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Kink and solitary wave solutions for the (3+1) dimensional Kadomtsev - Petviashvili equation

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Abstract In this work, we study the (3+1) dimensional Kadomtsev-Petviashvili (KP) equation by the effective tanh-coth method. The tanh-coth method is a direct algebraic approach, which can be used for constructing explicit exact kink and solitary wave solutions of a given nonlinear partial differential equation.

1 Introduction

Nonlinear model equations play the central role in various fields of mathematics, fluid dynamics, plasma physics, and engineering. Recently, several systematic powerful methods used to searching solutions of nonlinear partial differential equations (NLPDE) such as, Painleve analysis [1], Bäcklund transformation [1], Ansatz method [2, 3], Hirota bilinear formalism [3], Lie symmetry [1, 7], Homoclinic test method [11], Jacobi elliptic function method [12] and so on. Completely integrable KdV equation extended from (1+1) dimension to (2+1) dimension, which is called the Kadomtsev Petviashvili [5] equation or the KP equation.The analytical and numerical approach of (2+1) and (3+1) dimensional KP equations extensively studied variety of methods, see references [4, 6, 9, 10]. Applying the effective tanh - coth method [2] for nonlinear partial differential equations, which is a direct algebraic approach to establish exact solitary wave solutions. In this paper, we employ the tanh-coth method to (3+1) dimensional KP equation for constructing kink and exact solitary wave solutions. The complicated and tedious algebraic calculations managed by computer algebra system such as Mathematica.

2 The tanh-coth Method

The tanh - coth method introduced by wazwaz [2].

A wave variable $\xi = x - ct$ converts any nonlinear Partial Differential Equation (NLPDE)

$$P(u, u_t, u_x, u_{xx}, u_{xxx}, ...) = 0, \tag{2.1}$$

to an Ordinary Differential Equation (ODE)

$$Q(u, u', u'', u''', ...) = 0. \tag{2.2}$$

Eq. (2.2) is then integrated as long as all terms contains derivatives where integration constants are considered zeros. The standard tanh method is developed by Malfliet [8], where the tanh is used as a new variable, since all derivatives of a tanh are represented by tanh itself. Introducing a new independent variable,

$$Y = \tanh(\mu\xi), \qquad \xi = x - ct, \tag{2.3}$$

where, μ is the wave number, leads to the change of derivatives:

$$\begin{aligned} \frac{d}{d\xi} &= \mu(1 - Y^2)\frac{d}{dY}, \\ \frac{d^2}{d\xi^2} &= -2\mu^2 Y(1 - Y^2)\frac{d}{dY} + \mu^2(1 - Y^2)^2\frac{d^2}{dY^2}. \end{aligned} \tag{2.4}$$

The tanh – coth method admits the use of the finite expansion

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{M} a_k Y^k + \sum_{k=1}^{M} b_k Y^{-k}, \tag{2.5}$$

and

$$Y' = \mu(1 - Y^2). \tag{2.6}$$

where, M is a positive integer, in most cases, that will be determined by Homogeneous Balance Method (HBM). Substituting (2.5) and (2.6) into the reduced ODE results in an algebraic equation in powers of Y. Balancing the highest order the linear term with the highest order nonlinear term to determine the parameter M. We then collect all coefficients of powers of Y in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters a_k, b_k, μ, and c. Having determined these parameters, we obtain an analytic solution $u(x, t)$ in a closed form. The solutions we obtain may be solitons, travelling wave, and periodic solutions as well.

