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Primal and weakly primal ideals in C(X)

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Abstract Let X be a completely regular Hasudorff space, and consider the ring of continuous functions $C(X)$. A z-filter $\mathcal{F}$ on X is called primal if the set of all elements of $Z(X)$ that are not prime to $\mathcal{F}$ forms a z-filter on X: here an element $Z \in Z(X)$ is called prime to $\mathcal{F}$ if $Z \cup Z' \in \mathcal{F}$ implies that $Z' \in \mathcal{F}$. In this paper we consider the primal z-filters on X, and then we discuss on the relations between this class of z-filters on X and the class of primal ideals of $C(X)$. We also define the concept weakly prime and weakly primal z-filters on X and show that there is a one-to-one correspondence between the weakly prime (resp. weakly primal) z-filters on X and the weakly prime (resp. weakly primal) ideals of $C(X)$.

1 introduction

Prime z-filters play a central role in the study of the rings of continuous functions. Of course, a prime z-filter $\mathcal{F}$ on a completely regular Hausdorff space X is z-filter $\mathcal{F}$ on X with the property that

$$Z, Z' \in Z(X), Z \cup Z' \in \mathcal{F} \Rightarrow Z \in \mathcal{F} \text{ or } Z' \in \mathcal{F}.$$

There are several ways to generalize the notion of a prime z-filters. We could either restrict or enlarge where Z and/or Z' lie or restrict or enlarge where $Z \cup Z'$ lies. In this paper we will be mostly interested in generalizations obtained by restricting where $Z \cup Z'$ lies.

Let R be a commutative ring, I an ideal of R and J a subset of R. We denote by $(I :_R J)$ the set of all elements $r \in R$ with $ra \in I$ for every $a \in J$. Then the annihilator of J, denoted by $Ann_R(J)$ is just $(0 :_R J)$. An element $a \in R$ is called a zero-divisor of R provided that $Ann_(a) \neq 0$. We denote by $Z(R)$, the set of all zero-divisors of R.

We recall from [4] that an element $a \in R$ is called prime to I if $ra \in I$ (where $r \in R$) implies that $r \in I$, that is $(I :_R a) = I$. Denote by $S(I)$ the set of elements of R that are not prime to I, that is

$$S(I) = \{a \in R | ra \in I \text{ for some } r \in R \setminus I\}.$$

Then I is said to be primal if $S(I)$ forms an ideal of R; this ideal is always a prime ideal, called the adjoint prime ideal P of I. In this case we also say that I is a P-primal ideal of R ([4]). It is easy to see that I is a P-primal ideal of R if and only if $Z(R/I) = P/I$. Then ring R is called coprimal if the zero ideal of R is primal.

The concept of weakly primal ideals in a commutative ring R studied in [2]. An element $a \in R$ is called weakly prime to the ideal I if $0 \neq ra \in I$ ($r \in R$) implies $r \in I$. Clearly 0 is always weakly prime to I. Denote by $W(I)$ the set of all elements of R which are not weakly prime to I, that is

$$W(I) = \{a \in R | 0 \neq ra \in I \text{ for some } r \in R \setminus I\}.$$

I is called weakly primal provided that the set $P := W(I) \cup \{0\}$ forms an ideal of R. Then the ideal is a weakly prime ideal of R, called the weakly prime adjoint ideal of R. We also say that I is P-weakly primal ideal. Of Course a proper ideal P of R is said to be weakly prime if $0 \neq ab \in R$ implies either $a \in P$ or $b \in P$ [1].

Throughout this paper X is a completely regular Hausdorff space. Let $C(X)$ be the ring of all real-valued continuous functions on X. We list here some standard facts, terminology and notation for reference. The set of all positive integers is denoted by N. In any ring $C(X)$, the constant function whose value is r is designated by **r**. For any $f \in C(X)$, we write $Z(f)$ for the set

$$\{x \in X : f(x) = o\}$$

$Z(f)$ is called a zero-set in X. For $C' \subseteq C(X)$, we write $Z[C']$ to designate the family of zero-sets in C', that is

$$Z[C'] = \{Z(f) : f \in C'\}.$$

On the other hand, the family $Z[C(X)]$ of all zero-sets in X will also be denoted, for simplicity, by $Z(X)$.

