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MATHEMATICAL MODELING AND STABILITY ANALYSIS OF HIV/AIDS-TB CO-INFECTION

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Abstract. In this investigation, a nonlinear compartment model has been developed for the awareness of media coverage on controlling and prevention of infectious diseases, namely HIV/AIDS and TB within a population of varying size. The population has been divided into four sub-classes of susceptible, TB infected, HIV infected and AIDS patients. Using the stability analysis, the model has been studied quantitatively by taking small perturbation about equilibrium points. The developed mathematical model shows four equilibrium points, namely disease free, population free from HIV/AIDS, population free from TB and co-infection equilibrium point. It has been shown that the co-infection equilibrium point is always locally stable. The concept of the basic reproduction number has also been introduced in the model. It is predicted that the disease dies out if a reproduction number is less than unity and it becomes endemic if it is greater than unity.

1 Introduction

In the health sector, the problem of the HIV/AIDS and TB epidemic is a great challenge. It is widely accepted that infectious diseases are the major cause of human mortality, particularly in the developing countries like India. In the human being, TB is caused by Mycobacterium tuberculosis and it is an air-borne transmitted disease. Mycobacterium droplets are released into the air by an infected individual through sneezing or by coughing. Tubercle bacillus carried by such droplets lives in the air for a short duration of time and therefore, it is believed that occasional contact with an infectious person rarely leads to transmission.

Presently the world is experiencing the devastating effects of HIV/AIDS epidemic. It is well established that both TB and HIV/AIDS fuel each other. Preventive treatment of TB in HIV-infected individual is highly recommended and could dramatically reduce the impact of HIV on TB epidemiology, however, at present its implementation is limited in developing countries because of complex logistic and practical difficulties. TB and HIV/AIDS are severe diseases which have emerged as a global challenge in the twenty-first century, exhibit some distinct features such as rapid spread. It is, therefore, important to modify classical mathematical models to reflect these features by adding the new dimensions of massive media awareness and fast information flow that have a great influence not only on the individual but also on the formation and implementation of the public intervention and control policies.

It is seen that when an infectious disease breaks out in a particular region, the disease control and prevention department would take all possible measures to control and prevent the disease. One of the key measures is to tell the masses about the correct preventive measures of the disease through education and the media. With the exponential growth of information technology, the media can play a dominant role in educating the people about TB and HIV/AIDS epidemics. According to a survey, the odds of awareness among higher educated women and men were 46.7 and 77.73 times of non-educated women and men respectively. In addition, both women and men who regularly watch TV was 8.6 times more likely to be aware of acquired immunodeficiency syndrome (AIDS) compared to those who never watch TV.

Some efforts have been made by many authors to explore the effects of media coverage on transmission dynamics of infectious diseases. SIR mathematical models for epidemics where the contact rate is a monotone increasing function of the population density and the background death rate also depends on the population density have been formulated by Greenhalagh and Das (1995). A statistical analysis of media and education role in the awareness of AIDS among married couples in Bangladesh has been made by Khan et al., (1997). The effects of treatments and vaccination on the HIV transmission dynamics in homosexuals with genetic heterogeneity can be found in the work of Hsu (2000). Lloyd and Jansen (2004) developed multi-patch models which are also known as metapopulation models. They provided a simple framework within which the role of spatial processes in disease transmission can be examined. They considered an n-patch model which distinguishes between k different classes of individuals. Although immunity gained by experiencing the disease is permanent, vaccine-induced immunity is only temporary and a fixed time after vaccination individuals return to the susceptible class (Greenhalagh et al., 2005). (cf. Bhunu et al., 2009; Agusto and Adekunle 2014; Bolarin and Omatola 2016; Roeger et al., 2009; Naresh and Tripathi 2005) developed a nonlinear mathematical models for the transmission of HIV/AIDS-TB co-infection within a population of varying size, but they have not discussed the role of the media awareness of the diseases. A simple, ordinary differential equation model to study the epidemiological consequences of the drift mechanism for influenza. A viruses improving over the classical SIR approach by introducing a fourth class (C) for the cross-immune individuals in the population, i.e., those that recovered after being infected by different strains of the same viral subtype in the past years, has been studied by Casagrandi et al., (2006). A compartmental model to illustrate a possible mechanism for multiple outbreaks or even sustained periodic oscillations of emerging infectious diseases due to the psychological impact of the reported numbers of infectious and hospitalized individuals has been developed by Liu et al., (2007). The three-dimensional compartmental models have also been developed by some researchers to investigate the impact of media coverage on the spread and control of infectious diseases such as severe acute respiratory syndrome (SARS) in a given region/area (cf. Liu and Cui, 2008; Cui et al., 2008).

