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Characterization of lower quasi-modular extensions

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Abstract Let K/k be a purely inseparable extension of characteristic $p > 0$ and of finite size. We recall that K/k is modular if for every $n \in \mathbb{N}$, K^{p^n} and k are $k \cap K^{p^n}$-linearly disjoint. A natural generalization of this notion is to say that K/k is lq-modular if K is modular over a finite extension of k. Our main objective is to extend in definite form some results and definitions of the lq-modularity that have already been obtained in the case limited by the finiteness condition imposed on $[k : k^p]$ in a rather general framework (framework of extensions of finite size called also q-finite extensions). First, by means of invariants, we characterize the lq-modularity of a q-finite extension. Moreover, we give a necessary and sufficient condition for K/k to be lq-modular. As a consequence, the lq-modularity is stable up to a finite extension of the choice of the ground field. This makes it possible to reduce the study of lq-modularity to the case of relatively perfect extensions.

1 Introduction

Let K/k be a purely inseparable extension of characteristic $p > 0$. A subset B of K is called an r-basis (relative p-basis) of K/k if $K = k(K^p)(B)$ and for every $x \in B$, $x \notin k(K^p)(B \setminus \{x\})$. By virtue of ([1], III, p. 49, Corollary 3) and the exchange property of r-independence, we deduce that any extension admits an r-basis and that the cardinality of any r-basis is invariant. If K/k has an exponent, we immediately check that B is an r-basis of K/k if and only if B is a minimal generator of K/k. Taking account of ([1], III, p. 25, Proposition 2), we can control the size of any purely inseparable extension K/k by means of the irrationality degree of K/k defined by $di(K/k) = \sup_{n \in \mathbb{N}}(|G_n|)$ where G_n is a minimal generator of $k^{p^{-n}} \cap K/k$. In particular, the size measurement of an extension is an increasing function with respect to inclusion. More precisely, for every chain of purely inseparable extensions, $k \subseteq L \subseteq L' \subseteq K$, we have $di(L'/L) \leq di(K/k)$. Henceforth, any extension of finite size will be called a q-finite extension. It is clear that the q-finite extensions contain strictly all extensions of k whose degree $[k : k^p]$ is finite. Moreover, we show that any decreasing family of q-finite extensions is stationary. We also recall that K/k is said to be modular if and only if for any $n \in \mathbb{N}$, K^{p^n} and k are $K^{p^n} \cap k$-linearly disjoint. A natural generalization of this notion is to say that K/k is lq-modular if K is modular over a finite extension of k. Knowing that [7], [4] and [6] are entirely devoted to the study of this notion in the local case delimited by the hypothesis $[k : k^p]$ is finite, in this paper we want to extend some results of the lq-modularity that have already been obtained locally in a fairly renovated framework (this is the framework of the q-finite extensions) in a definitive form.

First, we begin by characterizing the lq-modularity of a q-finite extension by means of invariants. In this regard, we give a necessary and sufficient condition for K/k to be lq-modular, and consequently the lq-modularity is stable up to a finite extension of the choice of the ground field. This makes it possible to reduce the study of lq-modularity to the case of relatively perfect extensions.

It should be pointed that k always designates a commutative field of characteristic $p > 0$, Ω an algebraic closure of k and out that all the extensions involved in this paper are purely inseparable subextensions of Ω. It is also convenient to denote sometimes $[k, K]$ the set of intermediate field of an extension K/k.

2 Irrationality degree of a purely inseparable extension

Definition 2.1 ([11], Definition 1.3). Let K/k be a purely inseparable extension. A subset G of K is said to be an r-generator (generator) of K/k if $K = k(G)$, and if moreover for each $x \in G$, $x \notin k(G\backslash x)$, then G will be called a minimal r-generator of K/k.

