\rceil, \ldots, \left\lceil \frac{\delta + \sum_{i=\delta}^{\Delta}(n_i)}{\Delta+1} \right\rceil\right\}. \tag{1.2}$$

Also Nurdin et al. [11] posed the following conjecture.

Conjecture:1.2 [11] Let G be a connected graph having n_i vertices of a degree i ($i = \delta, \delta + 1, \delta + 2, \ldots, \Delta$) where δ and Δ are the minimum and the maximum degree of G, respectively. Then

$$tvs(G) = max\left\{\left\lceil \frac{\delta + n_\delta}{\delta+1} \right\rceil, \left\lceil \frac{\delta + n_\delta + n_{\delta+1}}{\delta+2} \right\rceil, \ldots, \left\lceil \frac{\delta + \sum_{i=\delta}^{\Delta}(n_i)}{\Delta+1} \right\rceil\right\}. \tag{1.3}$$

Conjecture 1.2 has been verified by several authors for several families of graphs. Nurdin et al. [11, 12] found the exact values of total vertex irregularity strength of trees, several types of trees and disjoint union of t copies of path. Slamin et al. [14] determined the total vertex irregularity strength of disjoint union of sun graphs. In [2] Ahmad, Bača and Numan determined the total vertex irregularity strength of disjoint union of friendship graphs. Ashfaq Ahmad et al. [4] found the exact value of the total vertex irregularity strength of ladder related graphs. We use the following definitions in the subsequent section.

Definition 1.2. The cycle quadrilateral snake CQ_n is obtained from the cycle C_n by identifying each edge of C_n with an edge of C_4.

Definition 1.3. The sun flower graph SF_n is obtained from the flower graph of F_n by adding n pendant edges to the central vertex. Thus the vertex set of SF_n is $V(SF_n) = \{v, a_i, b_i, c_i : 1 \leq i \leq n\}$ and the edge set of SF_n is $E(SF_n) = \{va_i, vb_i, vc_i, a_ia_{i+1}, a_ib_i : 1 \leq i \leq n\}$ with indices taken modulo n.

Definition 1.4. A double-wheel graph DW_n of size $4n$ can be composed of $2C_n + K_1$, that is it consists of two cycles of size n, where all the vertices of the two cycles are connected to a common hub.

Definition 1.5. A fungus graph Fg_n is obtained from a wheel $W_n, n \geq 3$ by attaching pendent vertices to the central vertex of W_n.

Definition 1.6. The book graph B_m is defined as the Cartesian product S_m X P_2 where S_m is a star graph on $m + 1$ vertices and P_2 is the path graph on two vertices.

2 Main Results

In this section we determine exact values of the total vertex irregularity strength of cycle quadrilateral snake, sunflower, double wheel, fungus, triangular book and quadrilateral book.

Theorem 2.1. $tvs(CQ_n) = \left\lceil \frac{2n+2}{3} \right\rceil, n \geq 3.$

Proof. Let $V(CQ_n) = \{u_i, a_i, b_i : 1 \leq i \leq n\}$ and $E(CQ_n) = \{a_ib_i, u_ia_i, u_iu_{i+1}, b_iu_{i+1} : 1 \leq i \leq n\}$ with indices taken modulo n. Let $k = \left\lceil \frac{2n+2}{3} \right\rceil$, then from (1.2) it follows that, $tvs(CQ_n) \geq max\left\{\left\lceil \frac{2n+2}{3} \right\rceil, \left\lceil \frac{3n+2}{5} \right\rceil\right\} = \left\lceil \frac{2n+2}{3} \right\rceil$. That is $tvs(CQ_n) \geq k$. To prove the reverse inequality, we define a function f from $V \cup E$ to $\{1, 2, 3, ..., k\}$ as follows:

$$f(u_1) = 1;$$

$$f(u_i) = \begin{cases} k + 1 - i, & if \;\; 2 \leq i \leq k \\ 1 + i - k, & if \;\; k + 1 \leq i \leq n; \end{cases}$$