We shall occasionally refer to the ring $C(X)$ itself as an improper ideal. Thus, the word *ideal*, unmodified, will always mean *proper* ideal. For any ideal I in $C(X)$ and $f \in C(X)$, the residue class of f modulo I is written $I(f)$. The ideal I is called a z-ideal if $Z(f) \in Z[I]$ implies that $f \in I$. By a prime z-filter, we shall mean a z-filter $\mathcal{F}$ on X with the property that whenever the union of two zero-sets belongs to $\mathcal{F}$, then at least one of them belongs to $\mathcal{F}$. For any undefined terms here the reader may consult [6].

The structure of the family of prime ideals in $C(X)$ has been extensively studied in [10, 11, 12]. In this paper we first study the basic properties of the family of primal ideals of the ring $C(X)$. We define primal, weakly prime and weakly primal z-filters on X. Then we show that there exists a one-to-one correspondence between the set of all primal z-ideals (resp. weakly prime, weakly primal) of $C(X)$ and the set of all primal (resp. weakly prime, weakly primal) z-filters on X.

2 Primal ideals in C(X)

In this section we discuss on primal ideals of $C(X)$ and consider the relations between primal ideals of $C(X)$ and primal z-filters on X.

Definition 2.1. Let $\mathcal{F}$ be a z-filter on X. An element Z in $Z(X)$ is called z-prime to $\mathcal{F}$ provided that $Z \cup Z' \in \mathcal{F}$ ($Z' \in Z(X)$) implies that $Z' \in \mathcal{F}$.

Lemma 2.2. *Let $\mathcal{F}$ be a z-filter on X and denote by $T(\mathcal{F})$ the set of all elements of $Z(X)$ that are not z-prime to $\mathcal{F}$, that is*

$$T(\mathcal{F}) = \{Z \in Z(X) | Z \cup Z' \in \mathcal{F} \textit{ for some } Z' \in Z(X) \setminus \mathcal{F}\}.$$

Then:

(1) $\mathcal{F} \subseteq T(\mathcal{F})$, and

(2) If $T(\mathcal{F})$ forms a z-filter on X, then it is a prime z-filter.

Proof. (1) For every $Z(f) \in \mathcal{F}$, the relations

$$Z(f) \cup Z(1) \in \mathcal{F} \text{ with } Z(1) = \emptyset \notin \mathcal{F}$$

imply that $Z(f)$ is not z-prime to $\mathcal{F}$. Hence $Z(f) \in T(\mathcal{F})$ and so $\mathcal{F} \subseteq T(\mathcal{F})$.

(2) Assume that $Z(f) \cup Z(g) \in T(\mathcal{F})$ but $Z(f) \notin T(\mathcal{F})$. There exists $Z(h) \in Z(X) \setminus \mathcal{F}$ such that $Z(f) \cup Z(g) \cup Z(h) \in \mathcal{F}$. As $Z(f)$ is z-prime to $\mathcal{F}$, we get $Z(g) \cup Z(h) \in \mathcal{F}$ with $Z(h) \in Z(X) \setminus \mathcal{F}$, that is $Z(g)$ is not z-prime to $\mathcal{F}$. So $Z(g) \in T(\mathcal{F})$. Therefore $T(\mathcal{F})$ is a prime z-filter.

□

Definition 2.3. A z-filter $\mathcal{F}$ on X is called a primal z-filter if $T(\mathcal{F})$ forms a z-filter on X. In this case, by Lemma 2.2, the z-filter $\mathcal{G} := T(\mathcal{F})$ is a prime z-filter, called the adjoint prime z-filter of $\mathcal{F}$. We will also say that $\mathcal{F}$ is a $\mathcal{G}$-primal z-filter.

