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ON THE STABILITY OF A FUNCTIONAL EQUATION[†]

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Abstract. In this paper, we study the stability of the functional equation

$$\sum_{i=1}^{n}\sum_{j=1}^{m} T(p_i q_j) = \sum_{i=1}^{n} T(p_i)\sum_{j=1}^{m} T(q_j) + (m-n)T(0)\sum_{j=1}^{m} T(q_j) + m(n-1)T(0)$$

in which $T : I \to \mathbb{R}$, $(p_1, \ldots, p_n) \in \Gamma_n$, $(q_1, \ldots, q_m) \in \Gamma_m$, $n \geq 3$, $m \geq 3$ being fixed integers.

1 Introduction

For $n = 1, 2, \ldots$; let $\Gamma_n = \{(p_1, \ldots, p_n) : p_i \geq 0,\ i = 1, \ldots, n; \sum_{i=1}^{n} p_i = 1\}$ denote the set of all n-component discrete probability distributions with nonnegative elements.

A mapping $a : \mathbb{R} \to \mathbb{R}$ is said to be additive on I or on the unit triangle $\Delta = \{(x, y) : 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 1,\ 0 \leqslant x + y \leqslant 1\}$ if it satisfies the equation $a(x + y) = a(x) + a(y)$ for all $(x, y) \in \Delta$; $I = \{x \in \mathbb{R} : 0 \leqslant x \leqslant 1\}$, $\mathbb{R}$ denoting the set of all real numbers. It is known [1] that if a mapping $a : I \to \mathbb{R}$ is additive on the unit triangle Δ, then there exists one and only one mapping $A : \mathbb{R} \to \mathbb{R}$ which is an extension of $a : I \to \mathbb{R}$ in the sense that $A(x) = a(x)$ for all $x \in I$ and is additive on $\mathbb{R}$, that is $A(x + y) = A(x) + A(y)$ for all $x \in \mathbb{R}$, $y \in \mathbb{R}$.

A mapping $M : I \to \mathbb{R}$ is said to be multiplicative if $M(pq) = M(p)\,M(q)$ for all $p \in I$, $q \in I$.

Suppose a mapping $T : I \to \mathbb{R}$ satisfies the functional equation

$$\sum_{i=1}^{n}\sum_{j=1}^{m} T(p_i q_j) = \sum_{i=1}^{n} T(p_i)\sum_{j=1}^{m} T(q_j) + (m-n)T(0)\sum_{j=1}^{m} T(q_j) + m(n-1)T(0) \tag{1.1}$$

for all $(p_1, \ldots, p_n) \in \Gamma_n$ and $(q_1, \ldots, q_m) \in \Gamma_m$; $n \geq 3$, $m \geq 3$ being fixed integers.

The functional equation (1.1) has been considered by Nath and Singh [6]. They determined its general solutions for fixed integers $n \geq 3$, $m \geq 3$.

The functional equation (1.1) plays an important role in finding the general solutions of several multiplicative and nonmultiplicative type sum form functional equations with atleast two unknown mappings (see [6] to [11]). Also, their solutions are related to the Shannon [13] entropy and the entropies of degree α [2].

Result 1.1 ([6]). Let $n \geqslant 3$, $m \geqslant 3$ be fixed integers. If a mapping $T : I \to \mathbb{R}$ satisfies the functional equation (1.1) for all $(p_1, \ldots, p_n) \in \Gamma_n$, $(q_1, \ldots, q_m) \in \Gamma_m$, then either

$$T(p) = a(p) + T(0)$$

where $a : \mathbb{R} \to \mathbb{R}$ is an additive mapping with

$$a(1) = \begin{cases} -mT(0) & \text{if } T(1) + (m-1)T(0) \neq 1 \\ 1 - mT(0) & \text{if } T(1) + (m-1)T(0) = 1 \end{cases}$$

or

$$T(p) = M(p) - b(p) + T(0)$$

in which $b : \mathbb{R} \to \mathbb{R}$ is an additive mapping with $b(1) = mT(0)$ and $M : I \to \mathbb{R}$ is a nonadditive multiplicative mapping with $M(0) = 0$, $M(1) = 1$.