Recently Singh et al., (2016) studied the transmission dynamics of the HIV/AIDS epidemic by considering the three latent compartments for the slow, medium and fast progress of developing the AIDS. Bhunu et al., (2008) studied the effects of chemoprophylaxis which is used for the treatment of TB infection. They have suggested the holistic approach of the intervention strategies in controlling TB. A number of theoretical studies have been carried out by many researchers by developing the mathematical models of TB transmission dynamics (cf. Aparicio et al., 2000a, 2000b; Blower et al., 1995, 1996; Chintu and Mwinga, 1999; Feng et al., 2000; Porco and Blower, 1998; Song et al., 2002; Chavez and Feng, 1997). A deterministic compartmental sex-structured model to evaluate the potential impact of male circumcision on the transmission dynamics of HIV/AIDS has been designed by Podder et al., (2007). Ali et al., (2016) developed a non-linear mathematical model for the transmission dynamics of HIV/AIDS and analyze disease free and endemic equilibria.

In the present paper, our investigation is focused on the mathematical analysis of the role of media coverage in the transmission dynamics of HIV/AIDS-TB co-infection by developing a compartment mathematical model. The framework of the paper is as follows. In section 2, we formulate a mathematical model by stating requisite notations and assumptions. In section 3, we determine the reproduction number and examine the local stability of disease-free equilibrium point and endemic equilibriums. Section 4 is devoted to numerical simulation. Finally in section 5, the discussion & conclusion has been drawn which summarize the research investigation.

2 Model Descriptions

To study the role of awareness in control and prevention of HIV/AIDS-TB co-infection, we consider the population of size at a time with the constant recruitment rate. The whole population is divided into four sub-classes, namely susceptible(S), tuberculosis-infected(T), HIV-infected(H) and AIDS infected(A). The susceptible becomes tuberculosis infected at the rate β_1 which is contact rate before media alert. The term $\beta(T) = \left(\beta_1 - \beta_1^{'}\frac{T}{m+T}\right)$ represents the reduced value of the transmission rate of tuberculosis infection after media alert. It measures the spreading of

TB infection from the infected to the susceptible individual. If $m = 0$, the transmission rate is constant. Naturally, the contact transmission rate is not only related to the spreading ability of the disease, but also closely related to the alertness to the disease of each susceptible individual of the population. The parameter m reflects the impact of media coverage on the contact transmission. The reduced value of the transmission rate of HIV infection after media awareness is taken as $\beta(H) = \left(\beta_2 - \beta_2^{'}\frac{H}{n+H}\right)$. Here the parameter n is used to reflect the impact of media coverage on the contact transmission of HIV infection. The population in TB class is infected with HIV infection at a rate β_3. Since TB is a curable disease, as such some individuals of TB class are recovered at a rate λ and enter into the susceptible class. It is assumed that no challenging anti-HIV treatment is available within the population and therefore some members of HIV class are bound to develop full blown AIDS with rate δ and media cannot do anything to the population having full blown AIDS. In fact, once AIDS is developed in an individual, no awareness can help him to be cured. It is assumed that the population dies at a constant rate d. Let α represents the disease-induced death rate. The transmission diagram of model is depicted in Figure 1.

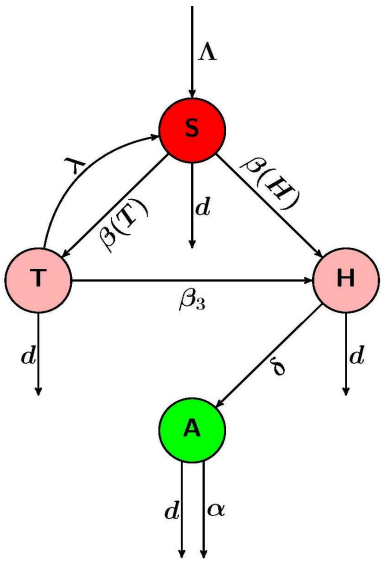

Figure 1. Transmission dynamics diagram of the model.

The mathematical model is described by the following system of non-linear differential equations

$$\frac{dS}{dt} = \Lambda - \left(\beta_1 - \beta_1^{'}\frac{T}{m+T}\right)\frac{ST}{N} - \left(\beta_2 - \beta_2^{'}\frac{H}{n+H}\right)\frac{SH}{N} - dS + \lambda T \tag{1}$$

$$\frac{dT}{dt} = \left(\beta_1 - \beta_1^{'}\frac{T}{m+T}\right)\frac{ST}{N} - \beta_3\frac{TH}{N} - (\lambda + d)T \tag{2}$$

$$\frac{dH}{dt} = \left(\beta_2 - \beta_2^{'}\frac{H}{n+H}\right)\frac{SH}{N} + \beta_3\frac{TH}{N} - (\delta + d)H \tag{3}$$

$$\frac{dA}{dt} = \delta H - (\alpha + d)A \tag{4}$$

with initial conditions

$$S(0) = S_0 \geq 0,\ T(0) = T_0 \geq 0,\ H(0) = H_0 \geq 0 \text{ and } A(0) = A_0 \geq 0 \tag{5}$$

