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A Study of Fuzzy Ideals in PO-Gamma-Semigroups

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Abstract. In this paper, we characterize the relationship between fuzzy ideals, fuzzy interior ideals, fuzzy bi-ideals and the characteristic function of fuzzy ideals in PO-Γ-semigroups. Also we proved the equivalent statements, necessary and sufficient conditions on partial ordered Γ-semigroups.

1 Introduction

The important concept of fuzzy set has been introduced by Lofti. A. Zadeh [22]. Since then many papers on fuzzy sets appeared showing its important fields of mathematics. Rosefneld [17] introduced the concept of fuzzy group. Semigroup is an algebraic structure consisting of a non-empty set S together with an associative binary operation. The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. Kuroki [8, 9, 10] characterized several class of semigroups in terms of fuzzy left, right and fuzzy bi-ideals. In [20,21] X. Y. Xie introduced the ideal extensions of fuzzy ideals in semigroups. The idea of fuzzy bi-ideals in semigroups has been introduced by W. J. Lie [12].
In 1984 the notion of Γ-semigroup was introduced M. K. Sen in [14]. In 1986 M. K. Sen and N. K. Saha [15] modified the definition of sen's Γ-semigroups. This newly defined Γ-semigroup is known as one sided Γ-semigroup. Γ-semigroup have been analyzed by a lot of mathematicians, for instance by Chattopadhyay [1], T. K. Dutta and N. C. Adhikari [2,3]. They defined operator semigroups of such type of Γ-semigroups and established many results and obtained many correspondence between a Γ-semigroups. In this paper we have considered both sided Γ-semigroups. N. Kehayopulu and M. Tsingelis [7] introduced the notion of fuzzy bi-ideals in PO-Γ-semigroups. S. K. Lee and J. H. Jung [11] introduced the notion of PO-semigroups and studied its related properties and interior ideals in PO-Γ-semigroups have been introduced. The notion of ordered Γ-semigroups have been introduced and studied varies properties by A. Iampan, N. Siripitukdlet and A. Kanlaya [4,5,6]. S. K. Mujemder and S. K. Sardar studied the properties of PO-Γ-semigroups in terms fuzzy ideals and fuzzy interior ideals [13]. Prince williams, Latha and Chandrasekeran [16] studied the notion of fuzzification of bi-ideals in Γ-semigroups and investigate some of their related properties. In this paper we studied some properties of fuzzy interior ideals of PO-Γ-semigroups.

2 Preliminaries

Definition 2.1. [4] Let S and Γ be two non-empty sets. Then S is called a Γ-*semigroup* if there exists a mapping from $S \times \Gamma \times S \to S$ written as $(a, \alpha, b) \mapsto a\alpha b$ satisfying the identity $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$.

Definition 2.2. [13] Let S be a Γ-semigroup. By Γ-*subsemigroup* of S we mean a non-empty subset A of S such that $A\Gamma A \subseteq A$.

Definition 2.3. [4] A Γ-semigroup S is called a *PO-Γ-semigroup* if for any $a, b, c \in S$ and for $\alpha \in \Gamma, a \leq b$ implies $a\alpha c \leq b\alpha c$ and $c\alpha a \leq c\alpha b$.

Definition 2.4. [18] Let S be a PO-Γ-semigroup. A non-empty subset A of S is said to be *right* (resp. left) *ideal* of S if
(i) $A\Gamma S \subseteq A$ (resp. $S\Gamma A \subseteq A$),
(ii) if $x \in A$ and $y \in S$ such that $y \leq x$, then $y \in A$.

Definition 2.5. [18] Let S be an PO-Γ-semigroup. A Γ-subsemigroup A of S is said to be *bi-ideal* of S if
(i) $A\Gamma S\Gamma A \subseteq A$,
(ii) if $x \in A$ and $y \in S$ such that $y \leq x$, then $y \in A$.

