SOME PROPERTIES OF SOFT GROUPS AND FUZZY SOFT GROUPS UNDER SOFT MAPPINGS

Sk. Nazmul

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Abstract. After the introduction of soft groups by Aktas and Cagman in 2007, many researchers have studied the properties of soft groups and fuzzy soft groups. But in the discussion of soft homomorphic image and preimage of these groups under soft mappings introduced by Kharal [7], they have considered a special type of soft mapping. In the present paper, the main aim to study the behaviour of soft homomorphic image and preimage of soft groups and fuzzy soft groups under the generalized soft mappings. Also it is noticed that in the discussion of functional image and preimage of soft sets, some authors defined constant soft mapping but the images of different soft sets under this type of constant soft mapping are different. For this, constant soft mapping is redefined, generalized soft elements, constant and pseudo constant soft elements are defined and the behaviour of functional image and preimage of soft sets under soft mappings are discussed.

1 Introduction

In 1999, Molodtsov [11] proposed a new tool called it soft sets for dealing uncertainties which is the most prevalent aspect in the natural occurance of events, taking a parameterized family of sets instead of the membership function introduced by Zadeh [19]. He established that Zadeh's fuzzy sets were special types of soft sets. In this respect it might be stated that the parameterization technique of soft set theory is more user friendly as compared to the membership function approach of fuzzy set theory in the field of their applications in real life problems. Afterwards Maji et al. [8] introduced several operations on soft sets. Since then researchers regarding soft sets and their hybrid generalizations have flourished in mamy directions such as fuzzy subgroups [17], soft group [1, 2, 12], fuzzy soft groups [12], intuitionistic L-fuzzy soft groups [13], soft mappings [7, 9], soft topology [4, 6, 18], soft topological group [14, 15, 16] etc. Soft set theory has potential applications in different fields viz. game theory, operations research, Fiemann integration, perron integration etc. But it is noticed that, in the discussion of functional image and preimage of soft groups and fuzzy soft groups, the corresponding authors have considered actually the crisp mappings. They have considered a special type of soft mappings [7], where the parameter set in the image and preimage soft sets remain unchanged as in the main soft sets i.e. they have considered the soft mapping f_φ where $\varphi : A \to A$, defined by $\varphi(\alpha) = \alpha$, $\forall \, \alpha \in A$. Also we observed that some authors have defined constant soft mapping, but the soft image of different soft sets under this constant soft mapping are different which violent the constantness. So, in the present paper we have investigated the behaviour of soft homomorphic image and preimage of soft groups and fuzzy soft groups under the generalized soft mappings f_φ where both the mappings $f : X \to Y$, $\varphi : A \to B$ are arbitrary. Also constant soft mapping, generalized soft element, constant and pseudo constant soft elements are introduced and the behaviour of functional image and preimage of soft sets under this type of constant soft mapping are discussed.

2 Preliminaries

soft sets:

In this section following [11, 8], some definitions and results of soft sets are given. Unless otherwise stated, X will be assumed to be an initial universal set, E will be taken to be a set of parameters, $P(X)$ denote the power set of X.

Definition 2.1. [11, 8] Let X be an initial universal set, E be the set of parameters, $P(X)$ denotes the power set of X and $A \subseteq E$. A pair (F, A) where F is a mapping from A to $P(X)$, is called a *soft set* over X. Also $S(X, E)$ denotes the set of all soft sets over X under the parameter E.

Definition 2.2. Let (F, A), $(G, B) \in S_1(X, E)$. Then
   (i) [11, 8] (F, A) is said to be *soft subset* of (G, B) if $A \subseteq B$ and $F(\alpha) \subseteq G(\alpha)$, $\forall\, \alpha \in A$. This relation is denoted by $(F, A) \,\widetilde{\subseteq}\, (G, B)$.
   (ii) [10] the *complement* of a soft set (F, A) is defined as $(F, A)^c = (F^c, A)$, where $F^c(\alpha) = (F(\alpha))^c = X - F(\alpha)$, $\forall\, \alpha \in A$.
   (iii) [11, 8] (F, A) is said to be *a null soft set* $\big($*an absolute soft set*$\big)$ if $F(\alpha) = \phi$ $\big(F(\alpha) = X\big)$, $\forall\, \alpha \in A$. This is denoted by $(\widetilde{\phi}, A)$ $\big((\widetilde{X}, A)\big)$.

Definition 2.3. [11, 8] Let (F, A), $(G, B) \in S(X, E)$. Then their
   (a) *Union*, is a soft set $(H, A \cup B) \in S(X, E)$, denoted by $(F, A) \,\widetilde{\cup}\, (G, B) = (H, A \cup B)$, is defined by $\forall\, \alpha \in (A \cup B)$

$$
H(\alpha) = \begin{cases} F(\alpha) & \text{if } \alpha \in (A - B) \\ G(\alpha) & \text{if } \alpha \in (B - A) \\ F(\alpha) \cup G(\alpha) & \text{if } \alpha \in (A \cap B) \end{cases}
$$

   (b) *Intersection*, is a soft set $(H, A \cap B) \in S(X, E)$, denoted by $(F, A) \,\widetilde{\cap}\, (G, B) = (H, A \cap B)$, is defined by $H(\alpha) = F(\alpha) \cap G(\alpha)$, $\forall\, \alpha \in (A \cap B)$.

   (c) *AND*, is a soft set $(H, A \times B) \in S(X, E \times E)$, denoted by $(F, A) \,\widetilde{\wedge}\, (G, B) = (H, A \times B)$, is defined by $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall\, (\alpha, \beta) \in (A \times B)$.

   (d) *OR*, is a soft set $(H, A \times B) \in S(X, E \times E)$, denoted by $(F, A) \,\widetilde{\vee}\, (G, B) = (H, A \times B)$, is defined by $H(\alpha, \beta) = F(\alpha) \cup G(\beta)$, $\forall\, (\alpha, \beta) \in (A \times B)$.

Remark 2.4. [10] For any $(F, A) \in S(X, E)$, construct a soft set $(H, E) \in S(X, E)$, where $\forall\, \alpha \in E$,

$$
H(\alpha) = \begin{cases} F(\alpha) & \text{if } \alpha \in A \\ \phi & \text{if } \alpha \in E/A \end{cases}
$$

Thus the soft sets (F, A) and (H, E) are equivalent to each other and the usual set operations of the soft sets $(F_i, A_i) \in S(X, E)$, $i \in I$ is the same as the corresponding soft sets $(H_i, E) \in S(X, E)$, $i \in I$.

So, in this paper we have considered the soft sets over the same parameter set A.
   Soft groups:

In this section, following [1, 12] the definitions and some results of soft groups are given in a slightly modified form. Throughout this section X, Y are taken to be groups, e_X, e_Y are the identity elements of X, Y respectively.

Definition 2.5. [1, 12] Let (F, A) be a soft set over X. Then (F, A) is said to be a *soft group* over X iff $F(\alpha)$ is a subgroup of X, $\forall\, \alpha \in A$ i.e. $F(\alpha) \leq X$, $\forall\, \alpha \in A$.

Proposition 2.6. *[1, 12] Let (F_1, A) and (F_2, A) be two soft groups over X. Then $(F_1, A) \,\widetilde{\cap}\, (F_2, A)$ and $(F_1, A) \,\widetilde{\wedge}\, (F_2, A)$ are soft groups over X.*

Definition 2.7. [1, 12] Let (F, A), (F_1, A) and (F_2, A) be soft groups over (X, A). Then

(i) (F, A) is said to be an *identity soft group* (*absolute soft group*) over X if $F(\alpha) = \{e_X\}$ $(F(\alpha) = X)$, $\forall \alpha \in A$.