$$f(a_i) = \begin{cases} 1, & if \;\; 1 \leq i \leq k \\ 2i - 2k + 1, & if \;\; k + 1 \leq i \leq n; \end{cases}$$

$$f(b_i) = \begin{cases} 1, & if \;\; 1 \leq i \leq k - 1 \\ 2 + 2i - 2k, & if \;\; k \leq i \leq n; \end{cases}$$

$$f(a_ib_i) = \begin{cases} i, & if \;\; 1 \leq i \leq k \\ k, & if \;\; k + 1 \leq i \leq n; \end{cases}$$

$$f(u_ia_i) = \begin{cases} i, & if \;\; 1 \leq i \leq k \\ k, & if \;\; k + 1 \leq i \leq n; \end{cases}$$

$$f(b_iu_{i+1}) = \begin{cases} i + 1, & if \;\; 1 \leq i \leq k - 1 \\ k, & if \;\; k \leq i \leq n; \end{cases}$$

$$f(u_iu_{i+1}) = k, 1 \leq i \leq n.$$

We observe that,

$$wt(a_i) = 2i + 1, 1 \leq i \leq n;$$

$$wt(b_i) = 2i + 2, 1 \leq i \leq n;$$

$$wt(u_i) = \begin{cases} 3k + 2, & if \;\; i = 1 \\ 3k + 1 + i, & if \;\; 2 \leq i \leq k \\ 3k + 1 + i, & if \;\; k + 1 \leq i \leq n. \end{cases}$$

It is easy to check that the weights of the vertices are distinct. This labeling construction shows

that $tvs(CQ_n) \leq k$. Combining this with the lower bound, we conclude that $tvs(CQ_n) = k$. Figure 1 shows the vertex irregular total labeling of CQ_6. □

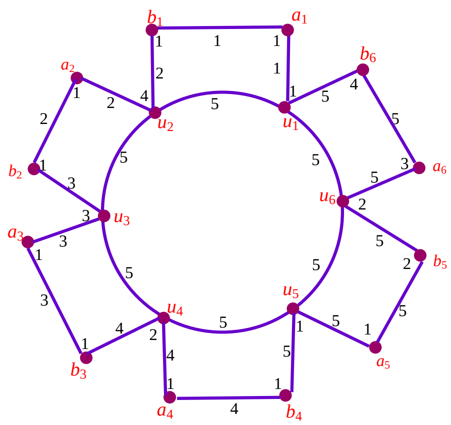

Figure 1.$tvs(CQ_6) = 5$.

Theorem 2.2. $tvs(SF_n) = \left\lceil \frac{2n+1}{3} \right\rceil, n \geq 3$.

Proof. Let $V(SF_n) = \{v, a_i, b_i, c_i : 1 \leq i \leq n\}$ and $E(SF_n) = \{va_i, vb_i, vc_i, a_ia_{i+1}, a_ib_i : 1 \leq i \leq n\}$ with indices taken modulo n. Let $k = \left\lceil \frac{2n+1}{3} \right\rceil$, then from (1.2) it follows that, $tvs(SF_n) \geq max\left\{\left\lceil \frac{n+1}{2} \right\rceil, \left\lceil \frac{2n+1}{3} \right\rceil, \left\lceil \frac{2n+1}{4} \right\rceil, \left\lceil \frac{3n+1}{5} \right\rceil\right\} = \left\lceil \frac{2n+1}{3} \right\rceil$. That is $tvs(SF_n) \geq \left\lceil \frac{2n+1}{3} \right\rceil = k$. To prove the reverse inequality, we define a function f from $V \cup E$ to $\{1, 2, 3, ..., k\}$ by considering the following two cases.

Case(i): $n = 3$.