Definition 2.6. [13] Let S be a PO-Γ-semigroup. A Γ-subsemigroup A of S is said to be *interior ideal* of S if
(i) $S\Gamma A\Gamma S \subseteq A$,
(ii) if $x \in A$ and $y \in S$ such that $y \leq x$, then $y \in A$.

Definition 2.7. A fuzzy subset μ of a non-empty set X is a function $\mu : X \to [0, 1]$.

Definition 2.8. [4] Let S be non-empty set and $A \subseteq S$. The characteristic mapping $\chi_A : S \to [0, 1]$ defined via

$$x \mapsto \chi_A(x) := \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

By the definition of a characteristic mapping, χ_A is a mapping of S into $\{0, 1\} \subset [0, 1]$. Hence χ_A is a fuzzy subset of S.

Definition 2.9. [12] A fuzzy subset μ of a PO-Γ-semigroup S is called a *fuzzy Γ-subsemigroup* of S if

$$\mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\} \text{ for all } x, y \in S \text{ and } \alpha \in \Gamma.$$

Definition 2.10. [5] A fuzzy subset μ of a PO-Γ-semigroup S is called a *fuzzy right* (resp. left) *ideal* of S if
(i) $x \leq y \Rightarrow \mu(x) \geq \mu(y)$ for all $x, y \in S$, and
(ii) $\mu(x\alpha y) \geq \mu(x)$ (resp. $\mu(x\alpha y) \geq \mu(y)$) for all $x, y \in S$ and $\alpha \in \Gamma$.
A fuzzy subset μ of a PO-Γ-semigroup S is called a *fuzzy ideal* of S, if it is both fuzzy left ideal and fuzzy right ideal.

Example 2.11. Let S be the set of all non-positive integers without zero and Γ be the set of all non-positive even integers without zero.Then S is a Γ-semigroup where $x\alpha y$ denote the usual multiplication of integers x, α, y with $x, y \in S$ and $\alpha \in \Gamma$. Then the routine calculation shows that S is a PO-Γ-semigroup. Let μ be fuzzy subset of S defined as follows:

$$\mu(x) = \begin{cases} 0.1 & \text{if } \mathrm{x = -1} \\ 0.3 & \text{if } x = -2 \\ 0.5 & \text{if } x < -2 \end{cases}$$

for each $x \in S$.
It is easy to verify that μ is fuzzy ideal of a PO-Γ-semigroup S.

Definition 2.12. [18] A fuzzy Γ-subsemigroup μ of a PO-Γ-semigroup S is called a *fuzzy bi-ideal* of S if
(i) $x \leq y \Rightarrow \mu(x) \geq \mu(y)$ for all $x, y \in S$, and
(ii) $\mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(z)\}$ for all $x, y, z \in S$ and $\alpha, \beta \in \Gamma$.

Definition 2.13. Let S be a PO-Γ-semigroup and μ, λ be two fuzzy subsets of S. Then the product $\mu\Gamma\lambda$ of μ and λ is defined as

$$(\mu\Gamma\lambda)(x) = \begin{cases} \sup\{\min\{\mu(y), \lambda(z)\}\} & \text{if } x = y\alpha z \text{ for } y, z \in S \text{ and } \alpha \in \Gamma, \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.14. [13] A fuzzy Γ-subsemigroup μ of a PO-Γ-semigroup S is called a *fuzzy interior ideal* of S if
(ii) $x \leq y \Rightarrow \mu(x) \geq \mu(y)$ for all $x, y \in S$, and
(i) $\mu(x\alpha a\beta y) \geq \mu(a)$ for all $x, y, a \in S$ and $\alpha, \beta \in \Gamma$.