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This paper deals with the stability of the sum form functional equation (1.1). For the meaning of stability of a functional equation, see Hyers and Rassias [3]. By the stability problem for the equation (1.1), we mean the following: Let $n \geq 3$, $m \geq 3$ be fixed integers and $0 \leq \epsilon \in \mathbb{R}$ be a fixed real number. Find all mappings $T : I \to \mathbb{R}$ satisfying the functional inequality

$$|\sum_{i=1}^{n}\sum_{j=1}^{m} T(p_i q_j) - \sum_{i=1}^{n} T(p_i)\sum_{j=1}^{m} T(q_j) - (m-n)T(0)\sum_{j=1}^{m} T(q_j) - m(n-1)T(0)| \leq \epsilon \quad (1.2)$$

for all $(p_1, \ldots, p_n) \in \Gamma_n$, $(q_1, \ldots, q_m) \in \Gamma_m$.

Now, we mention below some results needed for the development of the main result of this paper.

Result 1.2 ([4]). Let c be a given real constant. Suppose $\phi : I \to \mathbb{R}$ is a mapping which satisfies the functional equation $\sum_{i=1}^{n} \phi(p_i) = c$ for all $(p_1, \ldots, p_n) \in \Gamma_n$, $n \geq 3$ a fixed integer. Then there exists an additive mapping $a : \mathbb{R} \to \mathbb{R}$ such that $\phi(p) = a(p) - \frac{1}{n}a(1) + \frac{c}{n}$ for all $p \in I$.

Result 1.3 ([5]). Let $n \geqslant 3$ be a fixed integer and ϵ be a fixed nonnegative real number. Suppose a mapping $\psi : I \to \mathbb{R}$ satisfies the functional inequality $|\sum_{i=1}^{n} \psi(p_i)| \leq \epsilon$ for all $(p_1, \ldots, p_n) \in \Gamma_n$. Then there exist an additive mapping $A : \mathbb{R} \to \mathbb{R}$ and a bounded mapping $B : \mathbb{R} \to \mathbb{R}$ satisfying the conditions $B(0) = 0$ and $|B(p)| \leq 18\epsilon$ such that $\psi(p) - \psi(0) = A(p) + B(p)$ for all $p \in I$.

2 The Main Result

Theorem 2.1. *Let $n \geqslant 3$, $m \geqslant 3$ be fixed integers and ϵ be a given nonnegative real constant. Suppose the mapping $T : I \to \mathbb{R}$ satisfies the inequality* (1.2) *for all $(p_1, \ldots, p_n) \in \Gamma_n$ and $(q_1, \ldots, q_m) \in \Gamma_m$. Then either*

$$T(p) = a(p) + b(p) \quad (2.1)$$

for all $p \in I$ or

$$T(p) = M(p) - B(p) + T(0) \quad (2.2)$$

for all $p \in I$; where $a : \mathbb{R} \to \mathbb{R}$, $B : \mathbb{R} \to \mathbb{R}$ are additive mappings; $B(1) = mT(0)$; $b : \mathbb{R} \to \mathbb{R}$ is a bounded mapping; and $M : I \to \mathbb{R}$ is a multiplicative mapping which is not additive and $M(0) = 0$, $M(1) = 1$.