Since $N(t) = S(t) + T(t) + H(t) + A(t)$, the above set of equations (1)-(5) can be modified as:

$$\frac{dN}{dt} = \Lambda - dN - \alpha A \tag{6}$$

$$\frac{dT}{dt} = \left(\beta_1 - \beta_1^{'}\frac{T}{m+T}\right)\frac{N-T-H-A}{N}T - \beta_3 TH - (\lambda + d)T \tag{7}$$

$$\frac{dH}{dt} = \left(\beta_2 - \beta_2^{'}\frac{H}{n+H}\right)\frac{N-T-H-A}{N}H - \beta_3 TH + (\delta + d)H \tag{8}$$

$$\frac{dA}{dt} = \delta H - (\alpha + d)\,A \tag{9}$$

3 The Analysis

The analysis of the model has been done by evaluating the equilibrium points and using the perturbation theory.

3.1 Equilibrium Analysis: To obtain the equilibrium points, we set left-hand sides of all equations (6)-(9) to zeros, so that we get

$$\Lambda - dN - \alpha A = 0 \tag{10}$$

$$\left(\beta_1 - \beta_1^{'}\frac{T}{m+T}\right)\frac{N-T-H-A}{N}T - \beta_3 TH - (\lambda + d)T = 0 \tag{11}$$

$$\left(\beta_2 - \beta_2^{'}\frac{H}{n+H}\right)\frac{N-T-H-A}{N}H - \beta_3 TH + (\delta + d)H = 0 \tag{12}$$

$$\delta H - (\alpha + d)\,A = 0 \tag{13}$$

Using equations (10)-(13), the four possible equilibrium points are obtained as follows:

(i) The diseases free equilibrium point $E_0(\overline{N}, 0, 0, 0)$ exists for all parameter values as

$$\overline{N} = \frac{\Lambda}{d},\ \overline{T} = 0,\ \overline{H} = 0,\ \overline{A} = 0.$$

(ii) The HIV/AIDS infection-free equilibrium point $E_1(N^*, T^*, 0, 0)$ is given by

$$N^* = \frac{\Lambda}{d},\ H^* = 0,\ A^* = 0,\ A_1 T^{*2} + B_1 T^* + C_1 = 0$$

where

$$A_1 = -(\beta_1 - \beta_1^{'})\,\frac{d}{\Lambda},\ B_1 = -\frac{d}{\Lambda}m\beta_1 + (\beta_1 - \beta_1^{'}) - (\lambda + d),\ C_1 = m(\beta_1 - (\lambda + d)).$$

We know that $C_1 > 0$, if $\beta_1 - (\lambda + d) > 0$, the hence system has unique HIV/AIDS free equilibrium point.

(iii) The TB infection-free equilibrium point $E_2\left(\hat{N}, 0, \hat{H}, \hat{A}\right)$ is

$$\hat{N} = \frac{1}{d}\left(\Lambda - \frac{\alpha\delta\hat{H}}{\alpha + d}\right), \hat{T} = 0, \hat{A} = \frac{\delta\hat{H}}{\alpha + d}, A_2\hat{H}^2 + B_2\hat{H} + C_2 = 0$$

where

$$A_2 = -\left(\beta_2 - \beta_2'\right)\left(\frac{\delta}{\alpha + d}\left(\frac{1+d}{d}\right) + 1 - \frac{\alpha\delta\,(\delta + d)}{d\,(\alpha + d)}\right),$$
$$B_2 = -\frac{n\beta_2\delta}{\alpha + d}\left(\frac{1+d}{d}\right) - n\beta_2 + \left(\beta_2 - \beta_2'\right)\frac{\Lambda}{d} + \frac{n\alpha\delta\,(\delta + d)}{d\,(\alpha + d)},$$

$$C_2 = \frac{n\Lambda}{d}\left(\beta_2 - (\delta + d)\right) + \frac{\Lambda}{d}(\delta + d).$$

Clearly $C_2 > 0$, if $\beta_2 > (\delta + d)$, then system has unique TB infection-free equilibrium point.

(iv) The endemic co-infection equilibrium point $E_3\left(\tilde{N}, \tilde{T}, \tilde{H}, \tilde{A}\right)$ is given by

$$\tilde{N} = \frac{1}{d}\left(\Lambda - \frac{\alpha\delta\tilde{H}}{\alpha + d}\right), \tilde{A} = \frac{\delta\tilde{H}}{\alpha + d},$$

$$\tilde{T} = \frac{\frac{\Lambda}{d}\left[(\beta_1 - \beta_1' p_T) - (\lambda + d)\right] - \left[(\beta_1 - \beta_1' p_T)\left\{\frac{\alpha+d+\delta}{\alpha+d}\right\} + \beta_3 + \alpha\delta\left\{\frac{\left(\beta_1 - \beta_1' p_T\right) - (\lambda+d)}{d(\alpha+d)}\right\}\right]\tilde{H}}{(\beta_1 - \beta_1' p_T)}$$