Theorem 2.4. *(1) Let I and P be z-ideals of $C(X)$. If I is a P-primal of $C(X)$, then $Z[I]$ is a primal z-filter on X.*

(2) If $\mathcal{F}$ is a primal z-filter on X, then $Z^{\leftarrow}[\mathcal{F}]$ is a primal z-ideal of $C(X)$.

Proof. (1) We know that P is a prime ideal of $C(X)$. So, by [6, Theorem P. 29], $Z[P]$ is a prime z-filter on X. Assume that $Z(f) \in Z(X)$ is not z-prime to $Z[I]$. There exists $Z(g) \in Z(X)\backslash Z[I]$ such that $Z(f) \cup Z(g) \in Z[I]$. So $Z(fg) \in Z[I]$ and I z-ideal gives $fg \in I$. This implies that $g \in C(X)\backslash I$ with $fg \in I$, that is f is not prime to I. So $f \in P$ and so $Z(f) \in Z[P]$. Now assume that $Z(h) \in Z[P]$. As P is a z-ideal, $h \in P$. Therefore $hk \in I$ for some $k \in C(X)\backslash I$. It follows that $Z(h) \cup Z(k) \in Z[I]$ where $Z(k) \in Z(X)\backslash Z[I]$. Thus $Z(h)$ is not z-prime to $Z[I]$. We have already shown that $Z[P]$ consists exactly of elements of $Z(X)$ that are not z-prime to $Z[I]$. This shows that $Z[I]$ is a $Z[P]$-primal z-filter.

(2) Clearly $Z^{\leftarrow}[\mathcal{F}]$ is a z-ideal of $C(X)$. Assume that $\mathcal{F}$ is $\mathcal{G}$-primal. By Lemma 2.2 and [6, Theorem p.29], $Z^{\leftarrow}[\mathcal{G}]$ is a prime z-ideal of $C(X)$. It is enough to show that $Z^{\leftarrow}[\mathcal{G}] = T(Z^{\leftarrow}[\mathcal{F}])$. If $f \in C(X)$ is not prime to $Z^{\leftarrow}[\mathcal{F}]$, then $fg \in Z^{\leftarrow}[\mathcal{F}]$ for some $g \in C(X)\backslash Z^{\leftarrow}[\mathcal{F}]$. This implies that $Z(f) \cup Z(g) \in \mathcal{F}$ with $Z(g) \notin \mathcal{F}$, that is $Z(f)$ is not z-prime to $\mathcal{F}$. Therefore $Z(f) \in \mathcal{G}$ and so $f \in Z^{\leftarrow}[\mathcal{G}]$. Conversely, assume that $h \in Z^{\leftarrow}[\mathcal{G}]$. Then as $Z(h)$ is not z-prime to $\mathcal{F}$, there exists $Z(k) \in Z(X)\backslash\mathcal{F}$ with $Z(h) \cup Z(k) \in \mathcal{F}$. It follows that $hk \in Z^{\leftarrow}[\mathcal{F}]$ with $k \in C(X)\backslash Z^{\leftarrow}[\mathcal{F}]$, that is h is not prime to $Z^{\leftarrow}[\mathcal{F}]$.

□

Lemma 2.5. *Every prime z-filter is primal.*

Proof. Let $\mathcal{F}$ be a prime Z-filter. Then $Z^{\leftarrow}[\mathcal{F}]$ is a prime z-ideal of $C(X)$ by [6, Theorem p. 29]. But in any commutative ring, every prime ideal is primal. Hence $Z^{\leftarrow}[\mathcal{F}]$ is a primal z-ideal of $C(X)$. Now the result follows from Theorem 2.4. □

An annihilator condition on a commutative ring R is property (A). R is said to have property (A) if every finitely generated ideal I contained in $Z(R)$ has a nonzero annihilator ([7]). Y. Quentel introduced property (A) in [14], calling it condition (C). Faith in [4] studied rings with property (A) and called such rings McCoy. An example of a McCoy ring is a Noetherian ring. However, the property (A) fails for some non-Noetherian rings [9, p. 63]. To avoid the ambiguity we call such rings F-McCoy.