3 Main results

Proposition 3.1. *[5] Let S be a PO-Γ-semigroup and $\emptyset \neq A \subseteq S$. Then $A = (A]$ if and only if the fuzzy subset χ_A of S has the following property:*

$$x, y \in S, x \leq y \Rightarrow f_A(x) \geq f_A(y).$$

Theorem 3.2. *[5] Let A be a non-empty subset of a PO-Γ-semigroup S and χ_A be the characteristic function of A. Then A is a left ideal (right ideal, ideal) of S if and only if χ_A is a fuzzy left ideal (resp. fuzzy right ideal, fuzzy ideal) of S.*

Theorem 3.3. *[6] Let A be a non-empty subset of a PO-Γ-semigroup S and χ_A be the characteristic function of A. Then A is an interior ideal of S if and only if χ_A is a fuzzy interior ideal of S.*

Theorem 3.4. *[6] Let A be a non-empty subset of a PO-Γ-semigroup S and χ_A be the characteristic function of A. Then A is a bi-ideal of S if and only if χ_A is a fuzzy bi-ideal of S.*

Theorem 3.5. *A fuzzy subset μ of a PO-Γ-semigroup S is a fuzzy Γ-subsemigroup of S if and only if $\mu\Gamma\mu \subseteq \mu$.*

Proof. Suppose μ is a fuzzy Γ-subsemigroup of S. For any $x \in S$,

$$(\mu\Gamma\mu)(x) = \begin{cases} \sup\{\min\{\mu(y), \mu(z)\}\} & \text{if } x = y\alpha z \text{ for } y, z \in S \text{ and } \alpha \in \Gamma, \\ 0 & \text{otherwise.} \end{cases}$$

If $(\mu\Gamma\mu)(x) = 0$, then $(\mu\Gamma\mu)(x) \leq \mu(x)$. Let $(\mu\Gamma\mu)(x) = \sup\limits_{x=y\alpha z} \{\min\{\mu(y), \mu(z)\}\}$. Since μ is a fuzzy Γ-subsemigroup of S, we have $\mu(y\alpha z) \geq \min\{\mu(y), \mu(z)\}$ for all $y, z \in S$ and $\alpha \in \Gamma$. In particular, $\mu(x) = \mu(y\alpha z) \geq \min\{\mu(y), \mu(z)\}$ for all $y, z \in S$ and $\alpha \in \Gamma$ with $x = y\alpha z$. Thus $\mu(x) \geq \sup\limits_{x=y\alpha z} \{\min\{\mu(y), \mu(z)\}\} = (\mu\Gamma\mu)(x)$. This implies $(\mu\Gamma\mu)(x) \leq \mu(x)$. Hence, $\mu\Gamma\mu \subseteq \mu$.
Conversely, let $\mu\Gamma\mu \subseteq \mu$. Then for any $x, y \in S$ and $\alpha \in \Gamma$, we have $\mu(x\alpha y) \geq (\mu\Gamma\mu)(x\alpha y) \geq \min\{\mu(x), \mu(y)\}$. Hence, μ is a fuzzy Γ-subsemigroup of S. □

Theorem 3.6. *Every fuzzy ideal of a PO-Γ-semigroup is fuzzy bi-ideal of a PO-Γ-semigroup.*

Proof. Let S be a PO-Γ-semigroup and μ be a fuzzy ideal of S. For any $x, y \in S$ with $x \leq y$, $\mu(x) \geq \mu(y)$.
Case(i): Suppose μ is fuzzy left ideal of a PO-Γ-semigroup S. Then $\mu(x\alpha y) \geq \mu(y)$ for all $x, y \in S$ and $\alpha \in \Gamma$. For any $x, y, z \in S$ and $\alpha, \beta \in \Gamma$, we have
$\mu(x\alpha y\beta z) = \mu(x\alpha(y\beta z)) \geq \mu(y\beta z) \geq \mu(z)$.