$f(v) = 3$, $f(a_1) = f(a_2) = f(a_3) = 1$, $f(a_1a_2) = f(a_2a_3) = f(a_3a_1) = 3$, $f(va_1) = 1$, $f(va_2) = 2$, $f(va_3) = 3$, $f(b_1) = f(b_2) = f(b_3) = 3$, $f(c_1) = f(c_2) = f(c_3) = 1$, $f(a_1b_1) = 1$, $f(a_2b_2) = 2$, $f(a_3b_3) = 3$, $f(vb_1) = f(vb_2) = f(vb_3) = 1$, $f(vc_1) = 1$, $f(vc_2) = 2$, $f(vc_3) = 3$.

Case(ii): $n > 3$.

$$f(a_i) = f(c_i) = \begin{cases} 1, & if \ \ 1 \leq i \leq k \\ 1 + i - k, & if \ \ k + 1 \leq i \leq n; \end{cases}$$

$$f(b_i) = k, 1 \leq i \leq n;$$

$$f(v) = k;$$

$$f(va_i) = 2(n - k), 1 \leq i \leq n;$$

$$f(vb_i) = \begin{cases} n + 1 - k, & if \ \ 1 \leq i \leq k \\ n + 1 - 2k + i, & if \ \ k + 1 \leq i \leq n; \end{cases}$$

$$f(vc_i) = \begin{cases} i, & if \ \ 1 \leq i \leq k \\ k, & if \ \ k + 1 \leq i \leq n; \end{cases}$$

$$f(a_ib_i) = \begin{cases} i, & if \ \ 1 \leq i \leq k \\ k, & if \ \ k + 1 \leq i \leq n; \end{cases}$$

$$f(a_ia_{i+1}) = k, 1 \leq i \leq n.$$

We observe that,

$$wt(c_i) = 1 + i, 1 \leq i \leq n;$$

$$wt(b_i) = n + 1 + i, 1 \leq i \leq n;$$

$$wt(a_i) = 2n + 1 + i, 1 \le i \le n;$$

$$wt(v) = 2(n^2 - k^2 + k) + \sum_{i=1}^{k}(i) + \sum_{i=k+1}^{n}(n + 1 - 2k + i).$$

It is easy to check that the weights of the vertices are distinct. This labeling construction shows that $tvs(SF_n) \le k$. Combining this with the lower bound, we conclude that $tvs(SF_n) = k$. Figure 2 shows the vertex irregular total labeling of SF_8. □

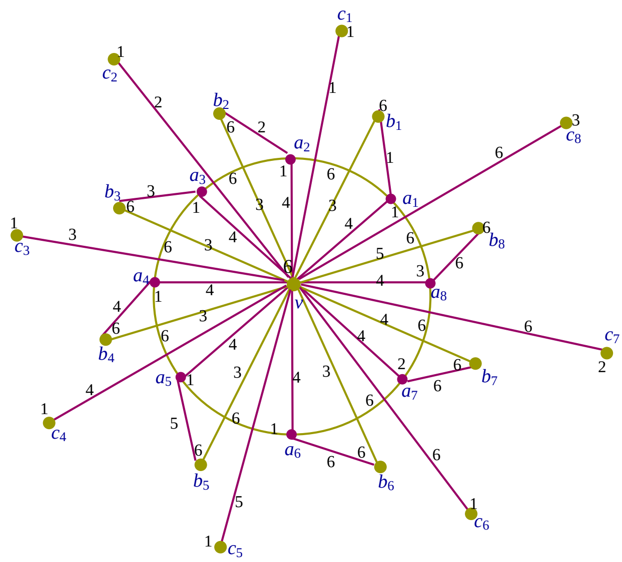

Figure 2.$tvs(SF_8) = 6$.