3 (3+1) dimensional KP - Equation

The (3+1) dimensional KP - Equation is

$$u_{xt} + u_{yt} + u_{xxxy} + 3(u_x u_y)_x - u_{zz} = 0, \tag{3.1}$$

$$u(x, t) = u(\xi), \qquad \xi = x + y + z - ct \tag{3.2}$$

using (3.2) into (3.1) and integrating we get on ODE

$$u''' - (2c + 1)u' + 3u'^2 = 0. \tag{3.3}$$

From Eq. (3.3) balancing the nonlinear term u'^2 with the higher order derivative term u''' we get the value $M = 1$ and hence,

$$u(x, y, z, t) = S(Y) = a_0 + a_1 Y + \frac{b_1}{Y}. \tag{3.4}$$

Substituting (3.4) into (3.3), collecting the coefficients of each power of Y^i, setting each coefficient to zero. Solving the resulting algebraic system with computer algebra software such as Mathematica, we obtain the following sets:

$$a_1 = \sqrt{1+2c}, \qquad b_1 = 0, \qquad \mu = \frac{1}{2}\sqrt{1+2c} \tag{3.5}$$

$$a_1 = 0, \qquad b_1 = \sqrt{1+2c}, \qquad \mu = \frac{1}{2}\sqrt{1+2c} \tag{3.6}$$

$$a_1 = \frac{1}{2}\sqrt{1+2c}, \qquad b_1 = \frac{1}{2}\sqrt{1+2c}, \qquad \mu = \frac{1}{4}\sqrt{1+2c} \tag{3.7}$$

$$a_1 = -\sqrt{-(1+2c)}, \qquad b_1 = 0, \qquad \mu = \frac{1}{2}\sqrt{-(1+2c)} \tag{3.8}$$

$$a_1 = 0, \qquad b_1 = \sqrt{-(1+2c)}, \qquad \mu = \frac{1}{2}\sqrt{-(1+2c)} \tag{3.9}$$

$$a_1 = -\frac{1}{2}\sqrt{-(1+2c)}, \qquad b_1 = \frac{1}{2}\sqrt{-(1+2c)}, \qquad \mu = \frac{1}{4}\sqrt{-(1+2c)} \tag{3.10}$$

Substituting Eqs.(2.3) and (3.5-3.10) into Eq. (3.4), we obtain the following solitary wave solutions

$$u_1(\xi) = \sqrt{1+2c}\tanh\left[\frac{1}{2}\sqrt{1+2c}(\xi)\right], \quad c>0, \tag{3.11}$$

$$u_2(\xi) = \sqrt{1+2c}\coth\left[\frac{1}{2}\sqrt{1+2c}(\xi)\right], \quad c>0, \tag{3.12}$$

$$u_3(\xi) = \frac{1}{2}\sqrt{1+2c}\left(\tanh\left[\frac{1}{4}\sqrt{1+2c}(\xi)\right] + \coth\left[\frac{1}{4}\sqrt{1+2c}(\xi)\right]\right), \quad c>0, \tag{3.13}$$

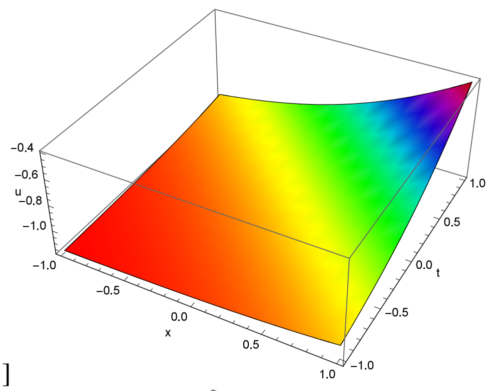
$$u_4(\xi) = -\sqrt{-(1+2c)}\tan\left[\frac{1}{2}\sqrt{-(1+2c)}(\xi)\right], \quad c<0, \tag{3.14}$$

$$u_5(\xi) = \sqrt{-(1+2c)}\cot\left[\frac{1}{2}\sqrt{-(1+2c)}(\xi)\right], \quad c<0, \tag{3.15}$$