Definition 2.2 ([11], Definition 1.2). Given an extension K/k of characteristic $p > 0$, and a subset B of K. We say that B is an r-basis (relative p-basis) of K/k, if B is a minimal r-generator of $K/k(K^p)$. B is said to be r-free (or r-independent) over k, if B is an r-basis of $k(B)/k$, in the opposite case B is said to be r-dependent over k.

It is known that, the r-dependence in $K/k(K^p)$ is a dependence relation (for example see [9], Lemma 6.1), and, consequently, according to ([9], Theorem 1.3) we obtain:

- *Every extension K/k has an r-basis and any two r-bases of K/k have the same cardinality.*
- *From any r-generator of $K/k(K^p)$, we can extract an r-basis of K/k.*
- *Any r-free subset of $k(K^p)$ can be completed to an r-basis of K/k.*
- *The r-basis of K/k is exactly the maximal subset of K which is r-independent over k.*
- *The r-basis of K/k is exactly the minimal r-generator of $K/k(K^p)$.*

Recall that K is said to have an exponent (or, to be of bounded exponent) over k, if there exists $e \in \mathbb{N}$ such that $K^{p^e} \subseteq k$, and the smallest integer that satisfies this relation will be called the exponent (or height) of K/k. Taking into account ([11], Corollary 1.6), if K/k has an exponent, a subset B of K is an r-basis of K/k if and only if B is a minimal r-generator of K/k. Let us consider a purely inseparable extension K/k of characteristic $p > 0$, clearly for any $n \in \mathbb{N}$, $k^{p^{-n}} \cap K/k$ has an exponent, and in addition, the cardinality of any minimal r-generator of $k^{p^{-n}} \cap K/k$ depends only on n.

Definition 2.3. The invariant $di(K/k) = \sup_{n\in\mathbb{N}}(|B_n|)$ (|.| designates the cardinality) will be called the irrationality degree of K/k.

Here the sup is taken in the sense of ([1], III, p. 25, Proposition 2). Furthermore, the size measurement of an extension grows as a function of inclusion. More specifically, for every purely inseparable extensions $k \subseteq L \subseteq L' \subseteq K$, $di(L'/L) \leq di(K/k)$ (cf. [8], Theorem 3.8). In addition, $di(K/k) = \sup(di(L/k))_{L\in[k,K]}$.

3 Quasi-finite extensions

Definition 3.1. Any extension of finite irrationality degree is called q-finite (quasi-finite) extension.

In the following, for each $n \in \mathbb{N}$, k_n always designates $k^{p^{-n}} \cap K$. It is immediately verified that:

(1) *K/k is finite if and only if K/k is q-finite of bounded exponent.*

(2) *The q-finitude is transitive. In particular, for every $n \in \mathbb{N}$, $K/k(K^{p^n})$ and k_n/k are finite.*

(3) *There exists $n_0 \in \mathbb{N}$, for every integer $n \geq n_0$, $di(k_n/k) = di(K/k)$.*

We recall that K/k is relatively perfect if $k(K^p) = K$. It is easy to verify that the property "to be relatively perfect" is stable by any product covering k, and as a result, there exists a largest subfield of K relatively perfect over k called the relatively perfect closure of K/k, and is denoted by $rp(K/k)$ (cf. [5], p. 50). In particular, for every $L \in [k : K]$, K/L is finite implies $rp(K/k) \subset L$, and if moreover K/k is relatively perfect, then K/L is finite involves $L = K$. In addition, we will see some immediate applications of ([5], Lemma 1.2) that will be useful later.