Proof. Let us write (1.2) in the form

$$|\sum_{i=1}^{n}\{\sum_{j=1}^{m} T(p_i q_j) - T(p_i)\sum_{j=1}^{m} T(q_j) - (m-n)T(0)p_i\sum_{j=1}^{m} T(q_j) - m(n-1)T(0)p_i\}| \leq \epsilon.$$

By Result 1.3, there exist a mapping $A_1 : \mathbb{R} \times \Gamma_m \to \mathbb{R}$ additive in the first variable and a bounded mapping $b_1 : \mathbb{R} \times \Gamma_m \to \mathbb{R}$ with $b_1(0; q_1, \ldots, q_m) = 0$ and $|b_1(x; q_1, \ldots, q_m)| \leq 18\epsilon$ for all $x \in \mathbb{R}$ such that

$$\begin{aligned} &\sum_{j=1}^{m} T(pq_j) - T(p)\sum_{j=1}^{m} T(q_j) - (m-n)\,T(0)\,p\sum_{j=1}^{m} T(q_j) - m(n-1)T(0)p \\ &- mT(0) + T(0)\sum_{j=1}^{m} T(q_j) = A_1(p; q_1, \ldots, q_m) + b_1(p; q_1, \ldots, q_m) \end{aligned} \quad (2.3)$$

for all $p \in I$. Let $x \in I$ and $(r_1, \ldots, r_m) \in \Gamma_m$. Putting successively $p = xr_t$, $t = 1, \ldots, m$ in

(2.3); adding the resulting m equations and using the additivity of A_1, we obtain

$$\begin{aligned}
&\sum_{t=1}^{m}\sum_{j=1}^{m} T(xr_tq_j) - \sum_{t=1}^{m} T(xr_t)\sum_{j=1}^{m} T(q_j) - (m-n)\,T(0)\,x\sum_{j=1}^{m} T(q_j)\\
&- \; m(n-1)T(0)x - m^2T(0) + mT(0)\sum_{j=1}^{m} T(q_j)\\
&= \; A_1(x;q_1,\ldots,q_m) + \sum_{t=1}^{m} b_1(xr_t;q_1,\ldots,q_m). \qquad (2.4)
\end{aligned}$$

Now put $p = x$, $q_1 = r_1, \ldots, q_m = r_m$ in (2.3). We obtain

$$\begin{aligned}
\sum_{t=1}^{m} T(xr_t) &= T(x)\sum_{t=1}^{m} T(r_t) + (m-n)\,T(0)\,x\sum_{t=1}^{m} T(r_t) + m(n-1)T(0)x\\
&+ \; mT(0) - T(0)\sum_{t=1}^{m} T(r_t) + A_1(x;r_1,\ldots,r_m) + b_1(x;r_1,\ldots,r_m). \qquad (2.5)
\end{aligned}$$

From (2.4) and (2.5), it follows that

$$\begin{aligned}
&\sum_{t=1}^{m}\sum_{j=1}^{m} T(xr_tq_j) - [T(x) + (m-n)\,T(0)\,x - T(0)]\sum_{t=1}^{m} T(r_t)\sum_{j=1}^{m} T(q_j) - m(n-1)T(0)x\\
&- \; m^2T(0) = [n(m-1)T(0)x + A_1(x;r_1,\ldots,r_m) + b_1(x;r_1,\ldots,r_m)]\sum_{j=1}^{m} T(q_j)\\
&\qquad + \; A_1(x;q_1,\ldots,q_m) + \sum_{t=1}^{m} b_1(xr_t;q_1,\ldots,q_m). \qquad (2.6)
\end{aligned}$$

The left hand side of (2.6) is symmetric in q_j and r_t; $j = 1,\ldots,m$; $t = 1,\ldots,m$. So, the right hand side of (2.6) should also be symmetric in q_j and r_t; $j = 1,\ldots,m$; $t = 1,\ldots,m$. This fact gives rise to the equation

$$\begin{aligned}
&[n(m-1)T(0)x + A_1(x;q_1,\ldots,q_m)][\sum_{t=1}^{m} T(r_t) - 1]\\
&\quad - \; [n(m-1)T(0)x + A_1(x;r_1,\ldots,r_m)][\sum_{j=1}^{m} T(q_j) - 1]\\
&\quad = \; b_1(x;r_1,\ldots,r_m)\sum_{j=1}^{m} T(q_j) + \sum_{t=1}^{m} b_1(xr_t;q_1,\ldots,q_m)\\
&\quad - \; b_1(x;q_1,\ldots,q_m)\sum_{t=1}^{m} T(r_t) - \sum_{j=1}^{m} b_1(xq_j;r_1,\ldots,r_m). \qquad (2.7)
\end{aligned}$$