$$\tilde{H} = \frac{\frac{\Lambda}{d}\left[\{(\beta_2 - \beta_2' p_H) - (\delta + d)\} + \left(\frac{\beta_3 - \left(\beta_2 - \beta_2' p_H\right)}{\left(\beta_1 - \beta_1' p_T\right)}\right)\{(\beta_1 - \beta_1' p_T) - (\lambda + d)\}\right]}{\left[\frac{\beta_3(\alpha+d+\delta)}{\alpha+d} + \beta_3\left(\frac{\beta_3 - \left(\beta_2 - \beta_2' p_H\right)}{\left(\beta_1 - \beta_1' p_T\right)}\right) + \frac{\alpha\delta}{d(\alpha+d)}\left[\{(\beta_1 - \beta_1' p_T) - (\lambda + d)\} + \{(\beta_2 - \beta_2' p_H) - (\delta + d)\}\right]\right]}$$

3.2. Linear Stability Analysis

We linearize the system of differential equations (6)-(9) by making small perturbation about equilibrium points by substituting

$$N(t) = N + n(t), T(t) = T + \tau(t), H(t) = H + h(t), A(t) = A + a(t) \qquad (14)$$

where $n(t)$, $\tau(t)$, $h(t)$ and $a(t)$ are the small perturbation made about equilibrium points.

3.2.1. Stability Analysis of Diseases Free Equilibrium Point E_0

The Jacobian matrix of the system of equations (6)-(9) about equilibrium point E_0 is given by

$$J\left(\frac{\Lambda}{d}, 0, 0, 0\right) = \begin{bmatrix} -d & 0 & 0 & -\alpha \\ 0 & \frac{\Lambda}{d}\left[\beta_1 - (\lambda + d)\right] & 0 & 0 \\ 0 & 0 & \frac{\Lambda}{d}\left[\beta_2 - (\delta + d)\right] & 0 \\ 0 & 0 & \delta & -(\alpha + d) \end{bmatrix}$$

The characteristic equation of the above matrix is $|J - \lambda' I| = 0$
i.e.

$$-(d + \lambda')\left(\frac{\Lambda}{d}\left[\beta_1 - (\lambda + d)\right] - \lambda'\right)\left(\frac{\Lambda}{d}\left[\beta_2 - (\delta + d)\right] - \lambda'\right)\left(-(\alpha + d + \lambda')\right) = 0 \qquad (15)$$

Now

(i) $\lambda' = -d \Rightarrow \lambda' < 0$

(ii) $\lambda' = \frac{\Lambda}{d}(\beta_1 - (\lambda + d)) \Rightarrow \lambda' < 0 \ \ if \ \ \beta_1 < (\lambda + d) \Rightarrow \frac{\beta_1}{(\lambda+d)} < 1.$
Here $\frac{\beta_1}{\lambda+d} = R_T$ is basic reproduction number for the TB epidemic and is defined as the number of secondary infections produced by a single TB-infected individual in the fully susceptible population. Also

(iii) $\lambda' = \frac{\Lambda}{d}(\beta_2 - (\delta + d)) \Rightarrow \lambda' < 0 \ \ if \ \ \beta_2 < (\delta + d) \Rightarrow \frac{\beta_2}{(\delta+d)} < 1.$
Again, here $\frac{\beta_2}{(\delta+d)} = R_H$is the basic reproduction number for HIV infection and is defined as the number of secondary infections produced by a single HIV-infected individual in the fully susceptible population.

(iv) $\lambda' = -(\alpha + d) \Rightarrow \lambda' < 0.$

We notice that the disease-free equilibrium is locally asymptotically stable if $R_T < 1$, $R_H < 1$ and the disease will die out and become no longer epidemic; and it becomes unstable if $R_T > 1$, $R_H > 1$, in this case, the diseases become an epidemic.

3.2.2. Stability Analysis of HIV/AIDS-Free Point E_1

The Jacobian matrix of the linearization of the system of equations (6)-(9) about second equilibrium point $E_1$is given as

$$J(N^*,\ T^*,\ 0,\ 0) = \begin{bmatrix} -d & 0 & 0 & -\alpha \\ D_1 & F & G_1 & H_1 \\ 0 & 0 & K_1 & 0 \\ 0 & 0 & \delta & -(\alpha + d) \end{bmatrix}$$

where

$D_1 = \left(\beta_1 - \beta_1' \frac{T^*}{m+T^*} - (\lambda + d)\right) T^*$

$F_1 = \left[\frac{\Lambda}{d}\left\{\frac{m\beta_1'}{(m+T^*)^2} + \frac{m\beta_1 + (\beta_1 - \beta_1')}{(m+T^*)} - (\lambda + d)\right\} + \left\{\frac{m\beta_1' T^*}{(m+T^*)^2} + 2\frac{(m\beta_1 + (\beta_1 - \beta_1'))}{(m+T^*)}\right\}\right] T^*$