Recently the concept of rings with property (A) has been generalized to noncommutative rings [8]. Let R be an associative ring with identity. We write $Z_l(R)$ and $Z_r(R)$ for the set of all left zero-divisors of R and the set of all right zero-divisors of R, respectively. Then the ring R has right (left) Property (A) if for every finitely generated two-sided ideal $I \subseteq Z_l(R)$ ($Z_r(R)$), there exists nonzero $a \in R$ ($b \in R$) such that $Ia = 0$ ($bI = 0$). A ring R is said to have Property (A) if R has right and left Property (A).

Nielsen in [13] defined another class of rings and called it McCoy. This paper is on the basis of some recent papers devoted to this new class of rings. Let R be an associative ring with 1 (not necessarily commutative). R is said to be right McCoy when the equation $f(x)g(x) = 0$ over $R[x]$, where $f(x), g(x) \neq 0$, implies there exists a nonzero $r \in R$ with $f(x)r = 0$. Left McCoy rings are defined similarly. If a ring is both left and right McCoy then R is called a McCoy ring. This class of McCoy rings includes properly the class of Armendariz rings introduced in [15], which is extensively studied in the last years.

Let R be a commutative ring with identity. Then concepts "F-McCoy ring" and "McCoy ring" are different concepts. In fact neither implies the other. For example, if R is a reduced ring, then it is McCoy by [13, Theorem 2]. But we know that there are reduced rings which are not F-McCoy. Also if we let Z_4 to be the ring of integers modulo 4, then, by [8, Theorem 2.1], $M_2(Z_4)$, the set of all 2×2 matrices over Z_4, has Property (A) but it is not right McCoy by [16].

The commutative ring R is called *strongly coprimal* (resp. Super coprimal) if for arbitrary $a, b \in Z(R)$ (resp. finite subset E of $Z(R)$) the annihilator of $\{a, b\}$ (resp. annihilator of E) in

R is non-zero. Clearly, R is a strongly coprimal if and only if R is both a coprimal and a F-McCoy ring. In the following Theorem, we give some conditions under which $C(X)$ is strongly coprimal (resp. super coprimal) [17].

Theorem 2.6. *(1) The ring $C(X)$ is strongly primal if and only if $intZ(f_1) \cap intZ(f_2) \neq \emptyset$ for every f_1 and f_2 in $Z(C(X))$.*

(2) The ring $C(X)$ is super primal if and only if $intZ(f_1) \cap intZ(f_2) \cap ... \cap intZ(f_n) \neq \emptyset$ for every $f_1, f_2, ..., f_n$ in $Z(C(X))$.

Proof. (1) Assume that $C(X)$ is strongly primal. Then, for every $f_1, f_2 \in Z(C(X))$, $Ann\{f_1, f_2\} \neq 0$. So there exists a nonzero element $g \in C(X)$ with $gf_1 = gf_2 = 0$. In this case, for every $x \in X$, if $g(x) \neq 0$ we have $f_1(x) = f_2(x) = 0$, that is $coZ(g) \subseteq Z(f_1) \cap Z(f_2)$. Therefore $intZ(f_1) \cap intZ(f_2) \neq \emptyset$. Now Suppose that $intZ(f_1) \cap intZ(f_2) \neq \emptyset$ for every $f_1, f_2 \in Z(C(X))$. Set $Y = intZ(f_1) \cap intZ(f_2)$ and define the map $g : X \to R$ as follows:

$$g(x) = \begin{cases} 1, & x \in Y; \\ 0, & x \in X - Y. \end{cases}$$

Then g is a continuous function. So $0 \neq C(X)$, and for every $x \in X$, $g(x)f_1(x) = 0$, $g(x)f_2(x) = 0$, that is $gf_1 = 0 = gf_2$. Consequently $Ann(\{f_1, f_2\} \neq 0$. Thus $C(X)$ is strongly primal.