(ii) (F_1, A) is said to be a *soft subgroup* (*soft normal subgroup*) of (F_2, A), denoted by $(F_1, A) \widetilde{\leq} (F_2, A)$ $((F_1, A) \widetilde{\lhd} (F_2, A))$ if $F_1(\alpha) \leq F_2(\alpha)$ $(F_1(\alpha) \lhd F_2(\alpha))$, $\forall \alpha \in A$.

Proposition 2.8. *[1, 12]   (i) Let (F, A) be a soft group over X and f be a homomorphism from the group X to a group Y. If $F(\alpha) = Kerf, \forall \alpha \in A$ then $(f(F), A)$ is an identity soft group over Y where $f(F)(\alpha) = f(F(\alpha)), \forall \alpha \in A$.   (ii) Let (F, A) be an absolute soft group over X and f be a homomorphism from the group X onto a group Y. Then $(f(F), A)$ is an absolute soft group over Y where $f(F)(\alpha) = f(F(\alpha)), \forall \alpha \in A$.*

Proposition 2.9. *[1, 12] Let (F_1, A) and (F_2, A) be two soft groups over X and $f : X \to Y$ be a homomorphism. Then*

(i) $(f(F_1), A)$ is a soft group over Y;

(ii) If $(F_1, A) \widetilde{\leq} (F_2, A)$, then $(f(F_1), A) \widetilde{\leq} (f(F_2), A)$;

(iii) If $(F_1, A) \widetilde{\lhd} (F_2, A)$, then $(f(F_1), A) \widetilde{\lhd} (f(F_2), A)$.

3   Some results on soft mappings

In this section, some important results of soft homomorphic image and preimage of soft groups and fuzzy soft groups under the soft mapping f_φ[7] are discussed. Constant soft mapping, generalized soft element (briefly g-soft element), constant and pseudo constant soft elements are introduced and the behaviour of functional image and preimage of soft sets under this soft mappings are studied.

Definition 3.1. [7] Let $S(X, A)$ and $S(Y, B)$ be the families of all soft sets over X and Y respectively. The mapping $f_\varphi : S(X, A) \to S(Y, B)$ is called a soft mapping from X to Y, where $f : X \to Y$ and $\varphi : A \to B$ are two mappings. Also

(i) the *image* of a soft set $(F, A) \in S(X, A)$ under the mapping f_φ is denoted by $f_\varphi[(F, A)] = (f_\varphi(F), B)$, and is defined by

$$[f_\varphi(F)](\beta) = \begin{cases} \bigcup_{\alpha \in \varphi^{-1}(\beta)} [f[F(\alpha)]] & \text{if } \varphi^{-1}(\beta) \neq \phi \\ \phi & \text{otherwise} \end{cases} , \ \forall \beta \in B.$$

(ii) the *inverse image* of a soft set $(G, B) \in S(Y, B)$ under the mapping f_φ is denoted by $f_\varphi^{-1}[(G, B)] = (f_\varphi^{-1}(G), A)$, and is defined by $[f_\varphi^{-1}(G)](\alpha) = f^{-1}[G[\varphi(\alpha)]]$, $\forall \alpha \in A$.

(iii) The soft mapping f_φ is called injective (surjective) if f and φ are both injcetive (surjective).

(iv) The soft mapping f_φ is identity soft mapping, if f and φ are both clasical identity mappings.

Proposition 3.2. *[7] Let X and Y be two nonempty sets and $f_\varphi : S(X, A) \to S(Y, B)$ be a soft mapping. If (F, A), $(F_i, A) \in S(X, A)$ and (G, B), $(G_i, B) \in S(Y, B)$, $i \in I$ then*

(i) $(F_1, A) \widetilde{\subseteq} (F_2, A) \Rightarrow f_\varphi[(F_1, A)] \widetilde{\subseteq} f_\varphi[(F_2, A)]$;

(ii) $(G_1, B) \widetilde{\subseteq} (G_2, B) \Rightarrow f_\varphi^{-1}[(G_1, B)] \widetilde{\subseteq} f_\varphi^{-1}[(G_2, B)]$;

(iii) $(F, A) \widetilde{\subseteq} f_\varphi^{-1}[f_\varphi(F, A)]$, the equality holds if f_φ is injective.

(iv) $f_\varphi[f_\varphi^{-1}(G, B)] \widetilde{\subseteq} (G, B)$, the equality holds if f_φ is surjective.

(v) $f_\varphi[\widetilde{\bigcup}_{i \in I}(F_i, A)] = \widetilde{\bigcup}_{i \in I}[f_\varphi(F_i, A)]$, and $f_\varphi^{-1}[\widetilde{\bigcup}_{i \in I}(G_i, A)] = \widetilde{\bigcup}_{i \in I}[f_\varphi^{-1}(G_i, A)]$;

(vi) $f_\varphi[\widetilde{\bigcap}_{i \in I}(F_i, A)] \widetilde{\subseteq} \widetilde{\bigcap}_{i \in I}[f_\varphi(F_i, A)]$, the equality holds if f_φ is injective and $f_\varphi^{-1}[\widetilde{\bigcap}_{i \in I}(G_i, A)] = \widetilde{\bigcap}_{i \in I}[f_\varphi^{-1}(G_i, A)]$.

Definition 3.3. [15] A soft set (F, A) in $S(X, A)$ is said to be a

(i) constant soft set if $F(\alpha) = X_0 \subseteq X$, $\forall \alpha \in A$. It is denoted by $(\widetilde{X_0}, A)$.

(ii) pseudo constant soft set if $F(\alpha) = X$ or ϕ, $\forall \alpha \in A$. The set of all pseudo constant soft set over X is denoted by $CS(X, A)$.

Proposition 3.4. *[15] Let X and Y be two nonempty sets and $f_\varphi : S(X, A) \to S(Y, B)$ be a soft mapping. If $(F, A) \in S(X, A)$ and $(G, B) \in S(Y, B)$ then*
  (i) $(G, B) \in CS(Y, B) \Rightarrow f_\varphi^{-1}[(G, B)] \in CS(X, A)$;
  (ii) $(F, A) \in CS(X, A)$ and f is surjective $\Rightarrow f_\varphi[(F, A)] \in CS(Y, B)$.

Definition 3.5. [5] A soft set (E, A) over X is called a *soft element* if $E(\alpha) \in X$, $\forall\, \alpha \in A$.

Definition 3.6. A soft set $(E, A) \neq (\widetilde{\phi}, A)$ over X is called a
  (i) *generalized soft element* (or briefly g-soft element) of X, if $E(\alpha) = \phi$ or $\exists\, x \in X$ such that $E(\alpha) = \{x\}$, $\forall\, \alpha \in A$.
  (ii) *constant $g - soft\ element$ of X, if $\exists\, x \in X$ such that $E(\alpha) = \{x\}$, $\forall\, \alpha \in A$.
  (iii) *pseudo constant $g-soft\ element$ of X, if $\exists\, x \in X$ such that $E(\alpha) = \phi$ or $\{x\}$, $\forall\, \alpha \in A$.
We denotes E_x, the constant soft element defined by $E(\alpha) = \{x\}$, $\forall\, \alpha \in A$ and E_x^α, the soft element defined by $E(\alpha) = \{x\}$ and $E(\beta) = \phi$, $\forall\, \beta(\neq \alpha) \in A$.

Definition 3.7. [3] The soft mapping $f_\varphi : S(X, A) \to S(Y, B)$ is called constant if f is constant.