$G_1 = -\left(\beta_1 - \beta_1' \frac{T^*}{m+T^*} - \beta_3\right) T^*$

$H_1 = -\left(\beta_1 - \beta_1' \frac{T^*}{m+T^*}\right) T^*$

$K_1 = \left(\frac{\Lambda}{d}(\beta_2 - (\delta + d)) - (\beta_2 + \beta_3) T^*\right)$

The characteristic polynomial of the Jacobian matrix is given

$$|J - \lambda' I| = 0$$

$$\lambda'^4 + a_1 \lambda'^3 + a_2 \lambda'^2 + a_3 \lambda' + a_4 = 0 \tag{16}$$

where

$a_1 = [2d - F_1 - K_1 + \delta], a_2 = [-d(F_1 - K_1) + F_1 K_1 + (d - F_1 - K_1)(\delta + d)]$

$a_3 = [F_1 K_1 d - (F_1 d + K_1 d - F_1 K_1)(\delta + d)], a_4 = d F_1 K_1 (\delta + d).$

The above equilibrium point is unstable due to positive values $F_1 > 0$ and $K_1 > 0$, hence $a_1 < 0$. This fails Routh-Hurwitz criterion for stability. By Routh-Hurwitz criterion, the above equilibrium point is locally asymptotically stable if $a_i > 0$ ($i = 1, 2, 3, 4$), $a_1 a_2 - a_3 > 0$ and $a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 > 0$, and unstable otherwise.

3.2.3. Stability Analysis of TB Free Equilibrium Point E_2

The Jacobian matrix of the linearized system of equations (6)-(9) about equilibrium point E_2 is

$$J\left(\hat{N}, 0, \hat{H}, \hat{A}\right) = \begin{bmatrix} -d & 0 & 0 & -\alpha \\ 0 & D_2 & 0 & 0 \\ F_1 & G_2 & H_2 & K_2 \\ 0 & 0 & \delta & -(\alpha + d) \end{bmatrix}$$

where

$D_2 = \frac{1}{d}\left(\Lambda - \frac{\alpha\delta\hat{H}}{(\alpha+d)}\right)(\beta_1 - (\lambda + d)) - \left(\beta_1 - \beta_3 - \beta_1 \frac{\delta}{(\alpha+d)}\right)\hat{H}$

$$F_2 = -\left[\left(\beta_2 - \beta_2' \frac{\hat{H}}{n+\hat{H}}\right) - (\delta + d)\right]\hat{H}$$
$$G_2 = -\left[\left(\beta_2 - \beta_2' \frac{\hat{H}}{n+\hat{H}}\right) + \beta_3\right]\hat{H}$$

$$H_2 = \frac{1}{d}\left(\Lambda - \frac{\alpha\delta\hat{H}}{(\alpha+d)}\right)\left[\left(\frac{n\beta_2'\hat{H}}{\left(n+\hat{H}\right)^2} + \left(\beta_2 - \beta_2'\frac{\hat{H}}{n+\hat{H}}\right)\right) - (\delta+d)\right] - \left[\frac{n\beta_2'}{\left(n+\hat{H}\right)^2} + 2\left(\beta_2 - \beta_2'\frac{\hat{H}}{n+\hat{H}}\right)\right]\hat{H}$$
$$- \left[\frac{n\beta_2'}{\left(n+\hat{H}\right)^2} + \left(\beta_2 - \beta_2'\frac{\hat{H}}{n+\hat{H}}\right)\right]\left(\frac{\delta\hat{H}}{(\alpha+d)}\right)$$

$$K_2 = -\left(\beta_2 - \beta_2'\frac{\hat{H}}{n+\hat{H}}\right)\hat{H}$$

The characteristic polynomial of the Jacobian matrix is given by:

$$|J - \lambda' I| = 0$$

$$\lambda'^4 + b_1\lambda'^3 + b_2\lambda'^2 + b_3\lambda' + b_4 = 0 \tag{17}$$

where

$b_1 = 2d - D_2 + \alpha, b_2 = [(d - D_2 - H_2)(d+\alpha) - (D_2 + H_2)d + (D_2 - 1)H_2]$

$b_3 = [(D_2H_2 - dD_2 - dH_2)(d+\alpha) + (D_2H_2 + \delta K_2)d - (D_2K_2 + \alpha F_2)\delta]$

$b_4 = [D_2H_2(\alpha + d)d - (D_2H_2d - D_2F_2\alpha)\delta].$

The above equilibrium point is unstable due to two positive eigenvalues and will be asymptotically stable if $\beta_1 < (\lambda + d)$, $\beta_2 < (\delta + d)$. By Ruth-Hurwitz criterion, the equilibrium point E_2 will be asymptotically stable if $b_i > 0$, ($i = 1,\ 2,\ 3,\ 4$), $b_1b_2 - b_3 > 0$ and $b_1b_2b_3 - b_3^2 - b_1^2b_4 > 0$, and unstable otherwise.