(2) The proof of this part is completely to that of part (1).

□

3 Weakly prime and Weakly primal ideals

The concept of weakly prime and weakly primal ideals in a commutative ring introduced in [1, 2]. In this section we define the weakly prime and weakly primal z-filters on X and then we investigate the relations among these classes of z-filters, weakly prime and weakly primal ideals.

Definition 3.1. Assume that $\mathcal{F}$ is a z-filter on X. $\mathcal{F}$ is said to be a weakly prime z-filter whenever, for $Z, Z' \in Z(X)$, $X \neq Z \cup Z' \in \mathcal{F}$ implies that either $Z \in \mathcal{F}$ or $Z' \in \mathcal{F}$.

Lemma 3.2. *Every prime z-filter is weakly prime.*

Theorem 3.3. *(1) If P is a weakly prime z-ideal in $C(X)$, then $Z[P]$ is a weakly prime z-filter on X.*

(2) If $\mathcal{F}$ is a weakly prime z-filter on X, then $Z^{\leftarrow}[\mathcal{F}]$ is a weakly prime z-ideal of $C(X)$.

Proof. (1) Let P be a weakly prime z-ideal in $C(X)$. Clearly $Z[P]$ is a z-filter on X. Assume that $X \neq Z(f) \cup Z(g) \in Z[P]$ for some $Z(f), Z(g) \in Z(X)$. Then $Z(0) \neq Z(fg) = Z(f) \cup Z(g) \in Z[P]$. Since P is a z-ideal, we have $0 \neq fg \in P$. Therefore either $f \in P$ or $g \in P$ since P is assumed to be weakly prime. It follows that either $Z(f) \in Z[P]$ or $Z(g) \in Z[P]$, that is $Z[P]$ is a weakly prime z-filter.

(2) Assume that $\mathcal{F}$ is a weakly prime z-filter on X. In this case $P = Z^{\leftarrow}[\mathcal{F}]$ is a z-ideal of $C(X)$. Suppose that $f, g \in C(X)$ are such that $0 \neq fg \in P$. Then, $X \neq Z(f) \cup Z(g) = Z(fg) \in Z[Z^{\leftarrow}[\mathcal{F}]] = \mathcal{F}$. Since $\mathcal{F}$ is weakly prime, either $Z(f) \in F$ or $Z(g) \in F$. Thus either $f \in P$ or $g \in P$, and this implies that P is a weakly prime z-ideal of $C(X)$.

□

Definition 3.4. Assume that $\mathcal{F}$ is a z-filter on X. An element Z in $Z(X)$ is called z-weakly prime to $\mathcal{F}$ provided that $X \neq Z \cup Z' \in \mathcal{F}$ ($Z' \in Z(X)$) implies that $Z' \in \mathcal{F}$.

Remark 3.5. Let $\mathcal{F}$ be a z-filter on X. Then:

(1) X (i.e. $Z(0)$) is always z-weakly prime to $\mathcal{F}$

(2) If $Z \in Z(X)$ is z-prime ro $\mathcal{F}$, then it is z-weakly prime to $\mathcal{F}$.

Lemma 3.6. *Let $\mathcal{F}$ be a z-filter on X and denote by $W(\mathcal{F})$ the set of all elements of $Z(X)$ that are not z-weakly prime to $\mathcal{F}$. Then:*

(1) $\mathcal{F} \subseteq W(\mathcal{F}) \cup \{X\}$, and

(2) If $W(\mathcal{F}) \cup \{X\}$ forms a z-filter on X, then it is a weakly prime z-filter.

