Palestine Journal of Mathematics

Vol. 6(Special Issue: II) (2017) , 102–111

STUDY OF RELIABILITY MEASURES OF SYSTEM CONSISTING OF TWO SUBSYSTEMS IN SERIES CONFIGURATION USING COPULA

V. V. Singh and Hamisu Ismail Ayagi

Communicated by H. M. Srivastava

MSC 2010 Classifications: Primary 20M99, 13F10; Secondary 62N05, 60K05.

Keywords and Phrases: Series Configuration, Redundant System, Availability, MTTF, Cost Analysis, Gumbel-Hougaard family copula and Supplementary variable. The authors would like to thank the editor and the anonymous referees for their useful comments and suggestions in this paper

Abstract This paper we deals with the study of a complex system which consists of two subsystems in a series configuration. The subsystem 2 is connected to subsystem 1 in a series configuration. The subsystem 1 has two identical units in parallel configuration and subsystem 2 has one unit connected in series with subsystem 1. A human operator operates the system, and human failure is considered to damage the entire system completely. All failure rates are assumed to follow exponential distribution and repairs follow two types of distribution general distribution and Gumbel-Hougaard family copula distribution. Supplementary variables technique is employed to studies the system, and the various measures of reliability are being discussed such as Availability of system, reliability, MTTF, and profit benefit by operation of the system. Some particular cases have been highlighted by taking different values of failure rates. Graphs demonstrate results, and consequently, conclusions have been done.

1 Introduction

The system reliability plays a vital role in almost all manufacturing industries and organizations, where different types of equipment are being produced or manufactured. In the past, many researchers and scientists have designed various types of mathematical models and proclaimed their validity under reliability characteristics. The system security has extensively studied by various researchers including Govil [1], Cui and Lin [6], Gupta and Sharma [10] and many others using the supplementary variable and Laplace transform. They have studied the reliability measures of complex systems by taking different failures and one repair facility. It is a well-known fact that whenever partial failure arises it degrades the efficiency system and there is also more possibility of complete damage to the entire system, which may harm the organization and even it might be a risk to the human life. In the other hand, if the failed system is not repaired in appropriate time it will cause a huge loss to the organization. Thus, assigning one repair to the failed system is not a good policy. There are many situations in real life, where more than one repair is possible between two adjacent transition states. When this possibility arises, the system should be repaired by employing copula. Redundancy plays an imperative role in system reliability. The system reliability can be improved by including some redundant units in the system together with the main operating unit. Various authors have studied the complex system under the redundant unit and have established that the system reliability increases by adjusting some redundant units with the system. Ibrahm Yusuf et al. [5] studied a three unit redundant system with three types of failure, and a comparative study of three different cases have done for various failure rates. Singh et al.[14] examined reliability characteristics of a complex system consisting 3 units as Super Priority, Priority and Ordinary under pre-emptive resume repair policy using supplementary variable technique under general repair. Singh et al.[15] studied availability, M.T.T.F and cost analysis of a complex system having two units in a series configuration with controllers and human failure under concept Gumbel-Hougaard family copula distribution. Most of the electronic equipment consists of switch and switch failure have its own importance in system configuration. In continuation to the study of redundant systems, Dalah et al. [2] studied reliability measure of a two unit standby system under the concept of switch failure using copula repair.Ram et al. [7] studied the stochastic analysis of a standby system with waiting repair strategy.

Redundancy and repair are the most important aspects of improving the reliability of the system. It is composed of techniques for increasing effectiveness through reducing failure and repair rates respectively. The researchers who extensively studied system security and the reliability measures of the repairable complex system considered various failures and one repair policy. A system is composed of subsystem and reliability of the entire system is depends on the reliability of subsystems and the configuration. It is a well-established fact that every subsystem must survive to survive the system. Negi and Singh[13] analyzed a non-reparable system with weighted subsystems connected in a series configuration with the concept of universal generating function. Singh et al. [18] studied a multi- state k-out-of-n type of system and emphasized on 2-out-of-3: G; system for computations as special cases. Warranty on the product is a most attractive business policy for an organization which attracts customer attention. Ram Niwas and M. S. Kadyan [12] studied the reliability modeling of a maintained system with warranty and degradation using supplementary variables technique. Kadyan et al. [8] studied availability and profit analysis of feeding system in sugar industry using the supplementary variables introducing method of conversion from non-Markova process to Markova process.