Theorem 2.3. $tvs(DW_n) = \left\lceil \frac{2n+3}{4} \right\rceil, n \ge 3.$

Proof. Let $V(DW_n) = \{a_i, b_i, c : 1 \le i \le n\}$ and $E(DW_n) = \{a_i a_{i+1}, b_i b_{i+1}, ca_i, cb_i : 1 \le i \le n\}$ with indices taken modulo n. Let $k = \left\lceil \frac{2n+3}{4} \right\rceil$, then from (1.2) it follows that, $tvs(DW_n) \ge max\left\{\left\lceil \frac{2n+3}{4} \right\rceil, \left\lceil \frac{2n+4}{2n+1} \right\rceil\right\} = \left\lceil \frac{2n+3}{4} \right\rceil$. That is $tvs(DW_n) \ge \left\lceil \frac{2n+3}{4} \right\rceil = k$. To prove the reverse inequality, we define a function f from $V \cup E$ to $\{1, 2, 3, ..., k\}$ as follows:

$$f(c) = k;$$

$$f(a_i) = f(cb_i) = \begin{cases} i, & if \;\; 1 \le i \le k \\ k, & if \;\; k+1 \le i \le n; \end{cases}$$

$$f(b_i) = f(ca_i) = \begin{cases} 1, & if \;\; 1 \le i \le k \\ 1 + i - k, & if \;\; k+1 \le i \le n; \end{cases}$$

$$f(b_i b_{i+1}) = 1, 1 \le i \le n;$$

$$f(a_i a_{i+1}) = k, 1 \le i \le n;$$

We observe that,

$$wt(b_i) = 3 + i, 1 \le i \le n;$$

$$wt(a_i) = 2k + 1 + i, 1 \le i \le n;$$

$$wt(c) = k(2 + n - k) + \sum_{i=1}^{k}(i) + \sum_{i=k+1}^{n}(1 + i - k).$$

It is easy to check that the weights of the vertices are distinct. This labeling construction shows that $tvs(DW_n) \le k$. Combining this with the lower bound, we conclude that $tvs(DW_n) = k$. Figure 3 shows the vertex irregular total labeling of DW_6. □

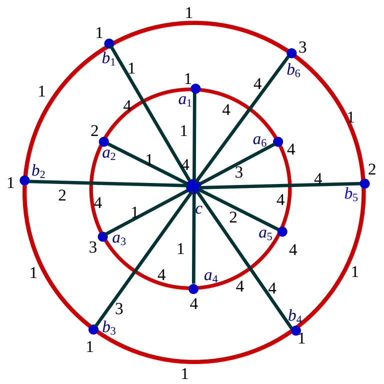

Figure 3.$tvs(DW_6) = 4$.

Theorem 2.4. $tvs(Fg_n) = \left\lceil \frac{n+1}{2} \right\rceil, n \geq 3.$

Proof. Let $V(Fg_n) = \{a_i, b_i, c : 1 \leq i \leq n\}$ and $E(Fg_n) = \{a_i a_{i+1}, ca_i, cb_i : 1 \leq i \leq n\}$ with indices taken modulo n. Let $k = \left\lceil \frac{n+1}{2} \right\rceil$, then from (1.2) it follows that, $tvs(Fg_n) \geq max\left\{\left\lceil \frac{n+1}{2} \right\rceil, \left\lceil \frac{2n+1}{4} \right\rceil, \left\lceil \frac{2n+2}{2n+1} \right\rceil\right\} = \left\lceil \frac{n+1}{2} \right\rceil$. That is $tvs(Fg_n) \geq \left\lceil \frac{n+1}{2} \right\rceil = k$. To prove the reverse inequality, we define a function f from $V \cup E$ to $\{1, 2, 3, ..., k\}$ as follows:

$$f(c) = 1;$$

$$f(a_i) = f(cb_i) = \begin{cases} i, & if \ \ 1 \leq i \leq k \\ k, & if \ \ k+1 \leq i \leq n; \end{cases}$$

$$f(b_i) = f(ca_i) = \begin{cases} 1, & if \ \ 1 \leq i \leq k \\ 1 + i - k, & if \ \ k+1 \leq i \leq n; \end{cases}$$

$$f(a_i a_{i+1}) = k, 1 \leq i \leq n;$$

We observe that,

$$wt(b_i) = 1 + i, 1 \leq i \leq n;$$

$$wt(a_i) = 2k + 1 + i, 1 \leq i \leq n;$$

$$wt(c) = 1 + k(1 + n - k) + \sum_{i=1}^{k}(i) + \sum_{i=k+1}^{n}(1 + i - k).$$

It is easy to check that the weights of the vertices are distinct. This labeling construction shows that $tvs(Fg_n) \leq k$. Combining this with the lower bound, we conclude that $tvs(Fg_n) = k$. Figure 4 shows the vertex irregular total labeling of Fg_8. □

