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MULTIPLICATIVE (GENERALIZED)-DERIVATIONS AND LEFT MULTIPLIERS IN SEMIPRIME RINGS

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Abstract Let R be a semiprime ring. A mapping $F : R \to R$ (not necessarily additive) is called a multiplicative (generalized)-derivation if there exists a map $f : R \to R$ (not necessarily a derivation nor an additive map) such that $F(xy) = F(x)y + xf(y)$ fulfilled for all $x, y \in R$. A map $H : R \to R$ (not necessarily additive) is called a multiplicative left multiplier if $H(xy) = H(x)y$ holds for all $x, y \in R$. The main objective of the present paper is to study the following situations: (i) $F(xy) \pm H(x)H(y) = 0$, (ii) $F(xy) \pm H(y)H(x) = 0$, (iii) $F(xy) \pm H(yx) = 0$, (iv) $F[x, y] \pm H[x, y] = 0$, (v) $F(xy) \pm [H(x), y] \in Z(R)$, (vi) $F(xy) \pm H(xy) \in Z(R)$ for all x, y in some appropriate subset of R.

1 Introduction

Throughout this paper, $Z(R)$ will denote the centre of an associative ring. The symbol $[x, y]$, where $x, y \in R$, stand for the commutator $xy - yx$. For any $a, b \in R$, a ring R is said to be prime if whenever $aRb = 0$ implies $a = 0$ or $b = 0$ and is semiprime if for any $a \in R$, $aRa = 0$ implies $a = 0$. An additive mapping $d : R \to R$ is said to be a derivation on R if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. Recall that an additive mapping f on R is said to be left multiplier if $f(xy) = f(x)y$ for all $x, y \in R$. An additive mapping $f : R \to R$ is said to be a generalized derivation if there exists a derivation $d : R \to R$ such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. In [8], Bresar introduced the notion of generalized derivation. Obviously, every derivation is a generalized derivation but the converse need not be true in general. Hence generalized derivations cover both the concepts of derivation and left multiplier maps. The concept of multiplicative derivation was introduced by Daif [10] and it was motivated by the work of Martindale [13]. According to Daif [10], the mapping $D : R \to R$ is said to be a multiplicative derivation if it satisfies $D(xy) = D(x)y + xD(y)$ for all $x, y \in R$. In the case of multiplicative derivations the mappings are not assumed to be additive. Further, Goldmann and Semrl [6] gave the complete description of these mappings. Daif and Tammam El-Sayiad [11] extended multiplicative derivations to multiplicative generalized derivations as follows: a mapping F on R is said to be multiplicative generalized derivation if there exists a derivation d on R such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. Recently, Dhara and Ali [2] gave a more precise definition of multiplicative (generalized)-derivation as follows: a mapping $F : R \to R$ is said to be a multiplicative (generalized)-derivation if there exists a map g on R such that $F(xy) = F(x)y+xg(y)$ for all $x, y \in R$, where g is any mapping on R (not necessarily additive). Hence the concepts of multiplicative (generalized)-derivation covers the concepts of multiplicative derivation and multiplicative generalized derivation. A mapping $H : R \to R$ (not necessarily additive) is said to be a multiplicative left multiplier (centralizer) if $H(xy) = H(x)y$ holds for all $x, y \in R$ ([12]). A multiplicative (generalized)- derivation associated with mapping $g = 0$ covers the concept of multiplicative left multipliers.

In [3] Dhara proved that if R is a semiprime ring and $F : R \to R$ be a non-zero generalized derivation of R associated with a derivation d such that $F(xy) = F(x)F(y)$ or $F(xy) = F(y)F(x)$ for all $x, y \in I$, then $d(I) = 0$ or R contains a nonzero central ideal. In particular, if R is a prime ring, then R is commutative and F is left multiplier mapping of R. Albas [4] studied the above mentioned identities in prime rings with central valued. More precisely Albas proved that if a prime ring R admits a nonzero generalized derivation F with associated derivation d such that $F(xy) - F(x)F(y) \in Z(R)$ or $F(xy) + F(x)F(y) \in Z(R)$ for all $x, y \in I$, then either R is commutative or $F = I_{id}$ or $F = -I_{id}$, where I_{id} denotes the identity map of the ring R.

In several papers all these identities are also investigated in some appropriate subsets of prime and semiprime rings. In this view we refer to ([1], [2], [4], [5], [9]) where further references can be found.