Researchers have designed various types of system and proclaimed the validity of those performances. Though the authors in the past have studied different kinds of systems and extend the research work done by their predecessor, there arises a need to study further in this field. Thinking this need in view in the present paper we decided to focused our study of a complex system which consisting of two subsystems Subsystem 1, and subsystem 2. The subsystem 2 is connected with the subsystem 1 in a series configuration. The subsystem 1 has two identical units, but the subsystem 2 has a single unit. Initially, in state S_0, both units of subsystem 1 and subsystem 2 are in good condition. After the failure of any unit of subsystem 1, the other takes a load of failed unit, and the failed unit gets a repair. If the second unit of the system is also failed before replacement of the previous unit, the system will be in complete failure mode. If both units of subsystem 1 are good and subsystem 2 fail then, it will also be in entire failed state. If one unit of subsystem 1 is in good and subsystem 2 failed then, the system will also be in failed state. A human operator operates the system then human failure may also be a cause of system failure. Whenever the system is in partially failed state i.e. operational with less efficiency, it is repaired by using general repair, but the entire failed state is needed to get repair quickly, so these states have been corrected by copula distribution more precisely Gumbel-Hougaard family copula distribution. The various interested and required measures of system reliability have been discussed. The results have computed for different values of failure and repair rates. The paper is organized in following the sections: Section 1 of the paper is introduction, section 2 of the paper leads to mathematical modeling of the paper, while section 3 of paper is analytical section in which the various reliability measures like Availability, Reliability, MTTF, and cost analysis has been calculated for different values of parameters. Table and graphs demonstrate results. Finally, in section 4 we concluded the study.

State Description

- S_0: In state S_0 both subsystems are in good working condition. The system is in perfect state operational state.

- S_1: The state S_1 represents that the first unit, of the subsystem 1 fail and subsystem 2 is in good condition. The state is under general repair, and the system is in operational mode.

- S_2: The state S_2 represents an entire failed state after failing both units of subsystem 1. The system is under repair and Copula repair {Gumbel-Hougaard} family copula is employed to the failed state.

- S_3: The state represents that the system is in the complete failed state due to failure in subsystem 2. Though the both units of subsystem 1 are in good condition. The system is under repair using copula distribution.

- S_4: The state represents that the system is in complete failed state due to failure in subsystem 2 after the failure of one unit in subsystem 1. The system is under repair using copula distribution.

- S_5: The state represents that the system is in a complete failed state due to human failure which is assumed to complete damage the system. The system is under repair using copula distribution.

2 Assumptions

The following presumptions have been taken throughout the study of Mathematical model.

- Initially, in state S_0 the system is in good working condition. Both subsystems are in good operational condition.

- The system can perform a task if any one unit of subsystem 1 is in good condition together with the subsystem 2.

- Human failure fails the system, and due to human failure, the system goes to complete failure mode.

- The system fails if both units of subsystem1 fail.

- It is assumed that repaired system works as a new and nothing is damaged during repair.

- Only one change is allowed in one state at a time.

- All failure rates are constant and assumed to follow exponential time distribution.

Notations

- t : Time variable on time scale.

- s : Laplace transform variable for all expressions.

- $2\lambda/\lambda//\lambda_1/\lambda_h$: Failure rates for the units of subsystem 1 / subsystem 2/failure rates due to human failure

- $\phi_(x)/\mu_0$: General repair rates for degraded states / complete failed states.

- P_0: The probability that the system is in perfect state S_0.

- $\bar{P}(s)$: Laplace transformation of state probability $P(t)$.

- $P_j(x,t)$: The probability that a system is in state S_j for $j = 1$to 5; the system is running under repair and elapsed repair time is x, t.

- $E_{p(t)}$: Expected profit during the time interval $[0, t)$.

- K_1, K_2 : Revenue and service cost per unit time respectively.

- $\mu_o(x)$: The expression of joint probability failed state Si to good state S_0 according to Gumbel-Hougaard family copula is given as $C_\theta(u_{1(x)}, u_{2(x)})$=$exp[x^\theta + \log\varphi(x)^\theta]^{\frac{1}{\theta}}$ where, $u_1 = \varphi(x)$, $u_2 = e^x$ and θ is a parameter $1 \leq \theta \leq \infty$.