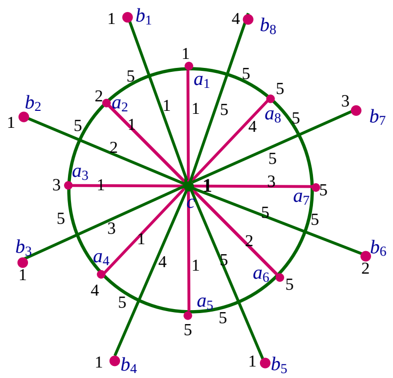

Figure 4. $tvs(Fg_8) = 5$.

Theorem 2.5. *The triangular book, that is Books with 3 sides (n copies of C_3 with an edge in common) admits a total vertex irregular labeling and* $tvs(B_n) = \left\lceil \frac{n+2}{3} \right\rceil, n \geq 2$.

Proof. Let $V(B_n) = \{v_1, v_2, a_i : 1 \leq i \leq n\}$ and $E(B_n) = \{v_1a_i, v_2a_i, v_1v_2 : 1 \leq i \leq n\}$. Let $k = \left\lceil \frac{n+2}{3} \right\rceil$, then from (1.2) it follows that, $tvs(B_n) \geq max\left\{\left\lceil \frac{n+2}{3} \right\rceil, \left\lceil \frac{n+4}{n+2} \right\rceil\right\} = \left\lceil \frac{n+2}{3} \right\rceil$. That is $tvs(B_n) \geq \left\lceil \frac{n+2}{3} \right\rceil = k$. To prove the reverse inequality, we define a function f from $V \cup E$ to $\{1, 2, 3, ..., k\}$ as follows:

$$f(v_1) = f(v_2) = f(v_1v_2) = k;$$

$$f(a_i) = \begin{cases} 1, & if \ \ 1 \leq i \leq 2k-1 \\ 2+i-2k, & if \ \ 2k \leq i \leq n; \end{cases}$$

$$f(v_1a_i) = \begin{cases} 1, & if \ \ 1 \leq i \leq k \\ 1+i-k, & if \ \ k+1 \leq i \leq 2k-1 \\ k, & if \ \ 2k \leq i \leq n; \end{cases}$$

$$f(v_2a_i) = \begin{cases} i, & if \ \ 1 \leq i \leq k \\ k, & if \ \ k+1 \leq i \leq n; \end{cases}$$

We observe that,

$$wt(a_i) = 2+i, 1 \leq i \leq n;$$

$$wt(v_1) = k(n+4) - 2k^2 + \sum_{i=k+1}^{2k-1} (1+i-k);$$

$$wt(v_2) = k(n+2) - k^2 + \sum_{i=1}^{k} i.$$

It is easy to check that the weights of the vertices are distinct. This labeling construction shows that $tvs(B_n) \leq k$. Combining this with the lower bound, we conclude that $tvs(B_n) = k$. Figure 5 shows the vertex irregular total labeling of B_4. □

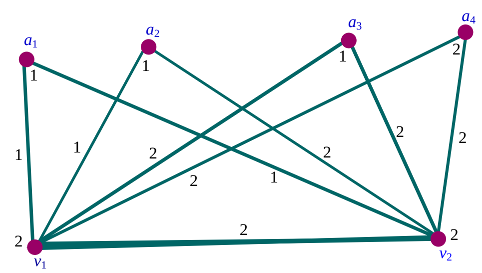

Figure 5.$tvs(B_4) = 2$.