Remark 3.8. In crisp sence, the image of any subset of the domain under constant mapping is constant but the following example shows that the functional image of soft sets under constant soft mapping defined in Definition 3.7 are not constant soft set i.e. $f_\varphi[(F, A)]$ is not constant $\forall\, (F, A) \in S(X, A)$.

Example 3.9. Let $X = \{x,\ y,\ z\}$, $Y = \{a,\ b,\ c,\ d\}$, $A = \{\alpha_1,\ \alpha_2\}$ and $B = \{\beta_1,\ \beta_2,\ \beta_3\}$. Also let $(F_1, A) = \{\{x,\ z\}/\alpha_1,\ \{\phi\}/\alpha_2\}$, $(F_2, A) = \{\{\phi\}/\alpha_1,\ \{y,\ z\}/\alpha_2\}$ be two soft sets in $S(X, A)$ and f_φ is a constant mapping such that $f(x) = f(y) = f(z) = c$.
If $\varphi(\alpha_1) = \beta_1$, $\varphi(\alpha_2) = \beta_2$, then $f_\varphi[(F_1, A)] = \{\{c\}/\beta_1,\ \{\phi\}/\beta_2,\ \{\phi\}/\beta_3\}$ and $f_\varphi[(F_2, A)] = \{\{\phi\}/\beta_1,\ \{c\}/\beta_2,\ \{\phi\}/\beta_3\}$.
Hence $f_\varphi[(F_1, A)] \neq f_\varphi[(F_2, A)]$.

Proposition 3.10. *Let $f_\varphi : S(X, A) \to S(Y, B)$ be a soft mapping such that $f : X \to Y$ is constant mapping and $f(x) = y_0(say)$. Then*
  (i) $\forall\, (F, A) \in S(X, A)$, $f_\varphi[(F, A)]$ is a pseudo constant soft element (E, B) of Y, where $E(\beta) = \phi$ or $\{y_0\}$;
  (ii) $f_\varphi^{-1}[(G, B)]$ is a pseudo constant soft set in $S(X, A)$, $\forall\, (G, B) \in S(Y, B)$.

Proof. Let $f : X \to Y$ be a constant mapping and $f(x) = y_0$, $\forall\, x \in X$.
  (i) Let (F, A) be any soft set in $S(X, A)$. Then $f_\varphi[(F, A)] = (f_\varphi(F), B) \in S(Y, B)$. Also $\forall\, \beta \in B$,

$$[f_\varphi(F)](\beta) = \begin{cases} \phi & \text{if } \varphi^{-1}(\beta) = \phi \\ \phi & \text{if } \varphi^{-1}(\beta) \neq \phi \text{ and } F(\alpha) = \phi,\ \forall\, \alpha \in \varphi^{-1}(\beta) \\ \{y_0\} & \text{if } \varphi^{-1}(\beta) \neq \phi \text{ and } \exists\, \alpha \in \varphi^{-1}(\beta) \text{ such that } F(\alpha) \neq \phi \end{cases}$$

Thus, $f_\varphi[(F, A)] = (E, B)$, where $E(\beta) = \phi$ or $\{y_0\}$, $\forall\, \beta \in B$.
Therefore, $\forall\, (F, A) \in S(X, A)$, $f_\varphi[(F, A)]$ is a pseudo constant soft element (E, B) of Y where $E(\beta) = \phi$ or $\{y_0\}$, $\forall\, \beta \in B$.
  (ii) Let (G, B) be any soft set in $S(Y, B)$. Then $f_\varphi^{-1}[(G, B)] = (f_\varphi^{-1}(G), A) \in S(X, A)$. Also $\forall\, \alpha \in A$,

$$[f_\varphi^{-1}(G)](\alpha) = f^{-1}\big[G[\varphi(\alpha)]\big] = \begin{cases} X & \text{if } y_0 \in G[\varphi(\alpha)] \\ \phi & \text{if } y_0 \notin G[\varphi(\alpha)] \end{cases}$$

Therefore $f_\varphi^{-1}[(G, B)]$ is a pseudo constant soft set in $S(X, A)$, $\forall\, (G, B) \in S(Y, B)$. □

Definition 3.11. The soft mapping $f_\varphi : S(X, A) \to S(Y, B)$ is called constant if $\exists$ a g-soft element $(E, B) \in S(Y, B)$ such that $f_\varphi[(F, A)] = (E, B)$, $\forall\, (F, A)[\neq (\widetilde{\phi}, A)] \in S(X, A)$.

Proposition 3.12. *If both the mappings $f : X \to Y$ and $\varphi : A \to B$ be constant then $f_\varphi : S(X, A) \to S(Y, B)$ is a constant soft mapping.*

Proof. Let $f : X \to Y$, $\varphi : A \to B$ be constant mappings and $f(x) = y_0$, $\forall\, x \in X$, $\varphi(\alpha) = \beta_0$, $\forall\, \alpha \in A$.

Let $(F, A) \neq (\widetilde{\phi}, A)$ be any soft set in $S(X, A)$. Then $f_\varphi[(F, A)] = (f_\varphi(F), B) \in S(Y, B)$. Also $\forall\, \beta \in B$,

$$[f_\varphi(F)](\beta) = \begin{cases} \phi & \text{if } \beta \neq \beta_0 \\ \{y_0\} & \text{if } \beta = \beta_0 \end{cases}$$

Hence $f_\varphi[(F, A)] = E_{y_0}^{\beta_0}$.

Therefore $f_\varphi : S(X, A) \to S(Y, B)$ is a constant soft mapping. $\square$

Remark 3.13. The following example shows that $\forall\, (F, A) \in S(X, A)$, $f_\varphi[(F, A)]$ is not constant if f is not constant.

Example 3.14. Let $X = \{x,\, y,\, z\}$, $Y = \{a,\, b,\, c,\, d\}$, $A = \{\alpha_1,\, \alpha_2\}$ and $B = \{\beta_1,\, \beta_2,\, \beta_3\}$. Also let $(F_1, A) = \{\{x\}/\alpha_1,\, \{\phi\}/\alpha_2\}$, $(F_2, A) = \{\{y\}/\alpha_1,\, \{y,\, z\}/\alpha_2\}$ be two soft sets in $S(X, A)$ and f_φ is a soft mapping such that φ is a constant mapping and let $\varphi(\alpha_1) = \varphi(\alpha_2) = \beta_3$. If $f(x) = a$, $f(y) = b$, $f(z) = b$, then $f_\varphi[(F_1, A)] = \{\{\phi\}/\beta_1,\, \{\phi\}/\beta_2,\, \{a\}/\beta_3\}$ and $f_\varphi[(F_2, A)] = \{\{\phi\}/\beta_1,\, \{\phi\}/\beta_2,\, \{b\}/\beta_3\}$. Hence $f_\varphi[(F_1, A)] \neq f_\varphi[(F_2, A)]$.

Remark 3.15. From Proposition 3.12, it can be seen that if both the mappings $f : X \to Y$ and $\varphi : A \to B$ are constant, then f_φ is a constant soft mapping. Thus, the question is whether the converse of the above is true or not. In the next Proposition, we get an affirmative answer to this question.

Proposition 3.16. *If $f_\varphi : S(X, A) \to S(Y, B)$ is a constant soft mapping. Then both the mappings $f : X \to Y$ and $\varphi : A \to B$ are constant.*

Proof. The proposition is proved if it is proved that, atleast one of the mappings $f : X \to Y$ and $\varphi : A \to B$ be not constant implies the soft mappings $f_\varphi : S(X, A) \to S(Y, B)$ is not constant. For this, first let f_φ be a soft mapping such that f is constant and φ is not constant.