3.2.4. Stability Analysis of Endemic Equilibrium Point E_3

The Jacobian matrix of the linearization of the system of equations (6)-(9) about an equilibrium point E_3

$$J\left(\tilde{N}, \tilde{T}, \tilde{H}, \tilde{A}\right) = \begin{bmatrix} -d & 0 & 0 & -\alpha \\ D_3 & F_3 & G_3 & H_3 \\ K_3 & L & M & N \\ 0 & 0 & \delta & -(\alpha + d) \end{bmatrix}$$

where

$$D_3 = \left[\left(\beta_1 - \beta_1'\frac{\tilde{T}}{m+\tilde{T}}\right) - (\lambda + d)\right]\tilde{T}$$

$$F_3 = \frac{1}{d}\left(\Lambda - \frac{\alpha\delta\tilde{H}}{(\alpha+d)}\right)\left[\left(\beta_1 - \beta_1'\frac{\tilde{T}}{m+\tilde{T}}\right) + \frac{m\beta_1'\tilde{T}}{\left(m+\tilde{T}\right)^2} - (\lambda+d)\right] - \left[\frac{m\beta_1'^2}{\left(m+\tilde{T}\right)^2} + 2\left(\beta_1 - \beta_1'\frac{\tilde{T}}{m+\tilde{T}}\right)\right]\tilde{T}$$
$$- \left[\left(\beta_1 - \beta_1'\frac{\tilde{T}}{m+\tilde{T}}\right) + \frac{m\beta_1'\tilde{T}}{\left(m+\tilde{T}\right)^2} - \beta_3\right]\tilde{H} - \left(\frac{\delta\tilde{H}}{(\alpha+d)}\right)\left[\left(\beta_1 - \beta_1'\frac{\tilde{T}}{m+\tilde{T}}\right) + \frac{m\beta_1'\tilde{T}}{\left(m+\tilde{T}\right)^2}\right]$$

$$G_3 = -\left[\left(\beta_1 - \beta_1'\frac{\tilde{T}}{m+\tilde{T}}\right) - \beta_3\right]\tilde{T}, H_3 = -\left(\beta_1 - \beta_1'\frac{\tilde{T}}{m+\tilde{T}}\right)\tilde{T}$$

$$K_3 = \left[\left(\beta_2 - \beta_2' \frac{\tilde{H}}{n+\tilde{H}}\right) - (\lambda + d)\right] \tilde{H}, L = -\left[\left(\beta_2 - \beta_2' \frac{\tilde{H}}{n+\tilde{H}}\right) - \beta_3\right] \tilde{H}$$

$$M = \frac{1}{d}\left(\Lambda - \frac{\alpha\delta\tilde{H}}{(\alpha+d)}\right)\left[\left(\frac{n\beta_2'\tilde{H}}{(n+\tilde{H})^2} + \left(\beta_2 - \beta_2' \frac{\tilde{H}}{n+\tilde{H}}\right)\right) - (\delta + d)\right] - \left[\frac{n\beta_2'}{(n+\tilde{H})^2} + \left(\beta_2 - \beta_2' \frac{\tilde{H}}{n+\tilde{H}}\right) - \beta_3\right] \tilde{T}$$
$$- \left[\frac{n\beta_2'}{(n+\tilde{H})^2} + 2\left(\beta_2 - \beta_2' \frac{\tilde{H}}{n+\tilde{H}}\right)\right] \tilde{H} - \left[\frac{n\beta_2'}{(n+\tilde{H})^2} + \left(\beta_2 - \beta_2' \frac{\tilde{H}}{n+\tilde{H}}\right)\right]\left(\frac{\delta\tilde{H}}{(\alpha+d)}\right)$$

$$N = -\left(\beta_2 - \beta_2' \frac{\tilde{H}}{n+\tilde{H}}\right) \tilde{H}$$

The characteristic polynomial of the Jacobian matrix is given as

$$|J - \lambda' I| = 0$$

$$\lambda'^4 + c_1\lambda'^3 + c_2\lambda'^2 + c_3\lambda' + c_4 = 0 \tag{18}$$

where
$c_1 = [2d - F_3 - M + (\alpha + d)], c_2 = [MF_3 - dF_3 - dM + (d - F_3 - M)(\alpha + d) - \delta N]$

$c_3 = [(MF_3 - \delta N)d - (dF_3 + dM + MF_3)(\alpha + d) + \delta N F_3], c_4 = dMF_3(\alpha + d) + d\delta N F_3$

Clearly, $c_i > 0$(where $i = 1, 2, 3, 4$). When $F_3 < 0 \Rightarrow \beta_1 < (\lambda + d)$, $M < 0 \Rightarrow \beta_2 < (\delta + d)$. It follows from Ruth-Hurwitz criteria that the polynomial (17) has negative real parts if and only if $\beta_1 < (\lambda + d)$, $\beta_2 < (\delta + d)$, $c_1c_2 - c_3 > 0$ and $c_1c_2c_3 - c_3^2 - c_1^2c_4 > 0$; under these conditions, the above equilibrium point will be asymptotically stable and unstable otherwise.