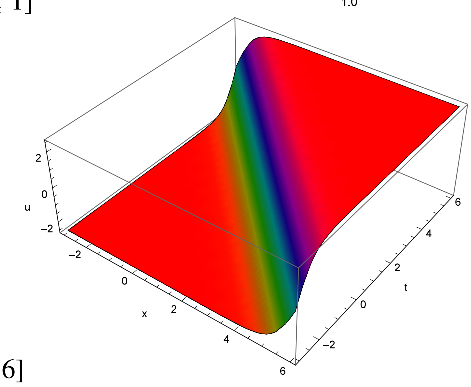
$$u_6(\xi) = \frac{1}{2}\sqrt{-(1+2c)}\left(\cot\left[\frac{1}{4}\sqrt{-(1+2c)}(\xi)\right] - \tan\left[\frac{1}{4}\sqrt{-(1+2c)}(\xi)\right]\right), \quad c<0. \tag{3.16}$$

where

$$\xi = x+y+z-ct$$

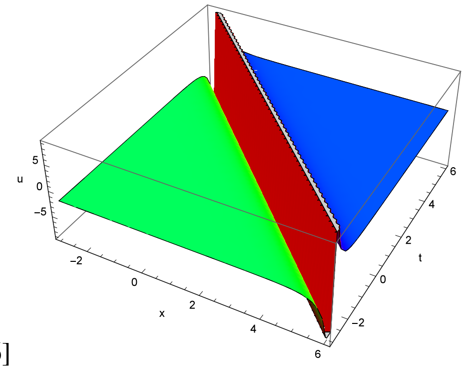


$[c = 0.2, t = 0.5, z = -2.5$ and $-1 \leqslant x, y \leqslant 1]$

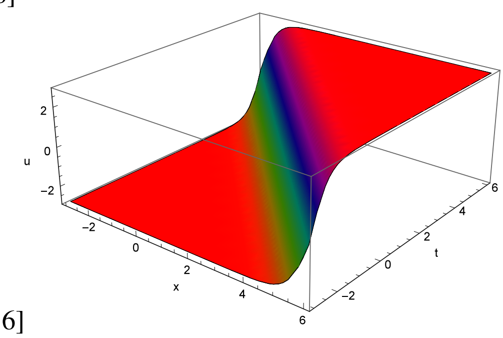


$[c = 2.75, t = 4, z = 8$ and $-3 \leqslant x, y \leqslant 6]$

Figure 1: Exact and Kink solution of $u_1(\xi)$

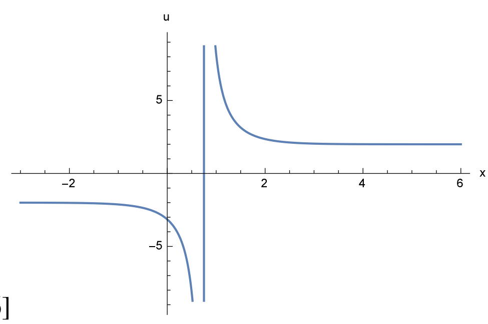


$[c = 1.8,\ t = 4,\ z = 4 \text{ and } -3 \leqslant x, y \leqslant 6]$

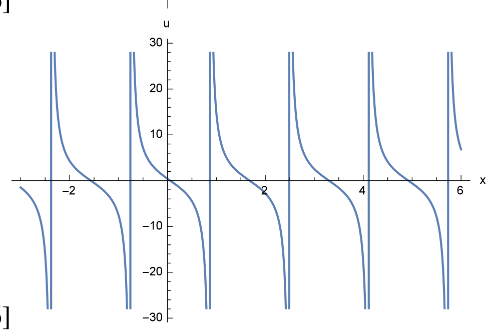


$[c = 3.5,\ t = 2.5,\ z = 5.7 \text{ and } -3 \leqslant x, y \leqslant 6]$

Figure 2: Rational and Kink solution of $u_3(\xi)$ and $u_4(\xi)$



$[c = 1.5,\ t = 0.5,\ y = z = 0 \text{ and } -3 \leqslant x \leqslant 6]$



$[c = -8,\ t = 0.5,\ y = z = 0 \text{ and } -3 \leqslant x \leqslant 6]$

Figure 3: 2D plot for solitary solution of $u_6(\xi)$

4 Conclusion

In this paper, we have studied the (3+1) dimensional KP equation employed by tanh-coth method with the help of Mathematica. This method gives the different physical structured kink and solitary wave solutions.

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