Since, f be constant, $\exists\, y_0 \in Y$ such that $f(x) = y_0$, $\forall\, x \in X$ and φ is not constant, $\exists\, \alpha_1,\, \alpha_2 \in A$ such that $\varphi(\alpha_1) \neq \varphi(\alpha_2)$. Let $\varphi(\alpha_1) = \beta_1$ and $\varphi(\alpha_2) = \beta_2$.

Now construct two soft sets (F_1, A), (F_2, A) in $S(X, A)$ by $F_1(\alpha_1) \neq \phi$, $F_1(\alpha) = \phi$, $\forall\, \alpha(\neq \alpha_1) \in A$ and $F_2(\alpha_2) \neq \phi$, $F_2(\alpha) = \phi$, $\forall\, \alpha(\neq \alpha_2) \in A$.

Now $\{f_\varphi[(F_1, A)]\}(\beta_1) = \cup_{\alpha \in \varphi^{-1}(\beta_1)} f[F_1(\alpha)] = \{y_0\}$, [since $\alpha_1 \in \varphi^{-1}(\beta_1) \neq \phi$] but $\{f_\varphi[(F_2, A)]\}(\beta_1) = \cup_{\alpha \in \varphi^{-1}(\beta_1)} f[F_2(\alpha)] = \phi$, [since $\alpha_2 \notin \varphi^{-1}(\beta_1) \neq \phi$].

Thus, $f_\varphi[(F_1, A)] \neq f_\varphi[(F_2, A)]$ and hence f_φ is not constant.

Secondly, let f_φ be a soft mapping such that f is not constant and φ is constant. Since, f be not constant, $\exists\, x_1,\, x_2 \in X$ such that $f(x_1) \neq f(x_2)$ and φ is constant, $\exists\, \beta_0 \in B$ such that $\varphi(\alpha) = \beta_0$ $\forall\, \alpha \in A$. Let $f(x_1) = y_1$ and $f(x_2) = y_2$.

Now construct two soft sets (F_1, A), (F_2, A) in $S(X, A)$ by $F_1(\alpha) = \{x_1\}$, $\forall\, \alpha \in A$ and $F_2(\alpha) = \{x_2\}$, $\forall\, \alpha \in A$.

Now $\{f_\varphi[(F_1, A)]\}(\beta_0) = \cup_{\alpha \in \varphi^{-1}(\beta_0)} f[F_1(\alpha)] = \cup_{\alpha \in A} f[F_1(\alpha)] = \{y_1\}$, but $\{f_\varphi[(F_2, A)]\}(\beta_0) = \cup_{\alpha \in \varphi^{-1}(\beta_0)} f[F_2(\alpha)] = \cup_{\alpha \in A} = \{y_2\}$.

Thus, $f_\varphi[(F_1, A)] \neq f_\varphi[(F_2, A)]$ and hence f_φ is not constant.

Therefore, if f_φ is a constant soft mapping, then both the mappings f and φ are constant. $\square$

Proposition 3.17. *The soft mapping $f_\varphi : S(X, A) \to S(Y, B)$ is constant iff the mappings $f : X \to Y$ and $\varphi : A \to B$ are constant.*

Proof. Proof follows from Proposition 3.16 and Proposition 3.12. $\square$

Proposition 3.18. *Let $f_\varphi : S(X, A) \to S(Y, B)$ be a soft mapping. If the mapping $\varphi : A \to B$ be constant then, $f_\varphi^{-1}[(G, B)]$ is a constant soft set $\forall\, (G, B) \in S(Y, B)$.*

Proof. Let $f_\varphi : S(X, A) \to S(Y, B)$ be a soft mapping such that $\varphi : A \to B$ is constant and $\varphi(\alpha) = \beta$(say), $\forall\, \alpha \in A$. Let (G, B) be any soft set in $S(Y, B)$. Then

$$[f_\varphi^{-1}(G, B)](\alpha) = f^{-1}\big[G[\varphi(\alpha)]\big] = f^{-1}[G(\beta)] = \begin{cases} \phi, \; \forall\, \alpha \in A & \text{if } G(\beta) = \phi \\ X_0 \subseteq X, \; \forall\, \alpha \in A & \text{if } G(\beta) \neq \phi \end{cases}$$

Hence, $f_\varphi^{-1}[(G, B)] = (\widetilde{\phi}, A)$ or $(\widetilde{X_0}, A)$.

Therefore, $f_\varphi^{-1}[(G, B)]$ is a constant soft set $\forall\, (G, B) \in S(Y, B)$. $\square$

Soft groups:

In this section, the behaviour of soft homomorphic image and preimage of soft groups under soft mapping are studied. Throughout this section X, Y are taken to be groups, e_X, e_Y are the identity elements of X, Y respectively.

Definition 3.19. [1, 12] A soft function $f_\varphi : S(X, A) \to S(Y, B)$ is said to be
  (i) soft homomorphism if $f : X \to Y$ is a algebraic homomorphism.
  (ii) soft isomorphism if $f : X \to Y$ is a algebraic isomorphism.

Proposition 3.20. *Let (F, A), (F_1, A), $(F_2, A))$ be soft groups over (X, A) and (G, B), (G_1, B), $(G_2, B))$ be soft groups over (Y, B). Also let $f_\varphi : S(X, A) \to S(Y, B)$ be a soft mapping.*
  (i) If (F, A) be identity soft group over (X, A) and f_φ is a soft homomorphism such that φ is onto, then $f_\varphi[(F, A)]$ is identity soft group over (Y, B).
  (ii) If (G, B) be identity soft group over (Y, B) and f_φ is a soft homomorphism, then $f_\varphi^{-1}[(G, B)]$ is the constant soft group $(\widetilde{ker(f)}, A)$ over (X, A). In perticular, if f is one-one then $f_\varphi^{-1}[(G, B)]$ is identity soft group over (X, A).
  (iii) If (F, A) be absolute soft group over (X, A) and f_φ is a soft homomorphism such that φ is onto, then $f_\varphi[(F, A)]$ is a constant soft group over (Y, B). Also if f_φ is onto, then $f_\varphi[(F, A)]$ is absolute soft group over (Y, B).
  (iv) If (G, B) be absolute soft group over (Y, B) and f_φ is a soft homomorphism, then $f_\varphi^{-1}[(G, B)]$ is absolute soft group over (X, A).

  (v) If f_φ is a soft homomorphism such that φ is onto, then $f_\varphi[(\widetilde{ker(f)}, A)] = (\widetilde{\{e_Y\}}, B)$.
  (vi) If (G, B) be a soft group over (Y, B) and f_φ is a soft homomorphism, then $f_\varphi^{-1}[(G, B)]$ is a soft group over (X, A).
  (vii) If (F, A) be a soft group over (X, A) and f_φ is a soft homomorphism such that φ is one-one, then $f_\varphi[(F, A)]$ is a soft group over (Y, B).
  (viii) If $(G_1, B) \widetilde{\leq} (G_2, B)$ and f_φ is a soft homomorphism, then $f_\varphi^{-1}[(G_1, B)] \widetilde{\leq} f_\varphi^{-1}[(G_2, B)]$.
  (ix) If $(F_1, A) \widetilde{\leq} (F_2, A)$ and f_φ is a soft homomorphism such that φ is one-one, then $f_\varphi[(F_1, A)] \widetilde{\leq} f_\varphi[(F_2, A)]$.
  (x) If $(G_1, B) \widetilde{\lhd} (G_2, B)$ and f_φ is a soft homomorphism, then $f_\varphi^{-1}[(G_1, B)] \widetilde{\lhd} f_\varphi^{-1}[(G_2, B)]$.
  (xi) If $(F_1, A) \widetilde{\lhd} (F_2, A)$ and f_φ is a soft homomorphism such that φ is one-one, then $f_\varphi[(F_1, A)] \widetilde{\lhd} f_\varphi[(F_2, A)]$.