For fixed $(q_1,\ldots,q_m) \in \Gamma_m$, $(r_1,\ldots,r_m) \in \Gamma_m$, the right hand side of (2.7) is a bounded mapping of x, $x \in I$. On the other hand, the left hand side is additive in x, $x \in I$. By using

Theorem 1.8 (see p-14 in [12]) and the Definition 1.2 (see p-4 in [12]), we have

$$[n(m-1)T(0)x + A_1(x;q_1,\ldots,q_m)][\sum_{t=1}^{m} T(r_t)-1]$$

$$- [n(m-1)T(0)x + A_1(x;r_1,\ldots,r_m)][\sum_{j=1}^{m} T(q_j)-1]$$

$$= x\{[n(m-1)T(0) + A_1(1;q_1,\ldots,q_m)][\sum_{t=1}^{m} T(r_t)-1]$$

$$- [n(m-1)T(0) + A_1(1;r_1,\ldots,r_m)][\sum_{j=1}^{m} T(q_j)-1]\}$$

which, on simplification, reduces to

$$A_1(x;q_1,\ldots,q_m) - xA_1(1;q_1,\ldots,q_m)][\sum_{t=1}^{m} T(r_t)-1]$$

$$= [A_1(x;r_1,\ldots,r_m) - xA_1(1;r_1,\ldots,r_m)][\sum_{j=1}^{m} T(q_j)-1]. \tag{2.8}$$

Now we divide our discussion into two cases:

Case 1. $\sum\limits_{t=1}^{m} T(r_t) - 1$ vanishes identically on Γ_m, that is,

$$\sum_{t=1}^{m} T(r_t) - 1 = 0$$

for all $(r_1,\ldots,r_m) \in \Gamma_m$. By Result 1.2, there exists an additive mapping $a : \mathbb{R} \to \mathbb{R}$ such that, for all $p \in I$,

$$T(p) = a(p) + T(0) \tag{2.9}$$

with $a(1) = 1 - mT(0)$. The solution (2.9) is included in (2.1) on defining a constant bounded mapping $b : \mathbb{R} \to \mathbb{R}$ as $b(p) = T(0)$.

Case 2. $\sum\limits_{t=1}^{m} T(r_t) - 1$ does not vanish identically on Γ_m.

In this case, there exists a probability distribution $(r_1^*,\ldots,r_m^*) \in \Gamma_m$ such that

$$\sum_{t=1}^{m} T(r_t^*) - 1 \neq 0\,. \tag{2.10}$$

Putting $r_1 = r_1^*,\ldots,r_m = r_m^*$ in (2.8) and making use of (2.10), it follows that

$$A_1(x;q_1,\ldots,q_m) = A_2(x)\left[\sum_{j=1}^{m} T(q_j) - 1\right] + xA_1(1;q_1,\ldots,q_m) \tag{2.11}$$

where $A_2 : \mathbb{R} \to \mathbb{R}$ is defined as

$$A_2(x) = \left[\sum_{t=1}^{m} T(r_t^*) - 1\right]^{-1} [A_1(x;r_1^*,\ldots,r_m^*) - xA_1(1;r_1^*,\ldots,r_m^*)] \tag{2.12}$$

for all $x \in \mathbb{R}$. The mapping A_2 is additive and $A_2(1) = 0$. Putting $p = 1$ in (2.3), we obtain