4 Numerical Simulations

In this section, we provide the numerical simulation of the model using fourth order Runge-Kutta scheme to solve the system of differential equations. The computer program is coded in MATLAB software. For computation purpose, we chose various default parameters. To select the values of β_1' and β_2', we realize that it should be less than β_1 and β_2, respectively. The default parameter values are chosen for numerical illustration purpose is as follows:
$\Lambda = 20000$, $\beta_1 = 0.952$, $\beta_1' = 0.0018$, $\beta_2 = 0.536$, $\beta_2' = 0.0020$, $m = 30$, $\beta_3 = 1.10, \lambda = 0.5$, $\delta = 0.4$, $\alpha = 1.5$, $d = 0.03$, $n = 10$. The initial set is as $N(0) = 100000$, $T(0) = 4000$, $H(0) = 3000$, $A(0) = 600$.

Fig. 2 depicts the variation of TB infected population with recovery rate (λ). It has been observed that with the increase in the value of λ, the TB population decreases and the susceptible population first increases with time and then reaches its equilibrium position.

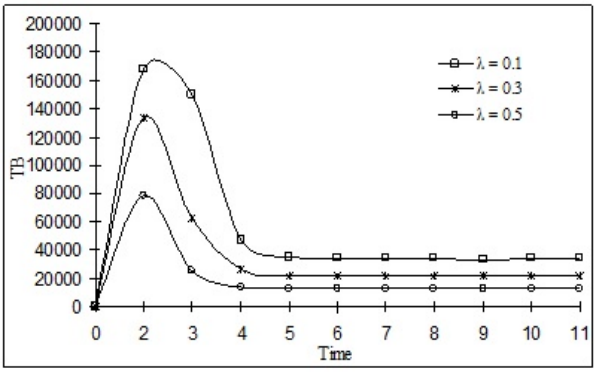

Figure 2. TB infected population with time for different values of λ

Fig. 3 exhibits the effect of the movements of δ on the HIV-infected population. It is seen that with the increase in the value of δ, the HIV-infected population decreases meaning thereby that it increases the full blown AIDS population.

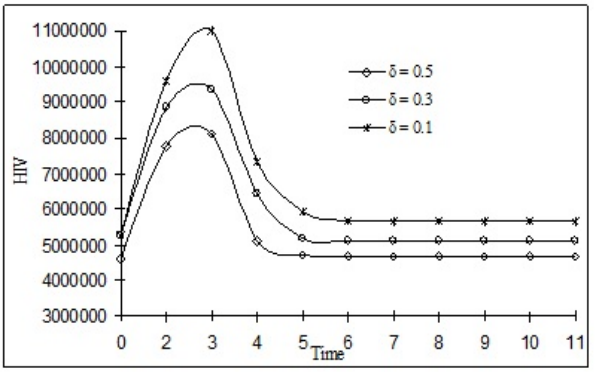

Figure 3. HIV-infected population with time for different values of δ.

In fig. 4, we demonstrate the effect of media parameter 'm' for TB infected population with varying the media parameter 'm' and fixed other parameters. We observe that with the increase in the value of 'm', the TB population decreases with time as media factor reduces the transmission rate of TB infection.

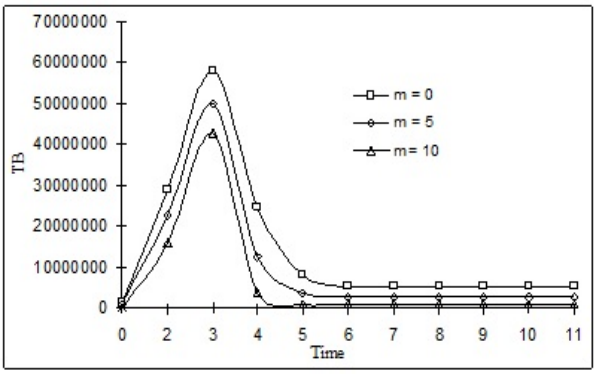

Figure 4. HIV-infected population with time for different values of m.

Fig. 5 shows the variation of the AIDS population with the disease induced death rate α. We note that with the increase in the induced death rate, the AIDS population decreases significantly.

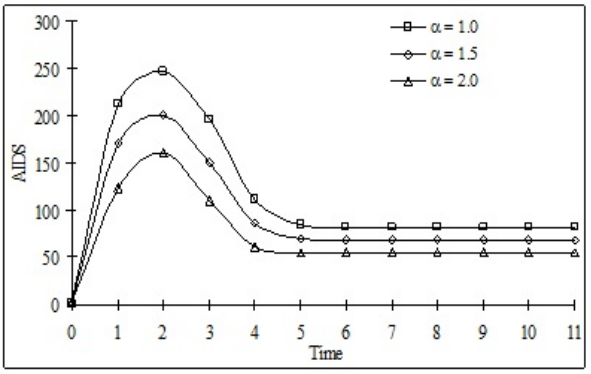

Figure 5. AIDS population with time for different values of α.