Theorem 2.6. *The quadrilateral book ,that is Books with 4 sides (n copies of C_4 with an edge in common) admits a total vertex irregular labeling and* $tvs(B_n) = \left\lceil \frac{2n+2}{3} \right\rceil, n \geq 2$.

Proof. Let $V(B_n) = \{v_1, v_2, a_i, b_i : 1 \leq i \leq n\}$ and $E(B_n) = \{v_1a_i, v_2b_i, v_1v_2, a_ib_i : 1 \leq i \leq n\}$. Let $k = \left\lceil \frac{2n+2}{3} \right\rceil$, then from (1.2) it follows that, $tvs(B_n) \geq max\left\{\left\lceil \frac{2n+2}{3} \right\rceil, \left\lceil \frac{2n+2}{n+2} \right\rceil\right\} = \left\lceil \frac{2n+2}{3} \right\rceil$. That is $tvs(B_n) \geq \left\lceil \frac{2n+2}{3} \right\rceil = k$. To prove the reverse inequality, we define a function f from $V \cup E$ to $\{1, 2, 3, ..., k\}$ in the following way.

$$f(v_1) = f(v_2) = f(v_1v_2) = k;$$

$$f(a_1) = f(v_2b_1) = 2;$$

$$f(a_i) = \begin{cases} i-1, & if \ \ 2 \leq i \leq k \\ k, & if \ \ k+1 \leq i \leq n; \end{cases}$$

$$f(b_i) = f(a_ib_i) = \begin{cases} i, & if \;\; 1 \le i \le k \\ k, & if \;\; k+1 \le i \le n; \end{cases}$$

$$f(v_1a_1) = 1;$$

$$f(v_1a_i) = \begin{cases} 2, & if \;\; 2 \le i \le k \\ 2i-2k+1, & if \;\; k+1 \le i \le n; \end{cases}$$

$$f(v_2b_i) = \begin{cases} 2, & if \;\; 2 \le i \le k \\ 2i-2k+2, & if \;\; k+1 \le i \le n. \end{cases}$$

We observe that,

$$wt(a_i) = 2i+1, 1 \le i \le n;$$

$$wt(b_i) = 2i+2, 1 \le i \le n;$$

$$wt(v_1) = 4k - 1 + \sum_{i=k+1}^{n}(2i-2k+1);$$

$$wt(v_2) = 4k + \sum_{i=k+1}^{n}(2i-2k+2).$$

It is easy to check that the weights of the vertices are distinct. This labeling construction shows that $tvs(B_n) \le k$. Combining this with the lower bound, we conclude that $tvs(B_n) = k$. Figure 6 shows the vertex irregular total labeling of B_4. □

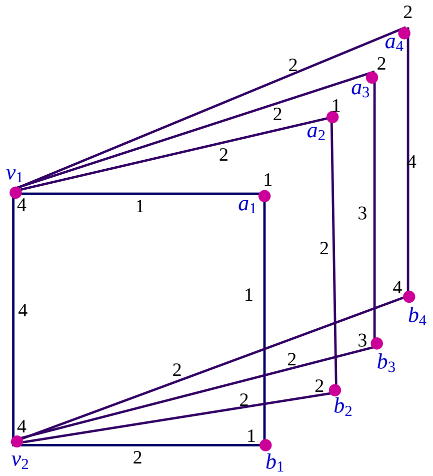

Figure 6. $tvs(B_4) = 4$.

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Author information

P.Jeyanthi, Research Centre, Department of Mathematics, Govindammal Aditanar College for Women, Tiruchendur - 628 215,Tamil Nadu, India.
E-mail: jeyajeyanthi@rediffmail.com

A.sudha, Department of Mathematics, Wavoo Wajeeha Women's college of Arts and Science, Kayalpatnam-628204, Tamil Nadu, India.
E-mail: sudhathanalakshmi@gmail.com

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