$$A_1(1;q_1,\ldots,q_m) = [1 - T(1) - (m-n)T(0) + T(0)]\sum_{j=1}^{m} T(q_j)$$

$$- mnT(0) - b_1(1;q_1,\ldots,q_m). \tag{2.13}$$

From (2.7), (2.11) and (2.13), we have

$$\{b_1(x;q_1,\ldots,q_m) - xb_1(1;q_1,\ldots,q_m) + x[1-T(1)-(m-1)T(0)]\}\sum_{t=1}^{m} T(r_t)$$
$$= \{b_1(x;r_1,\ldots,r_m) - xb_1(1;r_1,\ldots,r_m) + x\,[1-T(1)-(m-1)T(0)]\}\sum_{j=1}^{m} T(q_j)$$
$$+ \left[\sum_{t=1}^{m} b_1(xr_t;q_1,\ldots,q_m) - \sum_{j=1}^{m} b_1(xq_j;r_1,\ldots,r_m)\right]$$
$$- x\,[b_1(1;q_1,\ldots,q_m) - b_1(1;r_1,\ldots,r_m)] \quad (2.14)$$

for all $x \in I$, $(r_1,\ldots,r_m) \in \Gamma_m$ and $(q_1,\ldots,q_m) \in \Gamma_m$.

Case 2.1. The coefficient of $\sum_{t=1}^{m} T(r_t)$, in (2.14), does not vanish identically on $I \times \Gamma_m$.

In this case, there exist an element $x^* \in I$ and a probability distribution $(q_1^*,\ldots,q_m^*) \in \Gamma_m$ such that

$$\{b_1(x^*;q_1^*,\ldots,q_m^*) - x^* b_1(1;q_1^*,\ldots,q_m^*) + x^*\,[1-T(1)-(m-1)T(0)]\} \neq 0. \quad (2.15)$$

From (2.14), (2.15) and the boundedness of b_1, it follows that $|\sum_{t=1}^{m} T(r_t)| \leq \epsilon^*$ for some nonnegative real number ϵ^*. So, by Result 1.3, there exist an additive mapping $a : \mathbb{R} \to \mathbb{R}$ and a bounded mapping $b_2 : \mathbb{R} \to \mathbb{R}$ such that $T(p) - T(0) = a(p) + b_2(p)$ for all $p \in I$. This solution is included in (2.1) on defining a bounded mapping $b : \mathbb{R} \to \mathbb{R}$ as $b(p) = b_2(p) + T(0)$.

Case 2.2. The coefficient of $\sum_{t=1}^{m} T(r_t)$, in (2.14), vanishes identically on $I \times \Gamma_m$, that is,

$$b_1(x;q_1,\ldots,q_m) - xb_1(1;q_1,\ldots,q_m) + x\,[1-T(1)-(m-1)T(0)] = 0 \quad (2.16)$$

for all $x \in I$ and $(q_1,\ldots,q_m) \in \Gamma_m$.

From (2.11) and (2.13), we obtain

$$A_1(x;q_1,\ldots,q_m) = A_2(x)\left[\sum_{j=1}^{m} T(q_j) - 1\right] + x\{[1-T(1)-(m-n)T(0)$$
$$+ T(0)]\sum_{j=1}^{m} T(q_j) - mnT(0) - b_1(1;q_1,\ldots,q_m)\}. \quad (2.17)$$

Now, from (2.3), (2.16) and (2.17), one can derive

$$\sum_{j=1}^{m}[T(pq_j) + A_2(pq_j) + \{1-T(1)+T(0)\}\,pq_j - T(0)]$$
$$- [T(p) + A_2(p) + \{1-T(1)+T(0)\}p - T(0)]$$
$$\times \sum_{j=1}^{m}[T(q_j) + A_2(q_j) + \{1-T(1)+T(0)\}q_j - T(0)]$$
$$+ [T(p)+A_2(p)+\{1-T(1)+T(0)\}p-T(0)][1-T(1)-(m-1)T(0)]=0. \quad (2.18)$$

The substitution $p = 1$ in (2.18) gives (using $A_2(1) = 0$):

$$1 - T(1) + T(0) = mT(0). \quad (2.19)$$

Now, equations (2.18) and (2.19) give rise to

$$\sum_{j=1}^{m}[T(pq_j)+A_2(pq_j)+mT(0)\,pq_j-T(0)]$$
$$-\ [T(p)+A_2(p)+mT(0)p-T(0)]\sum_{j=1}^{m}[T(q_j)+A_2(q_j)+mT(0)q_j-T(0)]=0. \quad (2.20)$$