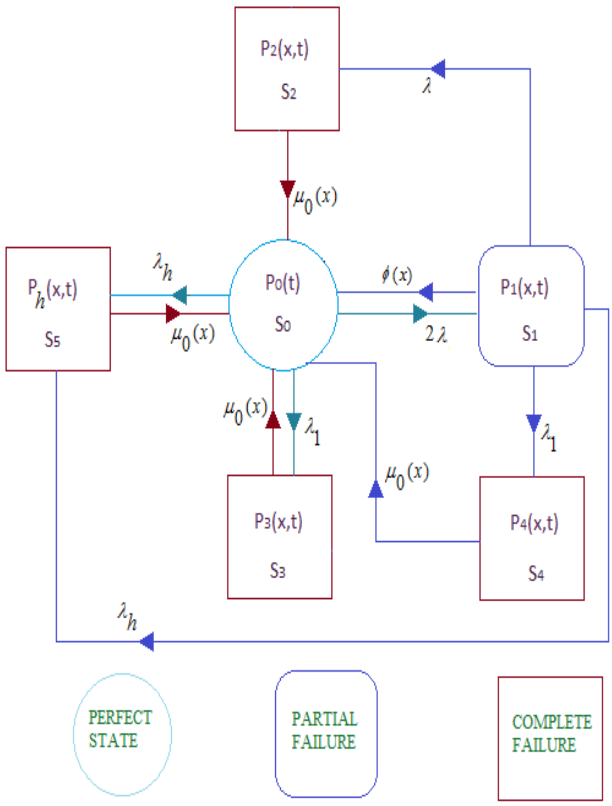

Figure 1. state Transition Diagram

3 Mathematical Formulation of Model

By the probability of considerations and continuity of arguments, we obtained the following set of differential equations governing the present mathematical model.

$$
\begin{aligned}
(\frac{\partial}{\partial t} + 2\lambda + \lambda_1 + \lambda_h)P_0(t) \quad &= \quad \int_0^\infty \varphi_s(x)P_1(x,t)dx + \int_0^\infty \mu_o(x)P_2(x,t)dx + \\
&\int_0^\infty \mu_o(x)P_3(x,t)dx + \int_0^\infty \mu_o(x)P_4(x,t)dx + \int_0^\infty \mu_o(x)P_h(x,t)dx
\end{aligned} \tag{3.1}
$$

$$
\begin{aligned}
(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda + \lambda_1 + \varphi_(x))P_1(x,t) &= 0 && (3.2) \\
(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_o(x))P_2(x,t) &= 0 && (3.3) \\
(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_o(x))P_3(x,t) &= 0 && (3.4) \\
(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\mu_o(x))P_4(x,t) &= 0 && (3.5) \\
(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_o(x))P_h(x,t) &= 0 && (3.6)
\end{aligned}
$$

The Boundary conditions are as follows

$$P_1(0,t) = 2\lambda P_0(t) \tag{3.7}$$
$$P_2(0,t) = 2\lambda^2 P_0(t) \tag{3.8}$$
$$P_3(0,t) = \lambda_1 P_0(t) \tag{3.9}$$
$$P_4(0,t) = 2\lambda\lambda_1 P_0(t) \tag{3.10}$$
$$P_h(0,t) = \lambda_h(1+2\lambda)P_0(t) \tag{3.11}$$

taking Laplace transform of equations (2.1)-(2.11) under the assumptions $P_0(0) = 0$ one can get the following results:

$$\begin{aligned}(s+2\lambda+\lambda_1\lambda_h)\overline{P_0(s)} &= 1+\int_0^\infty \varphi(x)\overline{P_1}(x,s)dx+\int_0^\infty \mu_0(x)\overline{P_2}(x,s)dx+ \\ &\int_0^\infty \mu_0(x)\overline{P_3}(x,s)dx+\int_0^\infty \mu_0(x)\overline{P_4}(x,s)dz+\int_0^\infty \mu_0(x)\overline{P_h}(x,s)dx\end{aligned} \tag{3.12}$$