Case(ii): Suppose μ is fuzzy right ideal of a PO-Γ-semigroup S. Then $\mu(x\alpha y) \geq \mu(x)$ for all $x, y \in S$ and $\alpha \in \Gamma$. For any $x, y, z \in S$ and $\alpha, \beta \in \Gamma$, we have
$\mu(x\alpha y\beta z) = \mu(x\alpha(y\beta z)) = \mu((x\alpha y)\beta z) \geq \mu(x\alpha y) \geq \mu(x)$.
From the both cases, we have $\mu(x\alpha y) \geq \mu(x) \wedge \mu(y) = \min\{\mu(x), \mu(y)\}$ and $\mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(z)\}$ for all $x, y, z \in S$ and $\alpha, \beta \in \Gamma$. Hence μ is fuzzy bi-ideal of S. This completes the proof. □

Proposition 3.7. *[6] Let S be a PO-Γ-semigroup and $\{f_i\}_{i \in I}$ a nonempty family of fuzzy subsets of S. Then $\bigwedge_{i \in I} f_i$ is a fuzzy subset of S.*

Proposition 3.8. *Let S be a PO-Γ-semigroup and $\{f_i\}_{i \in I}$ a nonempty family of fuzzy subsets of S. Then $\bigvee_{i \in I} f_i$ is a fuzzy subset of S.*

Proof. Let $x \in M$. Then the set $\{f_i(x)\}_{i\in I}$ is a nonempty bounded above subset of $\mathbb{R}$ By the Completeness axiom, there exists the $\sup\{f_i(x)\}_{i\in I}$ in $\mathbb{R}$. Since $0 \leq f_i(x) \leq 1$ for each $i \in I$, we have $0 \leq \sup\{f_i(x)\}_{i\in I} \leq 1$. Thus $0 \leq (\bigvee_{i\in I} f_i)(x) \leq 1$. If $x, y \in S$ is such that $x = y$, then $\{f_i(x)\}_{i\in I} = \{f_i(y)\}_{i\in I}$. Thus $\sup\{f_i(x)\}_{i\in I} = \sup\{f_i(y)\}_{i\in I}$, so $(\bigvee_{i\in I} f_i)(x) = (\bigvee_{i\in I} f_i)(y)$. Hence $\bigvee_{i\in I} f_i$ is a fuzzy subset of S. □

Proposition 3.9. *[6] Let S be a PO-Γ-semigroup and $\{f_i\}_{i\in I}$ a family of fuzzy Γ-subsemigroups of S. Then $\bigwedge_{i\in I} f_i$ is a fuzzy Γ-subsemigroup of S.*

Theorem 3.10. *Let S be a PO-Γ-semigroup and $\{f_i\}_{i\in I}$ a family of fuzzy bi-ideals of S. Then $\bigwedge_{i\in I} f_i$ is a fuzzy bi-ideal of S.*

Proof. By Proposition 3.9, we have $\bigwedge_{i\in I} f_i$ is a fuzzy Γ-subsemigroups of S. Now, let $x, y \in S$ be such that $x \leq y$. Since f_i is a fuzzy Γ-subsemigroup, $f_i(x) \geq f_i(y)$ for all $i \in I$. Thus $\sup\{f_i(x)\}_{i\in I} \geq f_i(x) \geq f_i(y)$ for all $i \in I$, so $\sup\{f_i(x)\}_{i\in I}$ is an upper bound of $\{f_i(y)\}_{i\in I}$. Hence $\sup\{f_i(x)\}_{i\in I} \geq \sup\{f_i(y)\}_{i\in I}$, so $(\bigvee_{i\in I} f_i)(x) \geq (\bigvee_{i\in I} f_i)(y)$. Let $x, y, z \in S$ and $\alpha, \beta \in \Gamma$. Since f_i is a fuzzy bi-ideal of S, we have $f_i(x\alpha y\beta z) \geq \min\{f_i(x), f_i(z)\}$ for all $i \in I$. Thus

$$\begin{aligned}
(\bigwedge_{i\in I} f_i)(x\alpha y\beta z) &= \inf\{f_i(x\alpha y\beta z)\}_{i\in I} \\
&\geq \inf\{\min\{f_i(x), f_i(z)\}\}_{i\in I} \\
&= \min\{\inf\{f_i(x)\}_{i\in I}, \inf\{f_i(z)\}_{i\in I}\} \\
&= \min\{(\bigwedge_{i\in I} f_i)(x), (\bigwedge_{i\in I} f_i)(z))\}.
\end{aligned}$$