Fig. 6 depicts the variance of the HIV population with β_3. It is seen that with the increase in the transmission rate of HIV population to the AIDS population, the HIV population decreases and in turn the AIDS population increases.

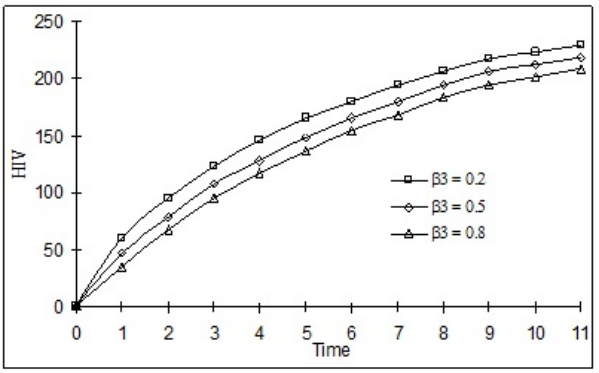

Figure 6. HIV population with time for different values of β_3.

In fig. 7, the impact of media parameter n on HIV population is displayed. It reveals that with the increase in the value of media parameter, the HIV population decreases.

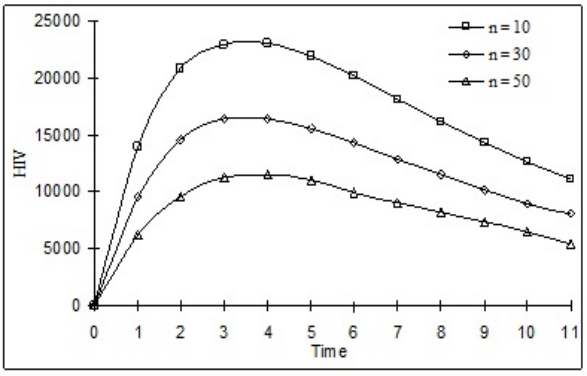

Figure 7. HIV population with time for different values of n.

5 Discussion and Conclusion

A nonlinear compartment model has developed to assess the role of awareness in the control and prevention of HIV/AIDS and TB co-infection. The stability of four equilibrium points is presented. The first disease free equilibrium point is stable if $\beta_1 < (\lambda + d)$ and $\beta_2 < (\delta + d)$. The second equilibrium point is for the case of HIV/AIDS free. The third point depicts the TB free population and is unstable due to the presence of positive eigenvalues. The fourth endemic point is locally asymptotically stable if it satisfies the conditions that $c_i > 0$,$(i = 1, 2, 3, 4)$, $c_1c_2 - c_3 > 0$ and $c_1c_2c_3 - c_3^2 - c_1^2c_4 > 0$. The reproduction numbers $R_T = \frac{\beta_1}{\lambda+d}$ and $R_H = \frac{\beta_2}{(\delta+d)}$ obtained for TB disease and HIV disease respectively, demonstrate that there arise three cases (i) If $R_T > 1$, the TB epidemic will become endemic (ii) if $R_T = 0$, the disease neither become epidemic nor die out (iii) if $R_T < 1$, then the TB will die out by leaving the system as susceptible. Similarly, the same case will happen to HIV epidemic. It is seen that if $\beta_1' = \beta_2' = 0$, there is no change in the reproduction numbers as they are independent of media effects. But it has been noticed that values of β_1', β_2', m and n change the equilibrium points and local stability of HIV/AIDS free point, and TB free point holds good when $R_H > 1$ and $R_T > 1$ respectively, otherwise these points become diseases free points. The two terms for media awareness for TB and HIV epidemics are $\beta(T) = \left(\beta_1 - \beta_1' \frac{T}{m+T}\right)$ and $\beta(H) = \left(\beta_2 - \beta_2' \frac{H}{n+H}\right)$ respectively. In fact $\frac{\partial \beta(T)}{\partial t} = \beta_1' \frac{T}{m+T} > 0$, which means that if m is smaller then the transmission rate will become

smaller too. It implies that when TB disease begins to spread, the media works for the awareness immediately and advises the masses to take protective measures to fight against the disease. Again when $\frac{\partial\beta(T)}{\partial\beta_1'} = -\frac{T}{m+T} < 0$, it is clear that for the bigger values of β_1', the transmission rate is smaller. The investigation done may be helpful to social workers, health organizations and practitioners in medical area in understanding the complex nature of dynamics of the fatal diseases viz. TB and HIV/AIDS; the result may be further used for the control and prevention of such diseases.

Overall, it is concluded that if TB infection is treated properly then HIV infection can be kept under control and the effective media awareness can lower the transmission dynamics of epidemics, not completely but to a greater extent.

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