$$(s+\frac{\partial}{\partial x}+\lambda+\lambda_1+\varphi_(x))\overline{P_1}(x,s) = 0 \tag{3.13}$$
$$(s+\frac{\partial}{\partial x}+\mu_0(x))\overline{P_2}(x,s) = 0 \tag{3.14}$$
$$(s+\frac{\partial}{\partial x}+\mu_0(x))\overline{P_3}(x,s) = 0 \tag{3.15}$$
$$(s+\frac{\partial}{\partial x}+\mu_0(x))\overline{P_4}(x,s) = 0 \tag{3.16}$$
$$(s+\frac{\partial}{\partial x}+\mu_0(x))\overline{P_h}(x,s) = 0 \tag{3.17}$$
$$\overline{P_1}(o,s) = 2\lambda\overline{P_o}(s) \tag{3.18}$$
$$\overline{P_2}(o,s) = 2\lambda^2\overline{P_o}(s) \tag{3.19}$$
$$\overline{P_3}(o,s) = \lambda_3\overline{P_o}(s) \tag{3.20}$$
$$\overline{P_4}(o,s) = 2\lambda\lambda_1\overline{P_o}(s) \tag{3.21}$$
$$\overline{P_h}(o,s) = \lambda_h(1+2\lambda)\overline{P_o}(s) \tag{3.22}$$

Solving equations (2.12)-(2.17) with help of equations (2.18)- (2.22), we have

$$\overline{P_0}(s) = \frac{1}{D(s)} \tag{3.23}$$
$$\overline{P_1}(s) = \frac{2\lambda}{D(s)}\frac{1-S_\phi(s+2\lambda+\lambda_1+\lambda_h)}{s+2\lambda+\lambda_1+\lambda_h} \tag{3.24}$$
$$\overline{P_2}(s) = \frac{2\lambda^2}{D(s)}\frac{1-\overline{S_{\mu_0}}(s)}{s} \tag{3.25}$$
$$\overline{P_3}(s) = \frac{\lambda_1}{D(s)}\frac{1-\overline{S_{\mu_0}}(s)}{s} \tag{3.26}$$
$$\overline{P_4}(s) = \frac{2\lambda_1^2}{D(s)}\frac{1-\overline{S_{\mu_0}}(s)}{s} \tag{3.27}$$
$$\overline{P_h}(s) = \frac{\lambda_h(1+2\lambda)}{D(s)}\frac{1-\overline{S_{\mu_0}}(s)}{s} \tag{3.28}$$

$$D(s) = B - 2\lambda A + 4\lambda^2 \overline{S_{\mu 0}}(s) + \lambda_h(1+2\lambda)\overline{S_{\mu 0}}(s)\overline{P_0}(s) \quad (3.29)$$

$$\overline{P}_{down} = 1 - \overline{P}_{up} \quad (3.30)$$

$$\overline{P}_{up} = \overline{P_0}(s) + \overline{P_1}(s) \quad (3.31)$$

$$A = S_{\phi_s}(s + \lambda_o + \lambda_{sw} + \lambda_D) \quad (3.32)$$

$$B = (s + 2\lambda + \lambda_1 + \lambda_h) \quad (3.33)$$

3.1 Availability Analysis

For particular cases the study of availability is focused on the following cases, when repair follows exponential distribution setting $S_{\mu_o}(s) = \overline{S}_e[x^\theta + \log\varphi(x)^\theta]^{\frac{1}{\theta}}(s)$, $S_{\mu_o}(s) = \frac{exp[x^\theta + \log\varphi(x)^\theta]^{\frac{1}{\theta}}}{s + exp[x^\theta + \log\varphi(x)^\theta]^{\frac{1}{\theta}}}$, $\overline{S}_\phi = \frac{\phi}{s+\phi}$, as $\lambda = 0.02, \lambda_1 = 0.02, \lambda_h = 0.012, \theta = 1, x = 1$, in equation (2.31), then taking the inverse Laplace transform, one can obtain

$$Availability = -0.040896e^{-1.029000t} - 0.005358e^{-2.7729t} + 0.03137e^{-1.01894t} + 1.00417e^{-0.01239t} \quad (3.34)$$