Further Atteya [9] continued these results on semiprime ring stated as: let R be a semiprime ring and I a non-zero ideal of R. Then R contains a non-zero central ideal if one of the following condition holds; (i) $F(xy) \pm xy \in Z(R)$, (ii) $F(xy) \pm yx \in Z(R)$, (iii) $F(x)F(y) \pm xy \in Z(R)$ for all $x, y \in I$, where F is a generalized derivation associated with a non-zero derivation d on R.

In this line of investigation, it is more interesting to study the identities involving multiplicative (generalized)-derivation and a multiplicative left multiplier. In the Present paper, our main object is to investigate the cases when a multiplicative (generalized)- derivation F and a multiplicative left multiplier H satisfies the identities: (i) $F(xy) \pm H(x)H(y) = 0$, (ii) $F(xy) \pm H(y)H(x) = 0$, (iii) $F(xy) \pm H(yx) = 0$, (iv) $F[x, y] \pm H[x, y] = 0$, (v) $F(xy) \pm [H(x), y] \in Z(R)$, (vi) $F(xy) \pm H(xy) \in Z(R)$ for all x, y in some appropriate subset of R.

2 Preliminary Results

Throughout the paper, we make extensive use of the basic commutator identities. For any $x, y, z \in R$,

$$[x, yz] = y[x, z] + [x, y]z \text{ and } [xy, z] = x[y, z] + [x, z]y.$$

For any subset S of R, we will denote $r_R(S)$ the right annihilator of S in R, that is $r_R(S) = \{x \in R | Sx = 0\}$ and by $l_R(S)$ the left annihilator of S in R, that is, $l_R(S) = \{x \in R | xS = 0\}$. If $r_R(S) = l_R(S)$, then $r_R(S)$ is called an annihilator ideal of R and is written as $ann_R(S)$. We know that if R is a semiprime ring and I is an ideal of R, then $r_R(I) = l_R(I)$.

Moreover, we shall require the following known results.

Lemma 2.1. [7, Lemma 1.1.5] If R is a semiprime ring, then centre of a non-zero one-sided ideal is contained in the centre of R; in particular, any commutative one-sided ideal is contained in the centre of R.

Lemma 2.2. [7, Corollary 2] If R is a semiprime ring and I is an ideal of R, then $I \cap ann_R(I) = 0$.

3 Main Results

Theorem 3.1. Let R be a semiprime ring and I a non-zero ideal of R. Let $F : R \to R$ be a multiplicative (generalized)-derivation associated with a map $f : R \to R$ and $H : R \to R$ be a multiplicative left multiplier such that $F(xy) \pm H(x)H(y) = 0$ for all $x, y \in I$. Then $[f(x), x] = 0$ and $[H(x), x] = 0$ for all $x \in I$.

Proof. Let

$$F(xy) - H(x)H(y) = 0 \text{ for all } x, y \in I. \tag{3.1}$$

Substituting yz for y in (3.1), we get

$$F(xy)z + xyf(z) - H(x)H(y)z = 0.$$

Using (3.1), we get

$$xyf(z) = 0 \text{ for all } x, y, z \in I. \tag{3.2}$$

Replacing z by x, we get

$$xyf(x) = 0 \text{ for all } x, y \in I. \tag{3.3}$$

Putting ry instead of y in (3.3), we obtain

$$xryf(x) = 0 \text{ for all } x, y \in I \text{ and } r \in R.$$

Substituting $f(x)r$ for r, we find

$$xf(x)Ryf(x) = 0.$$

Replacing y by x and using semiprimeness of R, we obtain $xf(x) = 0$, for all $x \in I$. In (3.3), left multiplying by $f(x)$ and right multiplying by x, we get $f(x)xyf(x)x = 0$ for all $x, y \in I$. This implies that $(f(x)xI)^2 = 0$ for all $x \in I$. Since semiprime ring contains no non-zero nilpotent ideal, we obtain that $f(x)xI = 0$ for all $x \in I$. Therefore, for all $x \in I$ we have $f(x)x \in I \cap ann_R(I)$. Since R is a semiprime, by Lemma 2.2 we conclude that $f(x)x = 0$ for all $x \in I$. Together of these two gives $[f(x), x] = 0$ for all $x \in I$. Now substituting xy for x in (3.1), we get

$$F(xy)y + xyf(y) - H(x)yH(y) + H(x)H(y)y - H(x)H(y)y = 0.$$

Using (3.1) and $xf(x) = 0$ for all $x \in I$, we get

$$H(x)[H(y), y] = 0 \text{ for all } x, y \in I. \tag{3.4}$$

Replacing x by xr in (3.4), we have

$$H(x)r[H(y), y] = 0 \text{ for all } x, y \in I. \tag{3.5}$$

Substituting x^2 for x in (3.5), we get

$$H(x)xr[H(y), y] = 0 \text{ for all } x, y \in I \text{ and } r \in R. \tag{3.6}$$

Left multiplying (3.5) by x and subtracting from (3.6), we get

$$[H(x), x]R[H(y), y] = 0 \text{ for all } x, y \in I. \tag{3.7}$$

Replacing y by x and using semiprimeness of R, we obtain $[H(x), x] = 0$ for all $x \in I$. By using similar argument, we arrive at the same conclusion for $F(xy) + H(x)H(y) = 0$ for all $x, y \in I$.