Proposition 3.2. *Let K/k be a q-finite extension. The sequence $(k(K^{p^n}))_{n\in\mathbb{N}}$ stops over $rp(K/k)$ from a n_0. In particular, $K/rp(K/k)$ is finite.*

We obtain in particular the following result:

Corollary 3.3. *The relatively perfect closure of a q-finite extension K/k is not trivial. More precisely, K/k is of unbounded exponent so is $rp(K/k)/k$.*

Proposition 3.4. *For every q-finite extension K/k, there exists $n \in \mathbb{N}$ such that K/k_n is relatively perfect. Moreover, $k_n(rp(K/k)) = K$.*

Proposition 3.5. *Let $k \subseteq L \subseteq K$ be q-finite and relatively perfect extensions over k. Then $L = K$ if and only if $di(L/k) = di(K/k)$.*

Proof. The result holds immediately since $di(K/k) = di(K/L) + di(L/k)$ (cf. [8], Proposition 4.8), and so $di(L/k) = di(K/k)$ is equivalent to $di(K/L) = 0$, or again $K = L$. □

Here is an extremely important application of the previous proposition

Proposition 3.6. *Any decreasing sequence of a q-finite extension is stationary.*

Proof. Let $(K_n/k)_{n\in\mathbb{N}}$ be a decreasing sequence of subextensions of K/k and $(F_i/k)_{i\in\mathbb{N}}$ the sequence associated with their relatively perfect closures. Taking into account ([8], Theorem 3.8), the sequence of integers $(di(F_i/k))_{i\in\mathbb{N}}$ is decreasing, hence stationary from an integer n_0, or again according to the previous proposition, for each integer $n \geq n_0$, $F_i = F_{n_0}$. By virtue of monotony, for every integer $n \geq n_0$, $[K_{n+1} : F_{n_0}] \leq [K_{n_0} : F_{n_0}]$. In other words, the sequence of integers $([K_n : F_{n_0}])_{n\geq n_0}$ is decreasing, hence stationary from an integer e, or again for each integer $n \geq e$, $[K_n : F_{n_0}] = [K_e : F_{n_0}]$. As for each integer $n \geq e$, $K_n \subseteq K_e$, we deduce that $K_n = K_e$ for every integer $n \geq e$. □

4 Lower quasi modularity

4.1 Invariant of the lq-modularity of an extension

Henceforth and unless otherwise stated, K/k denotes a q-finite extension of unbounded exponent, and for all $j \in \mathbb{N}$, $k_j = k^{p^{-j}} \cap K$ and $U_s^j(K/k) = j - o_s(k_j/k)$ for each $s \in \mathbb{N}^*$ where $o_s(k_j/k)$ designates the s-th exponent of k_j/k (refer to [2] or [3] for full details of exponents).

The definition below is similar to the one given in ([6], Definition 3.1).

Definition 4.1. The first natural integer i_0 for which the sequence $(U_{i_0}^j(K/k))_{j\in\mathbb{N}}$ is unbounded is called the invariant of the lq-modularity of K/k and is denoted $Ilqm(K/k)$.

We verify immediately that $2 \leq Ilqm(K/k)$. Moreover, the following result is an immediate consequence of ([3], Proposition 8.3).

Proposition 4.2. *For every positive integer s, the sequence $(U_s^j(K/k))_{j\in\mathbb{N}}$ is increasing.*

Proof. It is immediate that $o_s(k_n/k) \leq o_s(k_{n+1}/k) \leq o_s(k_n/k) + 1$ since ${k_{n+1}}^p \subseteq k_n$. Hence $n + 1 - o_s(k_{n+1}/k) \geq n - o_s(k_n/k)$; and consequently $(U_s^j(K/k))_{j\in\mathbb{N}}$ is increasing. □

It follows immediately that:

- For any integer $s \geq Ilqm(K/k)$, $\lim\limits_{n\to+\infty}(U_s^j(K/k)) = +\infty$.
- For every integer $s \in [1, Ilqm(K/k)[$, the sequence $(U_s^j(K/k))_{j\in\mathbb{N}}$ is bounded; and therefore, for any integer $n \geq \sup\limits_{j\in\mathbb{N}}(\sup(U_s^j(K/k)))$ (s<Ilqm(K/k)), we obtain $U_s^n(K/k) = U_s^{n+1}(K/k)$, or again, $o_s(k_{n+1}/k) = o_s(k_n/k) + 1$.