Proof.   (i) Let (F, A) be identity soft group over (X, A) and f_φ is a soft homomorphism such that φ is onto. Then $F(\alpha) = \{e_X\}$, $\forall\, \alpha \in A$ and $\varphi^{-1}(\beta) \neq \phi$, $\forall\, \beta \in B$. Now for any $\beta \in B$, $f_\varphi[(F, A)](\beta) = \cup_{\alpha \in \varphi^{-1}(\beta)} f[F(\alpha)] = \cup_{\alpha \in \varphi^{-1}(\beta)} f[\{e_X\}] = \{e_Y\}$.
Therefore, $f_\varphi[(F, A)] = (\widetilde{e_Y}, B)$ and hence $f_\varphi[(F, A)]$ is identity soft group over (Y, B).
  (ii) Let (G, B) be identity soft group over (Y, B) and f_φ is a soft homomorphism. Then $G(\beta) = \{e_Y\}$, $\forall\, \beta \in B$. Now for any $\alpha \in A$,
$f_\varphi^{-1}[(G, B)](\alpha) = f^{-1}(G[\varphi(\alpha)]) = f^{-1}[\{e_Y\}] = ker(f)$.
So, $f_\varphi^{-1}[(G, B)] = (\widetilde{ker(f)}, A)$.
Therefore, $f_\varphi^{-1}[(G, B)]$ is the constant soft group $(\widetilde{ker(f)}, A)$ over (X, A).

  Again if f is one-one, then $ker(f) = \{e_X\}$ and hence $f_\varphi^{-1}[(G, B)] = (\widetilde{ker(f)}, A) = (\widetilde{\{e_X\}}, A)$.

Thus, if φ is one-one, then $f_\varphi^{-1}[(G, B)]$ is identity soft group (X, A).

(iii) Let (F, A) be absolute soft group over (X, A) and f_φ is a soft homomorphism such that φ is onto, then $F(\alpha) = X$, $\forall \alpha \in A$ and $\varphi^{-1}(\beta) \neq \phi$, $\forall \beta \in B$. Also since f is a homomorphism, it follows that $f(X)$ is a subgroup of Y.

Now for any $\beta \in B$,
$$f_\varphi[(F, A)](\beta) = \cup_{\alpha \in \varphi^{-1}(\beta)} f[F(\alpha)] = \cup_{\alpha \in \varphi^{-1}(\beta)} f[\{X\}] = Y_0 \subseteq Y \text{ (say)}.$$
Therefore, $f_\varphi[(F, A)] = (\widetilde{Y_0}, B)$ and hence $f_\varphi[(F, A)]$ is a constant soft group over (Y, B).

Again if f_φ is onto, then for any $\beta \in B$,
$$f_\varphi[(F, A)](\beta) = \cup_{\alpha \in \varphi^{-1}(\beta)} f[F(\alpha)] = \cup_{\alpha \in \varphi^{-1}(\beta)} f[X] = Y \text{ and hence,}$$
$f_\varphi[(F, A)] = (\widetilde{Y}, B)$.
Thus, if f_φ is onto, then $f_\varphi[(F, A)]$ is absolute soft group over (Y, B).

(iv) Let (G, B) be absolute soft group over (Y, B) and f_φ is a soft homomorphism, then $G(\beta) = Y$, $\forall \beta \in B$. Now for any $\alpha \in A$, $f_\varphi^{-1}[(G, B)](\alpha) = f^{-1}(G[\varphi(\alpha)]) = f^{-1}[Y] = X$ and hence, $f_\varphi^{-1}[(G, B)] = (\widetilde{X}, A)$.
Therefore, $f_\varphi^{-1}[(G, B)]$ is absolute soft group over (X, A).

(v) Let f_φ be a soft homomorphism such that φ is onto, then $\varphi^{-1}(\beta) \neq \phi$.
Since $f[ker(f)] = \{e_Y\}$, it follows that
$$f_\varphi[(\widetilde{ker(f)}, A)](\beta) = \cup_{\alpha \in \varphi^{-1}(\beta)} f[\widetilde{ker(f)}[\varphi(\alpha)]] = f[ker(f)] = e_Y, \ \forall \ \beta \in B \text{ and hence,}$$
$f_\varphi[(\widetilde{ker(f)}, A)] = (\widetilde{\{e_Y\}}, B)$.
Therefore, $f_\varphi[(\widetilde{ker(f)}, A)]$ is identity soft group over (Y, B).

(vi) Let (G, B) be a soft group over (Y, B) and f_φ is a soft homomorphism, then $G(\beta)$ is a subgroup of Y, $\forall \ \beta \in B$ and $f^{-1}[G(\beta)]$ is a subgroup of X, $\forall \ \beta \in B$. Now for any $\alpha \in A$, $f_\varphi^{-1}[(G, B)](\alpha) = f^{-1}(G[\varphi(\alpha)]) = f^{-1}(G(\beta_0))$ where $\varphi(\alpha) = \beta_0$ (say) is a subgroup of X and hence $f_\varphi^{-1}[(G, B)]$ is a soft group over (X, A).

(vii) Let (F, A) be a soft group over (X, A) and f_φ is a soft homomorphism such that φ is one-one, then $F(\alpha)$ is a subgroup of X and hence $f[F(\alpha)]$ is a subgroup of Y, $\forall \ \alpha \in A$ and $\varphi^{-1}(\beta)$ is ϕ or a singleton set. Now for any $\beta \in B$ and $\varphi^{-1}(\beta) = \alpha$ (say),

$$f_\varphi[(F, A)](\beta) = \begin{cases} \cup_{\alpha \in \varphi^{-1}(\beta)} f[F(\alpha)] = f[F(\alpha)] & \text{if } \varphi^{-1}(\beta)[= \{\alpha\} \text{ (say)}] \neq \phi \\ \phi & \text{otherwise} \end{cases}$$

Thus $f_\varphi[(F, A)](\beta)$ is a subgroup of Y, $\forall \ \beta \in B$ and hence $f_\varphi[(F, A)]$ is a soft group over (Y, B).

(viii) Let $(G_1, B) \stackrel{\sim}{\leq} (G_2, B)$ and f_φ is a soft homomorphism, then $G_1(\beta) \leq G_2(\beta)$, $\forall \beta \in B$ and $f_\varphi^{-1}[(G_1, B)]$, $f_\varphi^{-1}[(G_2, B)]$ are soft groups over (X, A). Now for any $\alpha \in A$, $f_\varphi^{-1}[(G_1, B)](\alpha) = f^{-1}(G_1[(\varphi(\alpha))] \leq f^{-1}(G_2[(\varphi(\alpha))]) = f_\varphi^{-1}[(G_2, B)](\alpha)$ and hence $f_\varphi^{-1}[(G_1, B)] \stackrel{\sim}{\leq} f_\varphi^{-1}[(G_2, B)]$.