For, $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90$ units of time, one may get different values of availability as shown in table 1 and corresponding Fig.2

Table1. Variation of Availability with respect to time

Time(t)	Availability
0	1.000
10	0.887
20	0.783
30	0.692
40	0.612
50	0.540
60	0.477
70	0.421
80	0.372
90	0.329

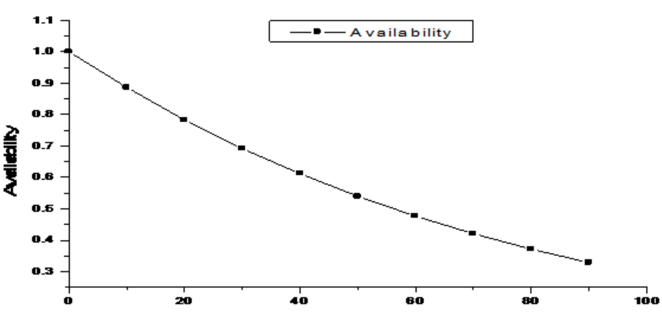

Figure 2. Availability as a function of time (t)

3.2 Reliability Analysis

Taking all repair equal to zero in equation (2.31) and taking inverse Laplace transform, we have expression for the reliability of system and for given values of failure rates $\lambda = 0.02$,

$\lambda_1 = 0.015, \lambda_h = 0.012$ in equation (2.31), one can get expression of reliability of system as

$$Reliability = -0.0727272e^{-0.0320000t} + 1.0727272e^{-0.045000} \tag{3.35}$$

Table 2 Variation of Reliability with respect to time

Time(t)	Reliability
0	1.000
10	0.681
20	0.436
30	0.270
40	0.177
50	0.011
60	0.072
70	0.045
80	0.029
90	0.018
10	0.011

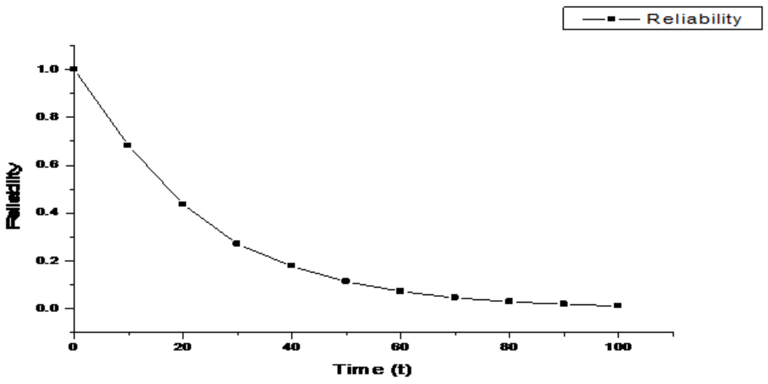

Figure 3. Reliability as a function of time

3.3 Mean Time to Failure Rate M.T.T.F

Taking all repairs zero and the limit as s tends to zero in equation 2.31 for the exponential distribution, one can obtain the M.T.T.F. as

$$M.T.T.F = \frac{1}{3\lambda + \lambda_h} \frac{3\lambda + \lambda_h + \lambda_A}{\lambda + \lambda_1 + \lambda_A} \tag{3.36}$$

Setting, $\lambda_1 = 0.02, \lambda_h = 0.12$ and varying λ as $0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09$ in equation (33) one may obtain Table 3 whose column 2 demonstrates the variation of MTTF with respect to failure rate λ. Setting λ=0.01, λ_h=0.012 and varying λ_1 as: $0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09$ in equation 33 one may obtain Table 2 in which the column 3 demonstrates variation of MTTF with respect to the failure rate λ_1. Setting $\lambda = 0.01$,$\lambda_1 = 0.02$ and varying λ_h as: $0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09$ in equation 33 one may obtain Table 2 and its column 4 demonstrates variation of MTTF with respect to failure rate λ_h.