This leads to:

Proposition 4.3. *Let $k \subseteq K_1 \subseteq K_2$ be q-finite extensions of unbounded exponent. For each $s \in \mathbb{N}^*$, for each $n \in \mathbb{N}$, we have $U_s^n(K_1/k) \geq U_s^n(K_2/k)$. In addition, $Ilqm(K_1/k) \leq Ilqm(K_2/k)$, and the equality holds if K_2/K_1 is finite.*

Proof. At first, for each $n \in \mathbb{N}$, we have $k^{p^{-n}} \cap K_1 \subseteq k^{p^{-n}} \cap K_2$, and by passing to exponents, we get $o_s(k^{p^{-n}} \cap K_1/k) \leq o_s(k^{p^{p^{-n}}} \cap K_2/k)$. Whence, $U_s^n(K_2/k) \leq U_s^n(K_1/k)$, and consequently $Ilqm(K_1/k) \leq Ilqm(K_2/k)$.

In the case where K_2/K_1 is finite, let $e = o_1(K_2/K_1)$, therefore for every integer $n > e$, $k(k^{p^{-n}} \cap K_2^{p^e}) \subseteq k^{p^{-n}} \cap K_1$. Taking into account ([3], Propositions 5.3 and 8.3), we obtain $o_s(k^{p^{-n-e}} \cap K_2/k) - e \leq o_s(k^{p^{-n}} \cap K_1/k)$; and consequently $e + n - o_s(k^{p^{-n-e}} \cap K_2/k) \geq n - o_s(k^{p^{-n}} \cap K_1/k)$, or again $U_s^n(K_1/k) \leq U_s^{n+e}(K_2/k)$. Hence $Ilqm(K_2/k) \leq Ilqm(K_1/k)$, and as a result, $Ilqm(K_1/k) = Ilqm(K_2/k)$. □

4.2 Characterization of the lq-modularity of a q-finite extension

We recall that an extension K/k is said to be modular if and only if for each $n \in \mathbb{N}$, K^{p^n} and k are $K^{p^n} \cap k$-linearly disjoint. This notion has been for the first time by Sweedleer in [12], she characterizes the purely inseparable extensions which are tensor product of simple extensions over k, it is the equivalent of the fundamental concept Galois theory. Furthermore, if there exists a subset B of a given field K such that $K \simeq \otimes_k(\otimes_k k(a))_{a\in B}$, necessarily B will be an r-basis of K/k and it will be called subsequently a modular r-basis (or a subbase) of K/k ([14], p. 435). In particular, according to Sweedleer's theorem, if K/k has an exponent, it is equivalent to say that:

(i) *K/k has a modular r-basis.*

(ii) *K/k is modular.*

Definition 4.4 ([4], Definition 5.3). Let K/k be a q-finite extension. K/k is said to be lq-modular if K is modular over a finite extension of k.

By virtue of ([13], Proposition 1.2), there exists a smallest subextension m/k of K/k such that K/m is modular, and from now on, we denote $m = lm(K/k)$. Obviously K/k is lq-modular if and only if $lm(K/k)/k$ is finite. Furthermore, some remarkable examples of lq-modular extensions are given in [4] in order to illustrate this notion. However, here is a nontrivial example of purely inseparable extension that is not lq-modular.

Example 4.5. Let k_0 be a perfect field of characteristic $p > 0$ and (X, Z_1, Z_2) an algebraically independent family over k_0. We note $k = k_0(X, Z_1, Z_2)$ and for each $n \in \mathbb{N}^*$, $K_n = k(X^{p^{-2n}}, \theta_n)$, where $\theta_1 = Z_1^{p^{-1}}X^{p^{-2}} + Z_2^{p^{-1}}$ and for every $n \geq 2$,

$$\theta_n = Z_1^{p^{-1}}X^{p^{-2n}} + (\theta_{n-1})^{p^{-1}} = Z_1^{p^{-1}}X^{p^{-2n}} + Z_1^{p^{-2}}X^{p^{-2n+1}} + \cdots + Z_1^{p^{-n}}X^{p^{-n-1}} + Z_2^{p^{-n}}.$$