Hence $\bigwedge\limits_{i\in I} f_i$ is a fuzzy bi-ideal of S. □

Theorem 3.11. *[6] Let S be a PO-Γ-semigroup and $\{f_i\}_{i\in I}$ a family of fuzzy left (resp. right) ideals of S. Then $\bigwedge_{i\in I} f_i$ is a fuzzy left (resp. right) ideal of S.*

Theorem 3.12. *Let S be a PO-Γ-semigroup and $\{f_i\}_{i\in I}$ a family of fuzzy left (resp. right) ideals of S. Then $\bigvee_{i\in I} f_i$ is a fuzzy left (resp. right) ideal of S.*

Proof. By Proposition 3.8, we have $\bigvee_{i\in I} f_i$ is a fuzzy subset of S. Now, let $x, y \in S$ be such that $x \leq y$. Since f_i is a fuzzy left ideal of M, $f_i(x) \geq f_i(y)$ for all $i \in I$. Thus $\sup\{f_i(x)\}_{i\in I} \geq f_i(x) \geq f_i(y)$ for all $i \in I$, so $\sup\{f_i(x)\}_{i\in I}$ is an upper bound of $\{f_i(y)\}_{i\in I}$. Hence $\sup\{f_i(x)\}_{i\in I} \geq \sup\{f_i(y)\}_{i\in I}$, so $(\bigvee_{i\in I} f_i)(x) \geq (\bigvee_{i\in I} f_i)(y)$. Finally, let $x, y \in S$ and $\alpha \in \Gamma$. Since f_i is a fuzzy left ideal of S, we have $f_i(x\alpha y) \geq f_i(y)$ for all $i \in I$. Thus

$$\begin{aligned}
(\bigvee_{i\in I} f_i)(x\alpha y) &= \sup\{f_i(x\alpha y)\}_{i\in I} \\
&\geq \sup\{f_i(y)\}_{i\in I} \\
&= (\bigvee_{i\in I} f_i)(y).
\end{aligned}$$

Hence $\bigvee\limits_{i\in I} f_i$ is a fuzzy left ideal of S. □

Theorem 3.13. *In a PO-Γ-semigroup S, the following statements are equivalent.*

(i) *μ is a fuzzy left ideal of S.*

(ii) *$\lambda\Gamma\mu \subseteq \mu$, and for any $x, y \in S, x \leq y$ implies $\mu(x) \geq \mu(y)$ where λ is the characteristic function of S.*

Proof. Assume that μ is a fuzzy left ideal of S. For any $x \in S$,

$$\begin{aligned}
(\lambda\Gamma\mu)(x) &= \begin{cases} \sup\{\min\{\lambda(y), \mu(z)\}\} & \text{if } x = y\alpha z \text{ for } y, z \in S \text{ and } \alpha \in \Gamma, \\ 0 & \text{otherwise.} \end{cases} \\
&= \begin{cases} \sup\{\min\{1, \mu(y\alpha z)\}\} & \text{if } x = y\alpha z \text{ for } y, z \in S \text{ and } \alpha \in \Gamma, \\ 0 & \text{otherwise.} \end{cases} \\
&= \begin{cases} \sup\{\mu(y\alpha z)\} & \text{if } x = y\alpha z \text{ for } y, z \in S \text{ and } \alpha \in \Gamma, \\ 0 & \text{otherwise.} \end{cases} \\
&= \begin{cases} \mu(x) \\ 0 \end{cases} \\
&\leq \mu(x).
\end{aligned}$$

Thus $\lambda\Gamma\mu \subseteq \mu$. Since μ is a fuzzy left ideal of S, we have $x \leq y$ implies $\mu(x) \geq \mu(y)$ for all $x, y \in S$.