Table 3. Variation in values of MTTF with respect to failure rates

Failure Rate	MTTFλ	MTTFλ_1	MTTFλ_h
0.01	35.14	38.69	37.50
0.02	25.57	35.14	28.00
0.03	19.29	32.96	22.22
0.04	15.99	31.49	18.36
0.05	13.70	30.42	15.62
0.06	12.00	29.61	13.58
0.07	10.68	28.98	12.00
0.08	09.63	28.47	10.74

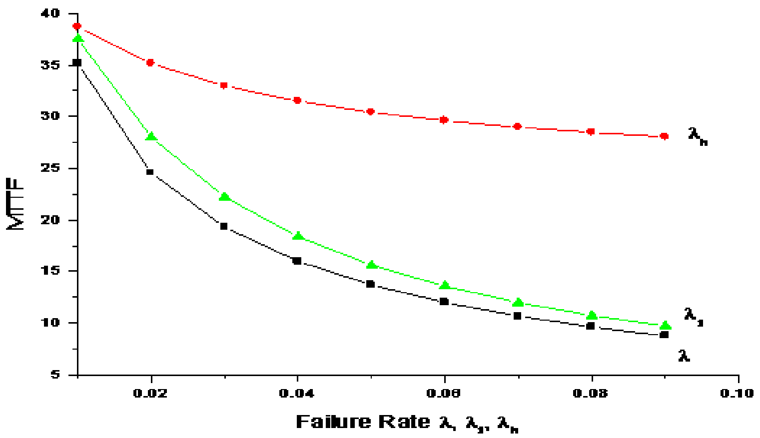

Figure 4. Variation of MTTF with respect to failure rates

3.4 Cost Analysis

Let the service facility be always available, then the expected profit during the interval $[0, t)$ is $E_p(t) = K_1 \int_0^t P_u p(t) dt - K_2 t$. For the set of values of parameter of equation 2.31, one can obtain the expression for expected profit of system in the interval $[0, t)$ as

$$E_p(t) = K_1 0.039744 e^{-1.0290t} + 0.0019604 e^{-2.7183t} \tag{3.37}$$
$$-0.0307527 e^{-1.01993t} - 81.00037 e^{-0.12397t} + 80.2212 - K2$$

Setting $K_1 = 1$ and K_2=0.5, 0.40, 0.30, 0.20, 0.1 respectively and varying $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$ units of time, the results for expected profit can be obtain as shown in Table 4 and the graphical representation in corresponding figure 5.

Table 4 Computation of expected profit for different values of time

Time (t)	$k_2 = 0.5$	$k_2 = 0.4$	$k_2 = 0.3$	$k_2 = 0.2$	$k_2 = 0.1$
0	0.0	0.0	0.0	0.0	0.0
10	3.665	4.665	5.665	6.665	7.665
20	7.008	9.008	11.008	13.008	15.008
30	9.378	12.378	15.378	18.378	21.379
40	10.889	14.889	18.889	22.889	26.889
50	11.641	16.641	21.641	26.641	31.641
60	11.722	17.722	23.722	29.722	35.722
70	11.211	18.211	25.211	32.722	39.211
80	10.176	18.177	26.176	34.176	42.176
90	8.679	17.677	26.679	35.679	44.679

Figure 5. Expected profit as function of time

3.5 Interpretation of Results and Conclusion

Fig.3 provides information how the availability of the repairable system changes with respect to the time, when failure rates are fixed at different values. When failure rates are fixed at lower values $\lambda = 0.01$, $\lambda_1 = 0.02$, $\lambda_h = 0.012$, availability of the system decreases and ultimately becomes steady to the value zero after a sufficient long interval of time. Hence, one can safely predict the future behavior of a complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model. Availability of system for which human failure is ignored is decreases up to time $t = 60$, but it again start increasing again. In fig.4 provides the variation in reliability of non-repairable system. Fig.5, yields the mean-time-to-failure M.T.T.F. of the system with respect to variation in λ, λ_1 and λ_h respectively, when the other parameters have been taken as constant. The variation in MTTF corresponding to failure rates λ_h are almost is very high but corresponding to λ, λ_1 it is very much close. When revenue cost per unit time K_1 is fixed at 1, service costs $K_2 =$ $0.5, 0.4, 0.3, 0.2, 0.1$, profit has been calculated and results are demonstrated by graphs in Fig.5. A critical examination from Fig.5 reveals that expected profit increases with respect to the time when the service cost K_2 fixed at minimum value 0.3. Finally, one can observe that as service cost increase, profit decrease. In general, for low service cost, the expected profit is high in comparison to high service cost.