Corollary 3.2. Let R be a semiprime ring and I a non-zero ideal of R. If $F : R \to R$ be a multiplicative (generalized)-derivation associated with a map $f : R \to R$ and $H : R \to R$ be a multiplicative left multiplier such that $F(xy) \pm H(x)H(y) = 0$ for all $x, y \in R$, then $F(xy) = F(x)y$ for all $x, y \in R$.

Proof. Replacing x by $f(z)$ in (3.2) and using semiprimeness of R, we have $f = 0$ and get the result.

Theorem 3.3. Let R be a semiprime ring and I a non-zero ideal of R. Let $F : R \to R$ be a multiplicative (generalized)-derivation associated with a map $f : R \to R$ and $H : R \to R$ be a multiplicative left multiplier such that $F(xy) \pm H(y)H(x) = 0$ for all $x, y \in I$. Then $f(I) \subseteq Z(R)$.

Proof. By hypothesis

$$F(xy) - H(y)H(x) = 0 \text{ for all } x, y \in I. \tag{3.8}$$

Replacing y with yz in (3.8), we get

$$F(xy)z + xyf(z) - H(y)zH(x) - H(y)H(x)z + H(y)H(x)z = 0.$$

Using (3.8), we have

$$xyf(z) + H(y)[H(x), z] = 0 \text{ for all } x, y, z \in I. \tag{3.9}$$

Substituting xz for x in (3.9), we get

$$xzyf(z) + H(y)[H(x)z, z] = 0,$$

$$xzyf(z) + H(y)[H(x), z]z = 0. \tag{3.10}$$

Multiplying (3.9) by z on the right and subtracting from (3.10), we have

$$x[yf(z), z] = 0 \text{ for all } x, y, z \in I.$$

This implies that

$$[yf(z), z] \in I \cap ann_R(I).$$

Since R is a semiprime, by Lemma 2.2 we conclude that

$$[yf(z), z] = 0 \text{ for all } y, z \in I. \tag{3.11}$$

Replacing y by $f(z)y$ in (3.11) and using (3.11), we find

$$[f(z), z]yf(z) = 0 \text{ for all } y, z \in I. \tag{3.12}$$

Substituting yz for y, we have

$$[f(z), z]yzf(z) = 0. \tag{3.13}$$

Right multiplying (3.12) by z and subtracting from (3.13), we get

$$[f(z), z]y[f(z), z] = 0 \text{ for all } y, z \in I.$$

This implies that $([f(z), z]I)^2 = 0$ for all $z \in I$. Since a semiprime ring contains no nonzero nilpotent ideals, we conclude that $[f(z), z]I = 0$ for all $z \in I$. Therefore, for all $z \in I$ we have $[f(z), z] \in I \cap ann_R(I)$. Since R is a semiprime, by Lemma 2.2 we obtain $[f(z), z] = 0$ for all $z \in I$. Replacing x by xr in (3.9), we get

$$xryf(z) + H(y)[H(x)r, z] = 0 \text{ for all } x, y, z \in I \text{ and } r \in R.$$

$$xryf(z) + H(y)H(x)[r, z] + H(y)[H(x), z]r = 0. \tag{3.14}$$

Multiplying (3.9) by r on the right and subtracting from (3.14), we get

$$x[yf(z), r] + H(y)H(x)[r, z] = 0.$$

Putting $r = f(z)$ and using $[f(z), z] = 0$, we find

$$x[y, f(z)]f(z) = 0 \text{ for all } x, y, z \in I.$$

Since R is a semiprime ring, we have

$$[f(z), y]f(z) = 0 \text{ for all } y, z \in I. \tag{3.15}$$

Replacing y by yw in (3.15) and using (3.15), we get

$$[f(z), y]wf(z) = 0 \text{ for all } y, z, w \in I. \tag{3.16}$$

Right multiplying (3.15) by w and subtracting from (3.16), we get

$$[f(z), y][f(z), w] = 0. \tag{3.17}$$

Substituting wy for w in (3.17) and using (3.17), we get

$$[f(z), y]w[f(z), y] = 0 \text{ for all } y, z, w \in I.$$

This implies that

$$[f(z), y]I[f(z), y] = 0 \text{ for all } y, z \in I.$$

Since R is a semiprime ring, we have $[f(z), y] = 0$ for $y, z \in I$. Applying Lemma 2.1, we get $f(I) \subseteq Z(R)$. Similarly we can prove the result for the case $F(xy) + H(y)H(x) = 0$ for all $x, y \in I$