Define a mapping $M: I \to \mathbb{R}$ as

$$M(x)=T(x)+A_2(x)+mT(0)\,x-T(0) \quad (2.21)$$

for all $x \in I$. Putting $x=0$ and $x=1$ respectively in (2.20) and using the fact that $A_2(1)=0$, we obtain

$$M(0)=0, M(1)=1. \quad (2.22)$$

Also (2.20) and (2.21) give

$$\sum_{j=1}^{m}[M(pq_j)-M(p)\,M(q_j)]=0.$$

By Result 1.2, there exists a mapping $E: I\times\mathbb{R}\to\mathbb{R}$, additive in second variable, such that

$$M(pq)-M(p)\,M(q)=E(p;q) \quad (2.23)$$

for all $p\in I$, $q\in I$ and $E(p;1)=0$. The symmetry of the left hand side of (2.23), in p and q, gives $E(p;q)=E(q;p)$ for all $p\in I$, $q\in I$. Consequently, E is also additive in the first variable. We may suppose that $E(\cdot\,;q)$ has been extended additively to the whole of $\mathbb{R}$.

For all $p,q,r\in I$, (2.23) gives

$$\begin{aligned} M(pqr)-M(p)\,M(q)\,M(r) &= E(pq;r)+M(r)\,E(p;q) \\ &= E(qr;p)+M(p)\,E(q;r). \end{aligned} \quad (2.24)$$

Now, we prove that $E(p;q)\equiv 0$ on $I\times I$. To the contrary, suppose that $E(p;q)\not\equiv 0$ on $I\times I$. Then, there exist $p^*\in I$ and $q^*\in I$ such that $E(p^*;q^*)\neq 0$. Substituting $p=p^*$, $q=q^*$ in (2.24) and using $E(p^*;q^*)\neq 0$, it follows that

$$M(r)=\left[E(p^*;q^*)\right]^{-1}\left[E(q^*r;p^*)+M(p^*)\,E(q^*;r)-E(p^*q^*;r)\right] \quad (2.25)$$

for all $r\in I$. The right hand side of (2.25) is additive. Hence M is also additive. Now, making use of (2.10), (2.21), (2.22) and the fact that $A_2(1)=0$, we have

$$1\neq\sum_{t=1}^{m}T(r_t^*)=\sum_{t=1}^{m}M(r_t^*)-A_2(1)-mT(0)+mT(0)=M(1)=1,$$

a contradiction. Hence our supposition "$E(p;q)\not\equiv 0$ on $I\times I$" is false. So, $E(p;q)=0$ for all $p\in I$, $q\in I$. Making use of this fact in (2.23), we conclude that M, defined by (2.21), is multiplicative with $M(0)=0$ and $M(1)=1$.

From (2.21), we have $T(x)=M(x)-A_2(x)-mT(0)\,x+T(0)$. Define a mapping $B:\mathbb{R}\to\mathbb{R}$ as $B(x)=A_2(x)+m\,T(0)\,x$ for all $x\in I$. Then B is additive with $B(1)=mT(0)$. Thus, we have obtained the solution (2.2).

If the multiplicative mapping $M: I\to\mathbb{R}$, with $M(0)=0$, $M(1)=1$, appearing in the solution (2.2), is also additive, then M is only of the form $M(p)=p$ for all $p\in I$. So, (2.2) reduces to $T(p)=p-B(p)+T(0)$. Making use of (2.10), we have

$$1\neq\sum_{t=1}^{m}T(r_t^*)=\sum_{t=1}^{m}[r_t^*-B(r_t^*)+T(0)]=1-B(1)+mT(0)=1,$$

a contradiction. Hence M is not additive. This completes the proof of the theorem.

□

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