We have $lm(K/k) = k(X^{p^{-\infty}})$, and so K/k is not lq-modular. In fact, let $m = lm(K/k)$. First, it's clear that $K/k(X^{p^{-\infty}})$ is modular, therefore $m \subseteq k(X^{p^{-\infty}})$. Suppose next the existence of a positive integer n such that $X^{p^{-n+1}} \in m$ and $X^{p^{-n}} \notin m$. By construction, we have $\theta_1{}^p = Z_1(X^{p^{-2}})^p + Z_2 = Z_1X^{p^{-1}} + Z_2$ and

$$[Z_1{}^{p^{-1}}X^{p^{-2n}} + \cdots + Z_1{}^{p^{-n}}X^{p^{-n-1}} + Z_2{}^{p^{-n}}]^{p^n} = Z_1{}^{p^{n-1}}X^{p^{-n}} + \cdots + Z_1X^{p^{-1}} + Z_2$$

for every integer $n \geq 2$, according to ([7], Lemma 3.7), we have $Z_1{}^{p^{-1}} \in K$. It follows that $Z_2{}^{p^{-1}} \in K$ since $Z_1{}^{p^{-1}}X^{p^{-2}} + Z_2{}^{p^{-1}} \in K$, and consequently $di(k(X^{p^{-1}}, Z_1{}^{p^{-1}}, Z_2{}^{p^{-1}})/k) = 3 \leq di(K/k) = 2$, contradiction.

The following result characterizes the lq-modular extensions by means of variation of exponents. More specifically, we have:

Theorem 4.6. *Let K/k be a q-finite extension and $t = di(rp(K/k)/k)$. The following statements are equivalent:*

(1) *K/k is lq-modular.*

(2) *There exists a natural number j such that $K/k^{p^{-j}} \cap K$ is modular.*

(3) *For each integer* $s \in [1, t]$*, the sequence* $(U_s^j(K/k))_{j\in\mathbb{N}}$ *is bounded.*

(4) $Ilqm(K/k) = t + 1$.

Proof. It's clear that $(2) \Leftrightarrow (3)$. Furthermore, taking into account Proposition 3.4, there exists a positive integer j_0 such that K/k_{j_0} is relatively perfect and $k_{j_0}(rp(K/k)) = K$. In particular, by ([8], Proposition 4.18), $di(K/k_{j_0}) = di(rp(K/k)/k) = t$. Suppose next that condition (1) holds. There are then two cases:

1^{st} *case:* If K/k is modular, by virtue of ([8], Proposition 6.3), for every integer $j \geq j_0$, we have k_j/k_{j_0} is equiexponential of exponent $j - j_0$ and $di(k_j/k_{j_0}) = t$. Hence for each $s \in \{1, \dots, t\}$, $U_s^j(K/k) = U_s^{j+1}(K/k)$.

2^{nd} *case:* If K is modular over a finite extension L of k, taking into account the finitude of L/k, there exists a natural number e_1 such that $L \subseteq k_{e_1}$. Therefore, $L^{p^{-j}} \cap K \subseteq k_{e_1+j}$, and by passing to exponents, for every $s \in \mathbb{N}^*$, $o_s(L^{p^{-j}} \cap K) \leq o_s(k_{e_1+j})$; let therefore $U_s^{e_1+j}(K/k) \leq e_1 + U_s^j(K/L)$. Hence, the sequence $(U_s^j(K/k))_{j\in\mathbb{N}}$ is stationary for each $s \in \{1, \dots, t\}$ (namely $rp(K/L) = L(rp(K/k))$ and L/k is finite, therefore $di(rp(K/L)/L) = di(L(rp(K/k))/L) = di(rp(K/k)/k) = t$ (cf. [8], Proposition 4.18)).