References

[1] A. K. Govil, Operational Behavior of a Complex System having shelf-life of the Components Under Preemptive-resume Repair Discipline, *Appl. Math.* **13**, 97–101 (2010).

[2] Chiwa Musa Dalah and V. V. Singh, Study of Reliability Measures of a Two Unit Standby System under the Concept of Switch Failure Using Copula Distribution, *American Journal of Computational and Applied Mathematics* **17**, 118–129 (2014).

[3] D. K. Rawal, M. Ram and V. V. Singh, Modeling and availability analysis of internet data center with various maintenance policies, *IJE Transactions A: Basics* **27**, 599–605 (2014).

[4] D. R. COX, The Analysis of Non-Markov Stochastic Processes by the Inclusion of Supplementary Variables, *Proc. Camb. Phil. Soc. Math. Phys. Sci.* **51**, 433–441 (1995).

[5] Ibrahim. Yusuf and N. Hussaini, A Comparative Analysis of Three Unit Redundant Systems with Three Types of Failures, *Arab J Sci Eng* , **51**, 3337–6083 (2014).

[6] Lirong Cui and Haijun Li, Analytical Method for Reliability and MTTF assessment of Coherent systems with Dependent Components, *Reliability Engineering and System Safety.* **92**, 300–307 (2007).

[7] M. Ram, S. B. Singh and V. V. Singh, Stochastic analysis of a Standby Complex System with waiting Repair Strategy, *International Journal of Reliability, Quality and Safety Engineering* **15**, 341–365 (2008).

[8] M. S. Kadyan and Ramesh Kumar, Availability and Profit analysis of a Feeding System in Sugar Industry, *International Journal of Engineering Science and Technology* **10**, – (2015).

[9] Mangey Ram, S. B. Singh and R. G. Varshey, Performance Improvement of a Parallel Redundant System with coverage Factor. *Appl. Math.* **8**, 344–350 (2013).

[10] P. P. Gupta and M. K. Sharma, Reliability and M.T.T.F Evaluation of a Two Duplex-unit Standby System with Two types of Repair, *Microelectronics Reliability* **33**, 291–295 (1993).

[11] R. B. Nelsen, An Introduction to Copulas, *New York, Springer* **33**, (2006).

[12] Ram Niwas and M. S. Kadyan, Reliability modeling of a Maintained System with Warranty and Degradation, *Journal of Reliability and Statistical Studies* **8**, 55-63 (2015).

[13] Seema Negi and S. B. Singh, Reliability Analysis of Non-repairable Complex System with Weihgted Subsystem connected in Series, *Applied Mathematics And Computation* **26**, 79–90 (2015).

[14] V. V. Singh, S.B. Singh, M. Ram and C. K. Goel,Availability, MTTF and Cost Analysis of a System Having Two Units in Series Configuration with Controller, *International Journal of Reliability and Applications* **11**, 41–53 (2010).

[15] V. V. Singh and Dilip Kumar Rawal, Availability Analysis of a System Having two units in series configuration with Controller and human failures under different repair policy, *International Journal of Sci. Eng. Res* **2**, 1–9 (2011).

[16] V. V. Singh, S.B Singh, M.Ram and C.K.Goel, Availability analysis of a System Having three units super priority, Priority and ordinary Under Preemptive Resume Repair Policy, *International Journal of System Assurance Engineering and Management* **4**, 341–352 (2012).

[17] V. V. Singh, M. Ram and D.K.Rawal, Cost Analysis of an Engineering System Involving Subsystems in Series Configuration, *IEEE Transactions on Automation Science and Engineering* **10**, 1124-1130 (2013).

[18] V. V. Singh and Mangey Ram, Multi -state k-out-of-n Type System Analysis,*Mathematics in Engineering, Science and Aerospace* **5**, 281-292 (2014).

Author information

V. V. Singh, Department of Mathematics, Faculty of Science, Northwest University, Kano, Nigeria.
E-mail: singhvijayvir68@gmail.com

Hamisu Ismail Ayagi, Department of Mathematics, Faculty of Science, Northwest University, Kano, Nigeria.
E-mail: ayagi2010@gmail.com

Received: August 26, 2016.

Accepted: November 11, 2016