Corollary 3.4. Let R be a prime ring and I a non-zero ideal of R. If $F : R \to R$ be a multiplicative (generalized)-derivation associated with a map $f : R \to R$ and $H : R \to R$ be a multiplicative left multiplier such that $F(xy) \pm H(y)H(x) = 0$ for all $x, y \in R$, then any one of the following holds:
(i) $F(xy) = F(x)y$.
(ii) f maps R into $Z(R)$.

Proof. In (3.16) using primeness of R, we have either $f(z) = 0$ for all $z \in R$ or $[f(z), y] = 0$ for all $y, z \in R$. If $f(R) = 0$, then $F(xy) = F(x)y$ for all $x, y \in R$ and later case yields that f maps R into $Z(R)$.

Theorem 3.5. Let R be a semiprime ring and I a non-zero ideal of R. Let $F : R \to R$ be a multiplicative (generalized)-derivation associated with a map $f : R \to R$ and $H : R \to R$ be a multiplicative left multiplier such that $F(xy) \pm H(yx) = 0$ for all $x, y \in I$. Then $f(I) \subseteq Z(R)$.

Proof. By assumption

$$F(xy) - H(yx) = 0 \text{ for all } x, y \in I. \tag{3.18}$$

Substituting yz for y in (3.18), we obtain

$$F(xy)z + xyf(z) - H(y)zx + H(y)xz - H(y)xz = 0,$$

$$(F(xy) - H(yx))z + xyf(z) + H(y)[x, z] = 0.$$

Using (3.18), we get

$$xyf(z) + H(y)[x, z] = 0. \tag{3.19}$$

Replacing z by x, we get

$$xyf(x) = 0 \text{ for all } x, y \in I. \tag{3.20}$$

Putting ry instead of y in (3.20), we obtain

$$xryf(x) = 0 \text{ for all } x, y \in I.$$

Substituting r by $f(x)r$, we find

$$xf(x)Ryf(x) = 0.$$

Replacing y by x and using semiprimeness of R, we obtain $xf(x) = 0$, for all $x \in I$. Now left multiplying (3.20) by $f(x)$ and right multiplying by x, we get

$$f(x)xyf(x)x = 0 \text{ for all } x, y \in I.$$

Since R is a semiprime ring, we get $f(x)x = 0$. Hence $[f(x), x] = 0$ for all $x \in I$. Substituting xr for x in (3.19), we obtain

$$xryf(z) + H(y)[xr, z] = 0 \text{ for all } x, y, z \in I \text{ and } r \in R,$$

$$xryf(z) + H(y)x[r, z] + H(y)[x, z]r + xyf(z)r - xyf(z)r = 0,$$

$$xryf(z) + H(y)x[r, z] - xyf(z)r + (xyf(z) + H(y)[x, z])r = 0.$$

Using (3.19), we get

$$x[r, yf(z)] + H(y)x[r, z] = 0. \tag{3.21}$$

Putting $r = f(z)$ in (3.21) and using $[f(z), z] = 0$, we obtain

$$x[f(z), y]f(z) = 0 \text{ for all } x, y, z \in I.$$

Arguing in the similar manner as in Theorem 3.3, we get the result.

Theorem 3.6. Let R be a semiprime ring and I a non-zero ideal of R. Let $F : R \to R$ be a multiplicative (generalized)-derivation associated with a map $f : R \to R$ and $H : R \to R$ be a multiplicative left multiplier such that $F[x,y] \pm H[x,y] = 0$ for all $x, y \in I$. Then $[f(x), x] = 0$ for all $x \in I$.

Proof. By hypothesis

$$F[x,y] \pm H[x,y] = 0 \text{ for all } x, y \in I. \tag{3.22}$$

Replacing y by yx in (3.22), we get

$$F[x,y]x + [x,y]f(x) \pm H[x,y]x = 0.$$

Using (3.22), we have

$$[x,y]f(x) = 0 \text{ for all } x, y \in I. \tag{3.23}$$

Substituting yx for y in (3.23), we obtain

$$[x,y]xf(x) = 0 \text{ for all } x, y \in I. \tag{3.24}$$

Right multiplying (3.23) by x and subtracting from (3.24), we get

$$[x,y][f(x), x] = 0. \tag{3.25}$$

Replacing y by $f(x)y$ in (3.25), we find

$$[x, f(x)]y[f(x), x] = 0 \text{ for all } x, y \in I.$$

Since R is a semiprime ring, we conclude that $[f(x), x] = 0$ for all $x \in I$.