(ix) Let $(F_1, A) \stackrel{\sim}{\leq} (F_2, A)$ and f_φ is a soft homomorphism such that φ is one-one, then $F_1(\alpha) \leq F_2(\alpha)$, $\forall \alpha \in A$ and $f_\varphi[(F_1, A)]$, $f_\varphi[(F_2, A)]$ are soft groups over (Y, B). Now for any $\beta \in B$,

$$\begin{aligned} f_\varphi[(F_1, A)](\beta) &= \begin{cases} \cup_{\alpha \in \varphi^{-1}(\beta)} f[F_1(\alpha)] & \text{if } \varphi^{-1}(\beta) \neq \phi \\ \phi & \text{otherwise} \end{cases} \\[2mm] &= \begin{cases} f[F_1(\alpha)] & \text{if } \varphi^{-1}(\beta)[= \{\alpha\} \text{ (say)}] \neq \phi \\ \phi & \text{otherwise} \end{cases} \\[2mm] &\leq \begin{cases} f[F_2(\alpha)] & \text{if } \varphi^{-1}(\beta) \neq \phi \\ \phi & \text{otherwise} \end{cases} \\[2mm] &= \begin{cases} \cup_{\alpha \in \varphi^{-1}(\beta)} f[F_2(\alpha)] & \text{if } \varphi^{-1}(\beta)[= \{\alpha\} \text{ (say)}] \neq \phi \\ \phi & \text{otherwise} \end{cases} \\[2mm] &= f_\varphi[(F_2, A)](\beta) \end{aligned}$$

Hence $f_\varphi[(F_1, A)] \stackrel{\sim}{\leq} f_\varphi[(F_2, A)]$.

(x) Proof is the same as part (viii).

(xi) Proof is the same as part (ix). □

Fuzzy soft groups:

In this section, the image and preimage of fuzzy soft sets under soft mapping are introduced and the behaviour of homomorphic image and preimage of fuzzy soft groups [12] under soft mapping are studied. Throughout this section X, Y are taken to be groups, e_X, e_Y are the identity elements of X, Y respectively, A be any non-empty set of parameters and $FS(X, A)$ be the family of all fuzzy soft sets over the soft universe (X, A).

Definition 3.21. [17] Let μ_1, μ_2 be two fuzzy subset of X. Then their union is denoted by $\mu_1 \cup \mu_2$ and defined by $(\mu_1 \cup \mu_2)(x) = \mu_1(x) \vee \mu_2(x)$, $\forall\, x \in X$.

Definition 3.22. [17] The fuzzy point p_x of X, is defined by the fuzzy subset μ where $\mu(x) = p$ and $\mu(x') = 0$, $\forall\, x'(\neq\ x) \in X$. Also $p_{\{X\}}$ is denoted by the fuzzy subset μ of X where $\mu(x) = p$, $\forall\, x \in X$.

Definition 3.23. [17] Let $f : X \to Y$ be a function and let μ, ν be fuzzy subsets over X, Y respectively. Then the fuzzy subsets $f(\mu) \in FP(Y)$ and $f^{-1}(\nu) \in FP(X)$ by
(a) the image of μ under f is the fuzzy subset $f(\mu)$ over Y and is defined by $\forall\, y \in Y$,

$$[f(\mu)](y) = \begin{cases} \vee\{\mu(x);\ f(x) = y,\ x \in X\} & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

(b) the preimage of ν under f is the fuzzy set $f^{-1}(\nu)$ over X and is defined by $\forall\, x \in X$, $[f^{-1}(\nu)](x) = \nu[f(x)]$.

Definition 3.24. [17] Let μ be a fuzzy subset of X. Then μ is called a *fuzzy subgroup* of X if
(1) $\mu(xy) \geq \mu(x) \wedge \mu(y)$, $\forall\, x,\ y \in X$ and
(2) $\mu(x^{-1}) \geq \mu(x)$, $\forall\, x \in X$.
If μ be a fuzzy subgroup of X. Then $\forall\, x \in X$,
(1) $\mu(e_X) \geq \mu(x)$,
(2) $\mu(x) = \mu(x^{-1})$.

Proposition 3.25. *[17] Let μ be a fuzzy subgroup of X and ν be a fuzzy subgroup of Y. Suppose that $f : X \to Y$ is a homomorphism. Then $f(\mu)$ is a fuzzy subgroup of Y and $f^{-1}(\nu)$ is a fuzzy subgroup of X.*

Definition 3.26. [12] A fuzzy soft set (F, A) in $FP(X, A)$ is said to be a *fuzzy soft group* over (X, A) iff $F(\alpha) \leq X$, i.e. $F(\alpha)$ is a fuzzy subgroup of X, $\forall\, \alpha \in A$.

Definition 3.27. [12] Let (F, A), (F_1, A) and (F_2, A) be fuzzy soft groups over (X, A). Then
(i) (F, A) is said to be *identity fuzzy soft group* over X if $F(\alpha) = 1_{\{e_X\}}, \forall \alpha \in A$.
(ii) (F, A) is said to be *absolute fuzzy soft group* over X if $F(\alpha) = 1_{\{X\}}, \forall \alpha \in A$.
(iii) (F_1, A) is said to be a *fuzzy soft subgroup* of (F_2, A), denoted by $(F_1, A) \widetilde{\leq} (F_2, A)$ if $F_1(\alpha) \leq F_2(\alpha)$, $\forall\, \alpha \in A$.
(iv) (F_1, A) is said to be a *fuzzy soft normal subgroup* of (F_2, A), denoted by $(F_1, A) \widetilde{\triangleleft} (F_2, A)$ if $F_1(\alpha) \triangleleft F_2(\alpha)$, $\forall\, \alpha \in A$.

Proposition 3.28. *[12]   (i) Let (F, A) be a fuzzy soft group over X and $f : X \to Y$ be a homomorphism. If $F(\alpha) = Kerf$, $\forall\, \alpha \in A$, then $(f(F), A)$ is an identity fuzzy soft group over Y where $f(F)(\alpha) = f(F(\alpha))$, $\forall\, \alpha \in A$.*

(ii) Let (F, A) be an absolute fuzzy soft group over X and $f : X \to Y$ be a onto homomorphism. Then $(f(F), A)$ is an absolute fuzzy soft group over Y where $f(F)(\alpha) = f(F(\alpha))$, $\forall\, \alpha \in A$.

Proposition 3.29. *[12] Let (F_1, A) and (F_2, A) be two fuzzy soft groups over X and $f : X \to Y$ be a homomorphism. Then*
(i) $(f(F_1), A)$ is a fuzzy soft group over Y;
(ii) If $(F_1, A) \widetilde{\leq} (F_2, A)$, then $(f(F_1), A) \widetilde{\leq} (f(F_2), A)$;
(iii) If $(F_1, A) \widetilde{\triangleleft} (F_2, A)$, then $(f(F_1), A) \widetilde{\triangleleft} (f(F_2), A)$.

Definition 3.30. Let f_φ is a soft mapping, where $f : X \to Y$ and $\varphi : A \to B$ are crisp mappings. Then

(i) the *image* of a fuzzy soft set $(F, A) \in FS(X, A)$ under the mapping f_φ is denoted by $f_\varphi[(F, A)] = (f_\varphi(F), B)$, and is defined by

$$[f_\varphi(F)](\beta) = \begin{cases} \bigcup_{\alpha \in \varphi^{-1}(\beta)} [f[F(\alpha)]] & \text{if } \varphi^{-1}(\beta) \neq \phi \\ 0_{\{Y\}} & \text{otherwise} \end{cases}$$

$\forall \, \beta \in B$.

(ii) the *inverse image* of a fuzzy soft set $(G, B) \in FS(Y, B)$ under the mapping f_φ is denoted by $f_\varphi^{-1}[(G, B)] = (f_\varphi^{-1}(G), A)$, and is defined by $[f_\varphi^{-1}(G)](\alpha) = f^{-1}[G[\varphi(\alpha)]]$, $\forall \, \alpha \in A$.