Conversely, if condition (2) is satisfied, there exists $m_0 \geq \sup(e(K/k), j_0)$, for every integer $j \geq m_0$, for every $s \in \{1, \dots, t\}$, we have $o_s(k_{j+1}/k) = o_s(k_j/k) + 1$ (and $di(k_j/k_{m_0}) = t$). We concluded that k_j/k_{j_0} is equiexponential, and thus modular. Hence $K = \bigcup_{j>m_0} k_j$ is modular over k_{j_0}. □

The fact that the sequence $(U_s^j(K/k))_{s\in\mathbb{N}^*}$ is increasing (j being a fixed natural integer), condition (3) of the above theorem reduces to $(U_t^j(K/k))_{j\in\mathbb{N}}$ is bounded, and consequently K/k is lq-modular if and only if the sequence $(U_t^j(K/k))_{j\in\mathbb{N}}$ is bounded.

Let k be a commutative field of characteristic $p > 0$ and Ω an algebraic closure of k. We define, as in ([4]), the relation $\sim$ on Ω as follows: $k_1 \sim k_2$ if and only if $k_1 \subseteq k_2$ and k_2/k_1 is finite or $k_2 \subseteq k_1$ and k_1/k_2 is finite, and we get similar results when we extend the lq-modularity to q-finite extensions. We first check that $\sim$ is reflexive, symmetric, however $\sim$ is generally nontransitive. Moreover, for any q-finite extension K_1/k, the application of lower modularity:

$$\begin{array}{rcl} lm : [k : K_1] & \longmapsto & [k : K_1] \\ L & \longrightarrow & lm(K_1/L), \end{array}$$

is compatible with the relation $\sim$. More specifically, we have:

Proposition 4.7. *Let* $k_1 \subseteq k_2 \subseteq K_1$ *be* q*-finite extensions. If* $k_1 \sim k_2$*, then* $lm(K_1/k_1) \sim lm(K_1/k_2)$.

Proof. It is enough to note that $lm(K_1/k_1) \subseteq lm(K_1/k_2)$, and if moreover $o_1(k_2/k_1) = e_1$, then $k_2 \subseteq (lm(K_1/k_1))^{p^{-e_1}} \cap K_1$ with $K_1/(lm(K_1/k_1))^{p^{-e_1}} \cap K_1$ is modular (cf. [10], Proposition 3). Let therefore $lm(K_1/k_2) \subseteq (lm(K_1/k_1))^{p^{-e_1}} \cap K_1$. □

As a consequence, the lq-modularity is stable up to a finite extension of the choice of the ground field, as specified by the following result:

Proposition 4.8. *Let* K/k *be a* q*-finite extension. We have:*

(1) *If* $k' \sim k$ *and* $k' \subset K$*,* K/k *is* lq*-modular if and only if the same holds for* K/k'*.*

(2) *If* $K \sim K'$ *and* $k \subset K'$*,* K/k *is* lq*-modular if and only if the same holds for* K'/k*.*

(3) *If* $k' \sim k$ *and* $K \sim K'$*, with* $k' \subset K'$*, then* K/k *is* lq*-modular if and only if the same holds for* K'/k'*.*

Proof. This results immediately from Propositions 4.3 and 4.7 and Theorem 4.6. □

As a consequence, the result below makes it possible to reduce the study of lq-modularity to the case of relatively perfect extensions.

Corollary 4.9. *Let K/k be a q-finite extension and H/k the relatively perfect closure of K/k. Then:*

(i) *K/k is lq-modular if and only if the same holds for H/k.*

(ii) *Let F/k be a subextension of K/k. K/F is lq-modular if and only if the same is true for $rp(K/k)/rp(F/k)$ and $K/rp(F/k)$.*

Proof. It is sufficient to note that $K \sim rp(K/k)$ and $F \sim rp(F/k)$. □

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