Proof. (1) For every $Z(f) \in \mathcal{F} - \{X\}$ we have:

$$X \neq Z(f) = Z(f) \cup Z(1) \in \mathcal{F}$$

with $Z(1) = \emptyset \notin \mathcal{F}$. This implies that $Z(f)$ is not z-weakly prime to $\mathcal{F}$. Hence $Z(f) \in W(\mathcal{F})$. Therefore $\mathcal{F} \subseteq W(\mathcal{F}) \cup \{X\}$.

(2) Let $Z(f), Z(g) \in Z(X)$ be such that $X \neq Z(f) \cup Z(g) \in W(\mathcal{F}) \cup \{X\}$. Suppose also that $Z(f) \notin W(\mathcal{F}) \cup \{X\}$, that is $Z(f)$ is z-weakly prime to $\mathcal{F}$. There exists $Z(h) \in Z(X) \backslash \mathcal{F}$ such that $X \neq Z(f) \cup Z(g) \cup Z(h) \in \mathcal{F}$. Now $Z(f)$ is z-weakly prime to $\mathcal{F}$ implies that $X \neq Z(g) \cup Z(h) \in \mathcal{F}$ with $Z(h) \in Z(X) \backslash \mathcal{F}$, that is $Z(g)$ is not z-weakly prime to $\mathcal{F}$. hence $Z(g) \in W(\mathcal{F}) \cup \{X\}$, that is $W(\mathcal{F}) \cup \{X\}$ is a weakly prime z-filter on X.

□

Definition 3.7. Assume that $\mathcal{F}$ is a z-filter on X. $\mathcal{F}$ is called a weakly primal z-filter on X if $W(\mathcal{F}) \cup \{X\}$ forms a z-filter on X. In this case, by Lemma 3.6, the z-filter $\mathcal{G} := W(\mathcal{F}) \cup \{X\}$ is a weakly prime z-filter, called the adjoint weakly prime z-filter of $\mathcal{F}$. In this case we sat that $\mathcal{F}$ is a $\mathcal{G}$-weakly primal z-filter.

Theorem 3.8. *Every weakly prime z-filter on X is weakly primal.*

Proof. Assume that $\mathcal{F}$ is a weakly prime z-filter on X. Then $\mathcal{F} \subseteq W(\mathcal{F}) \cup \{X\}$ by Lemma 3.6. Now pick an element $Z(f) \in W(\mathcal{F}) \cup \{X\}$. If $Z(f) = X$, then $Z(f) \in \mathcal{F}$. So assume that $Z(f) \neq X$. Then $Z(f)$ is z-weakly prime to $\mathcal{F}$. So there exists $Z(g) \in Z(X) - \mathcal{F}$ with $X \neq Z(f) \cup Z(g) \in \mathcal{F}$. Since $\mathcal{F}$ is a weakly prime z-filter we get $Z(f) \in \mathcal{F}$, that is $W(\mathcal{F}) \cup \{X\} \subseteq \mathcal{F}$. Hence $\mathcal{F} = W(\mathcal{F}) \cup \{X\}$, and this implies that $\mathcal{F}$ is an $\mathcal{F}$-weakly primal z-filter. □

Theorem 3.9. *(1) Let I be a P-weakly primal ideal of $C(X)$, where I and P are both z-ideals. Then $Z[I]$ is a primal z-filter on X with the weakly prime adjoint z-filter $Z[P]$.*

(2) If $\mathcal{F}$ is a $\mathcal{G}$-weakly primal z-filter on X, then $Z^{\leftarrow}[\mathcal{F}]$ is a weakly primal z-ideal of $C(X)$ with the weakly prime adjoint ideal $Z^{\leftarrow}[\mathcal{F}]$.

Proof. The proof is completely similar to that of Theorem 2.4 and we omit it. □

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