Proposition 3.31. *Let* (F, A), (F_1, A), $(F_2, A))$ *be fuzzy soft groups over* (X, A) *and* (G, B), (G_1, B), $(G_2, B))$ *be fuzzy soft groups over* (Y, B). *Also let* f_φ *be a soft mapping.*

(i) If (F, A) *be identity fuzzy soft group over* (X, A) *and* f_φ *is a soft homomorphism such that* φ *is onto, then* $f_\varphi[(F, A)]$ *is identity fuzzy soft group over* (Y, B).

(ii) If (G, B) *be identity fuzzy soft group over* (Y, B) *and* f_φ *is a soft homomorphism such that* $ker(f) = \{e_X\}$, *then* $f_\varphi^{-1}[(G, B)]$ *is identity fuzzy soft group over* (X, A).

(iii) If (F, A) *be absolute fuzzy soft group over* (X, A) *and* f_φ *is onto soft mapping, then* $f_\varphi[(F, A)]$ *is absolute fuzzy soft group over* (Y, B).

(iv) If (G, B) *be absolute fuzzy soft group over* (Y, B) *and* f_φ *is a soft mapping, then* $f_\varphi^{-1}[(G, B)]$ *is absolute soft group over* (X, A).

(v) If (G, B) *be a fuzzy soft group over* (Y, B) *and* f_φ *is a soft homomorphism, then* $f_\varphi^{-1}[(G, B)]$ *is a fuzzy soft group over* (X, A).

(vi) If (F, A) *be a fuzzy soft group over* (X, A) *and* f_φ *is a soft homomorphism such that* f *is onto and* φ *is one-one, then* $f_\varphi[(F, A)]$ *is a fuzzy soft group over* (Y, B).

(vii) If $(G_1, B) \widetilde{\leq} (G_2, B)$ *and* f_φ *is a soft homomorphism, then* $f_\varphi^{-1}[(G_1, B)] \widetilde{\leq} f_\varphi^{-1}[(G_2, B)]$.

(viii) If $(F_1, A) \widetilde{\leq} (F_2, A)$ *and* f_φ *is a soft homomorphism such that* f *is onto and* φ *is one-one, then* $f_\varphi[(F_1, A)] \widetilde{\leq} f_\varphi[(F_2, A)]$.

(ix) If $(G_1, B) \widetilde{\vartriangleleft} (G_2, B)$ *and* f_φ *is a soft homomorphism, then* $f_\varphi^{-1}[(G_1, B)] \widetilde{\vartriangleleft} f_\varphi^{-1}[(G_2, B)]$.

(x) If $(F_1, A) \widetilde{\vartriangleleft} (F_2, A)$ *and* f_φ *is a soft homomorphism such that* φ *is one-one, then* $f_\varphi[(F_1, A)] \widetilde{\vartriangleleft} f_\varphi[(F_2, A)]$.

Proof.   (i) Let (F, A) be identity fuzzy soft group over (X, A) and f_φ is a soft homomorphism such that φ is onto. Then $F(\alpha) = 1_{\{e_x\}}$, $\forall \, \alpha \in A$. Since f is a homomorphism, it follows that $f(e_X) = e_Y$ and for any $y \in Y$,

$$\{f[1_{\{e_x\}}]\}(y) = \begin{cases} \vee_{x \in f^{-1}(y)} [1_{\{e_x\}}](x) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{if otherwise} \end{cases} = \begin{cases} 1 & \text{if } y = e_Y \\ 0 & \text{if } y \neq e_Y \end{cases}$$

Again since φ is onto, it follows that $\varphi^{-1}(\beta) \neq \phi$, $\forall \, \beta \in B$ and hence for any $\beta \in B$,
$f_\varphi[(F, A)](\beta) = \cup_{\alpha \in \varphi^{-1}(\beta)} f[F(\alpha)] = \cup_{\alpha \in \varphi^{-1}(\beta)} f[1_{\{e_x\}}] = 1_{\{e_Y\}}$.
Therefore, $f_\varphi[(F, A)] = (\widetilde{1_{\{e_Y\}}}, B)$ and hence $f_\varphi[(F, A)]$ is identity fuzzy soft group over (Y, B).

(ii) Let (G, B) be identity fuzzy soft group over (Y, B) and f_φ is a soft homomorphism such that $ker(f) = \{e_X\}$. Then $G(\beta) = 1_{\{e_Y\}}$, $\forall \, \beta \in B$. Now for any $\alpha \in A$ and $x \in X$,

$$\{f^{-1}[G(\varphi(\alpha))]\}(x) = [G(\varphi(\alpha))](f(x)) = \begin{cases} [G(\varphi(\alpha))](e_Y) = 1 & \text{if } x = e_X \\ 0 & \text{if } x \neq e_X \end{cases}$$

and hence $\{f_\varphi^{-1}[(G, B)]\}(\alpha) = f^{-1}\{G[\varphi(\alpha)]\} = 1_{\{e_X\}}$.
So, $f_\varphi^{-1}[(G, B)] = (\widetilde{1_{\{e_X\}}}, A)$.
Therefore, $f_\varphi^{-1}[(G, B)]$ is identity fuzzy soft group over (X, A).

(iii) Let (F, A) be absolute fuzzy soft group over (X, A) and f_φ is a soft homomorphism such that f_φ is onto. Then $F(\alpha) = 1_{\{X\}}$, $\forall\, \alpha \in A$. Since f is onto, it follows that $f^{-1}(y) \neq \phi$, $\forall\, y \in Y$ and for any $y \in Y$,
$\{f[1_{\{X\}}]\}(y) = \vee_{x \in f^{-1}(y)}[1_{\{e_X\}}](x) = 1$
Again since φ is onto, it follows that $\varphi^{-1}(\beta) \neq \phi$, $\forall\, \beta \in B$ and hence for any $\beta \in B$,
$f_\varphi[(F, A)](\beta) = \cup_{\alpha \in \varphi^{-1}(\beta)} f[F(\alpha)] = \cup_{\alpha \in \varphi^{-1}(\beta)} f[1_{\{X\}}] = 1_{\{Y\}}$.
Therefore, $f_\varphi[(F, A)] = (\widetilde{1_{\{Y\}}}, B)$ and hence $f_\varphi[(F, A)]$ is absolute fuzzy soft group over (Y, B).

(iv) Let (G, B) be absolute fuzzy soft group over (Y, B) and f_φ is a soft mapping. Then $G(\beta) = 1_{\{Y\}}$, $\forall\, \beta \in B$. Now for any $\alpha \in A$ and $x \in X$,
$\{f^{-1}[G(\varphi(\alpha))]\}(x) = [G(\varphi(\alpha))](f(x)) = 1$.
Hence, $\{f_\varphi^{-1}[(G, B)]\}(\alpha) = f^{-1}\{G[\varphi(\alpha)]\} = 1_{\{X\}}$, $\forall\, \alpha \in A$.
So, $f_\varphi^{-1}[(G, B)] = (\widetilde{1_{\{X\}}}, A)$.
Therefore, $f_\varphi^{-1}[(G, B)]$ is absolute fuzzy soft group over (X, A).

(v) Let (G, B) be a fuzzy soft group over (Y, B) and f_φ is a soft homomorphism. Then $G(\beta)$ is a fuzzy subgroup of Y, $\forall\, \beta \in B$. Since f is a homomorphism, it follows that $f^{-1}[G(\beta)]$ is a fuzzy subgroup of X, $\forall\, \beta \in B$.
Hence, $\{f_\varphi^{-1}[(G, B)]\}(\alpha) = f^{-1}\{G[\varphi(\alpha)]\}$ is a fuzzy subgroup of X, $\forall\, \alpha \in A$.
Therefore, $f_\varphi^{-1}[(G, B)]$ is a fuzzy soft group over (X, A).