Conversely, let us assume the second statement of the theorem. Let $x, y \in S$. Since $\lambda\Gamma\mu \subseteq \mu$, we have

$$\begin{aligned}
\mu(x\alpha y) &\geq (\lambda\Gamma\mu)(x\alpha y) \\
&\geq \min\{\lambda(x), \mu(y)\} \\
&= \min\{1, \mu(y)\} \\
&= \mu(y).
\end{aligned}$$

Hence, μ is a fuzzy left ideal of S. □

Theorem 3.14. *In a PO-Γ-semigroup S, the following statements are equivalent.*

(i) μ is a fuzzy right ideal of S.

(ii) $\mu\Gamma\lambda \subseteq \mu$, and for any $x, y \in S, x \leq y$ implies $\mu(x) \geq \mu(y)$ where λ is the characteristic function of S.

Combining the above two theorems, we have the following.

Theorem 3.15. *In a PO-Γ-semigroup S, the following statements are equivalent.*

(i) μ is a fuzzy two sided ideal of S.

(ii) $\mu\Gamma\lambda \subseteq \mu, \lambda\Gamma\mu \subseteq \mu$, and for any $x, y \in S, x \leq y$ implies $\mu(x) \geq \mu(y)$ where λ is the characteristic function of S.

Theorem 3.16. *In a PO-Γ-semigroup S, the following statements are satisfied.*

(i) If μ is a fuzzy bi-ideal ideal of S, then $\mu\Gamma\mu \subseteq \mu$.

(ii) If $\mu\Gamma\mu \subseteq \mu, \mu\Gamma\lambda\Gamma\mu \subseteq \mu$, and for any $x, y \in S, x \leq y$ implies $\mu(x) \geq \mu(y)$ where λ is the characteristic function of S, then μ is a fuzzy bi-ideal ideal of S.

Proof. (i) Assume that μ is a fuzzy bi-ideal ideal of S. Since μ is a fuzzy Γ-subsemigroup of S, we have for any $x \in S$,

$$\begin{aligned}
(\mu\Gamma\mu)(x) &= \begin{cases} \sup\{\min\{\mu(y), \mu(z)\}\} & \text{if } x = y\alpha z \text{ for } y, z \in S \text{ and } \alpha \in \Gamma, \\ 0 & \text{otherwise.} \end{cases} \\
&\leq \begin{cases} \sup\{\mu(y\alpha z)\} & \text{if } x = y\alpha z \text{ for } y, z \in S \text{ and } \alpha \in \Gamma, \\ 0 & \text{otherwise.} \end{cases} \\
&= \begin{cases} \mu(x) \\ 0 \end{cases} \\
&\leq \mu(x).
\end{aligned}$$

Thus $\mu\Gamma\mu \subseteq \mu$.
(ii) Assume that $\mu\Gamma\mu \subseteq \mu, \mu\Gamma\lambda\Gamma\mu \subseteq \mu$, and for any $x, y \in S, x \leq y$ implies $\mu(x) \geq \mu(y)$ where λ is the characteristic function of S. Since $\mu\Gamma\mu \subseteq \mu$, we have for any $x, y \in S$ and $\alpha \in \Gamma$,

$$\mu(x\alpha y) \geq (\mu\Gamma\mu)(x\alpha y) \geq \min\{\mu(x), \mu(y)\}.$$

Thus μ is a fuzzy Γ-subsemigroup of S. Since $\mu\Gamma\lambda\Gamma\mu \subseteq \mu$, we have for any $x, y, z \in S$ and $\alpha, \beta \in \Gamma$,

$$\begin{aligned}
\mu(x\alpha y\beta z) &\geq (\mu\Gamma\lambda\Gamma\mu)(x\alpha y\beta z) \\
&\geq \min\{(\mu\Gamma\lambda)(x\alpha y), \mu(z)\} \\
&\geq \min\{\min\{\mu(x), \lambda(y)\}, \mu(z)\} \\
&= \min\{\min\{\mu(x), 1\}, \mu(z)\} \\
&= \min\{\mu(x), \mu(z)\}.
\end{aligned}$$

Hence, μ is a fuzzy bi-ideal of S. □

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