(vi) Let (F, A) be a fuzzy soft group over (X, A) and f_φ is a soft homomorphism such that f is onto and φ is one-one. Then $F(\alpha)$ is a fuzzy subgroup of X and hence $f[F(\alpha)]$ is a fuzzy subgroup of Y, $\forall\, \alpha \in A$. Again since φ is one-one, it follows that either $\varphi^{-1}(\beta)$ is ϕ or a singleton set. Now for any $\beta \in B$,

$$f_\varphi[(F, A)](\beta) = \begin{cases} \cup_{\alpha \in \varphi^{-1}(\beta)} f[F(\alpha)] = f[F(\alpha)] & \text{if } \varphi^{-1}(\beta)[= \{\alpha\}(\text{say})] \neq \phi \\ 0_{\{Y\}} & \text{otherwise} \end{cases}$$

Thus $f_\varphi[(F, A)](\beta)$ is a fuzzy subgroup of Y, $\forall\, \beta \in B$
therefore, $f_\varphi[(F, A)]$ is a fuzzy soft group over (Y, B).

(vii) Let (G_1, B), (G_2, B) be two fuzzy soft group over (Y, B) such that $(G_1, B) \widetilde{\leq} (G_2, B)$ and f_φ is a soft homomorphism. Then by part (v), we have $f_\varphi^{-1}[(G_1, B)]$, $f_\varphi^{-1}[(G_2, B)]$ are fuzzy soft groups over (X, A). Also $G_1(\beta) \leq G_2(\beta)$, $\forall\, \beta \in B$. Thus
$f_\varphi^{-1}[(G_1, B)](\alpha) = f^{-1}(G_1[(\varphi(\alpha)]) \leq f^{-1}(G_2[(\varphi(\alpha)]) = f_\varphi^{-1}[(G_2, B)](\alpha)$, $\alpha \in A$. Therefore, $f_\varphi^{-1}[(G_1, B)] \widetilde{\leq} f_\varphi^{-1}[(G_2, B)]$.

(viii) Let (F_1, A), (F_2, A) be two fuzzy soft groups over (X, A) such that $(F_1, A) \widetilde{\leq} (F_2, A)$ and f_φ is a soft homomorphism where f is onto and φ is one-one. Then by part (vi), $f_\varphi[(F_1, A)]$, $f_\varphi[(F_2, A)]$ are soft groups over (Y, B). Also $F_1(\alpha) \leq F_2(\alpha)$, $\forall\, \alpha \in A$ and Now for any $\beta \in B$,

$$
\begin{aligned}
f_\varphi[(F_1, A)](\beta) &= \begin{cases} \cup_{\alpha \in \varphi^{-1}(\beta)} f[F_1(\alpha)] & \text{if } \varphi^{-1}(\beta) \neq \phi \\ 0_{\{Y\}} & \text{otherwise} \end{cases} \\
&= \begin{cases} f[F_1(\alpha)] & \text{if } \varphi^{-1}(\beta)[= \{\alpha\}(\text{ say })] \neq \phi \\ 0_{\{Y\}} & \text{otherwise} \end{cases} \\
&\leq \begin{cases} f[F_2(\alpha)] & \text{if } \varphi^{-1}(\beta) \neq \phi \\ 0_{\{Y\}} & \text{otherwise} \end{cases} \\
&= \begin{cases} \cup_{\alpha \in \varphi^{-1}(\beta)} f[F_2(\alpha)] & \text{if } \varphi^{-1}(\beta)[= \{\alpha\}(\text{ say })] \neq \phi \\ 0_{\{Y\}} & \text{otherwise} \end{cases} \\
&= f_\varphi[(F_2, A)](\beta)
\end{aligned}
$$

Hence $f_\varphi[(F_1, A)] \widetilde{\leq} f_\varphi[(F_2, A)]$.
    (ix) Proof is the same as part (vii).

(x) Proof is the same as part (viii). □

4 Conclusion

In this paper, we have studied the detailed properties of soft functional image and preimage of soft sets, soft groups and fuzzy soft groups under soft mappings. This study has a great importance for further research in generalization of the different structures in soft set theory viz. soft topological group, soft topological vector space etc.

References

[1] H. Aktas and N. Cagman, *Soft sets and soft groups*, Inform. Sci., **177**, 2726-2735 (2007).

[2] M. Aslam and S. M. Qurashi, *Some contributions to soft groups*, Annl. Fuzzy Math. Inform., **4(1)**, 177-195 (2012).

[3] A. Aygunoglu and H. Aygun, *Some notes on soft topological spaces*, Neural Comput & Applic, **21** (Suppl 1), S113-S119 (2012).

[4] N. Cagman, S. Karatas and S. Enginoglu, *Soft topology*, Comput. Math. Appl., **62**, 351-358 (2011).

[5] , S. Das and S. K. samanta, *Soft metric*, Annl. Fuzzy Math. Inform., **6(1)**, 77-94 (2013).

[6] H. Hazra, P. Majumdar and S. K. Samanta, *soft topology*, Fuzzy Inform. Eng., **4(1)**, 105-115 (2012).

[7] A. Kharal and B. Ahmad, *Mappings on soft classes*, New Math. Nat. Comput., **7(3)** 471-481 (2010).

[8] P. K. Maji, R. Biswas and A. Roy, *Soft Set Theory*, Comput. Math. Appl., **45**, 555-562 (2003).

[9] P. Majumdar and S. K. Samanta, *On soft mappings*, Comput. Math. Appl. **60**, 2666-2672 (2010).

[10] P. Majumdar and S. K. Samanta, *Similarity measure of soft sets*, New Math. Nat. Comput., **4(1)**, 1-12 (2008).

[11] D. Molodtsov, *Soft Set Theory-First Results*, Comput. Math. Appl., **37**, 19-31 (1999).

[12] Sk. Nazmul and S. K. Samanta, *Fuzzy soft group*, J. Fuzzy Math., **19(1)**, 101-114 (2011).

[13] Sk. Nazmul and S. K. Samanta, *Intuitionistic L-Fuzzy soft sets and intuitionistic L-Fuzzy soft groups*, J. Fuzzy Math., **21(2)**, 371-386 (2013).

[14] Sk. Nazmul and S. K. Samanta, *Soft topological soft groups*, Mathematical Sciences, **6(66)**, 1-10 (2012).

[15] Sk. Nazmul and S. K. Samanta, *Generalized group soft topology*, Annl. Fuzzy Math. Inform., **9(5)**, 783-800 (2015).

[16] Sk. Nazmul and S. K. Samanta, *Some properties of soft topology and group soft topology*, Annl. Fuzzy Math. Inform., **8(4)**, 645-661 (2014).

[17] A. Rosenfeld, *Fuzzy groups*, J. Math. Anal. Appl., **35**, 512-517 (1971).

[18] M. Shabir and M. Naz, *On soft toplogical spaces*, Comput. Math. Appl., **61(7)**, 1786-1799 (2011).

[19] L. A. Zadeh, *Fuzzy sets*, Inform. Contrl, **8**, 338-353 (1965).

Author information

Sk. Nazmul, Department of Mathematics, Bankura University, Purandarpur, Bankura-722155, West Bengal, India.
E-mail: sk.nazmul₋math@yahoo.in