Theorem 3.7. Let R be a semiprime ring and I a nonzero ideal of R. Let $F : R \to R$ be a multiplicative (generalized)-derivation associated with a map $f : R \to R$ and $H : R \to R$ be a multiplicative left multiplier such that $F(xy) \pm [H(x), y] \in Z(R)$ for all $x, y \in I$. Then $[f(x), x] = 0$ for all $x \in I$.

Proof. By hypothesis

$$F(xy) + [H(x), y] \in Z(R) \text{ for all } x, y \in I. \tag{3.26}$$

Replacing y by yz, we get

$$F(xy)z + xyf(z) + y[H(x), z] + [H(x), y]z \in Z(R) \text{ for all } x, y, z \in I,$$

$$(F(xy) + [H(x), y])z + xyf(z) + y[H(x), z] \in Z(R). \tag{3.27}$$

Combining (3.26) and (3.27), we obtain

$$[xyf(z), z] + [y[H(x), z], z] = 0. \tag{3.28}$$

Replacing x by xz in (3.28), we find that

$$[xzyf(z), z] + [y[H(xz), z], z] = 0,$$

$$[xzyf(z), z] + [y[H(x)z, z], z] = 0,$$

$$[xzyf(z), z] + [y[H(x), z], z]z = 0. \tag{3.29}$$

Multiplying (3.28) by z on the right and subtracting from (3.29), we get

$$[x[yf(z), z], z] = 0 \text{ for all } x, y, z \in I. \tag{3.30}$$

Replacing x by wx in (3.30) and using (3.30), we obtain

$$[w,z]x[yf(z),z]=0 \text{ for all } x,y,z,w\in I.$$

Replacing w by $yf(z)$ and using semiprimeness of R, we get

$$[yf(z),z]=0 \text{ for all } y,z\in I. \tag{3.31}$$

Substituting $f(z)y$ instead of y in (3.31) and using (3.31), we obtain

$$[f(z),z]yf(z)=0 \text{ for all } y,z\in I.$$

This implies that

$$[f(z),z]y[f(z),z]=0.$$

Since R is semiprime ring, we find that

$$[f(z),z]=0 \text{ for all } z\in I.$$

By using similar argument we can get the result for the case $F(xy)-[H(x),y]\in Z(R)$ for all $x,y\in I$.

Theorem 3.8. Let R be a semiprime ring and I a non-zero ideal of R. Let $F:R\to R$ be a multiplicative (generalized)-derivation associated with a map $f:R\to R$ and $H:R\to R$ be a multiplicative left multiplier such that $F(xy)\pm H(xy)\in Z(R)$ for all $x,y\in I$. Then $[f(x),x]=0$ for all $x\in I$.

Proof. Suppose that

$$F(xy)\pm H(xy)\in Z(R) \text{ for all } x,y\in I. \tag{3.32}$$

Replacing y by yz in (3.32), we have

$$F(xy)z+xyf(z)\pm H(xy)z\in Z(R) \text{ for all } x,y,z\in I,$$

$$(F(xy)\pm H(xy))z+xyf(z)\in Z(R).$$

Using (3.32), we get

$$[xyf(z),z]=0 \text{ for all } x,y,z\in I. \tag{3.33}$$

Replacing x by $f(z)x$ in (3.33) and using (3.33), we find that

$$[f(z),z]xyf(z)=0.$$

Substituting $f(z)y$ for y, we get

$$[f(z),z]xf(z)yf(z)=0 \text{ for all } x,y,z\in I.$$

This implies that

$$[f(z),z]x[f(z),z]y[f(z),z]=0 \text{ for all } x,y,z\in I.$$

This gives that $([f(z),z]I)^3=0$ for all $z\in I$. Since there is no nilpotent ideal in a semiprime ring, we get $[f(z),z]I=0$ for all $z\in I$. This implies that $[f(z),z]\in I\cap ann_R(I)$. Application of Lemma 2.2, yields that $[f(z),z]=0$ for all $z\in I$.

The following examples demonstrate that Corollary 3.2 and 3.4 do not hold for arbitrary rings.

Example 3.9. Consider S be a set of integers. Let

$$R = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} \mid x, y, z \in S \right\}.$$

Define maps $F, f, H : R \to R$ by $F \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & m^2y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $f \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & mx^2 & my^2 \\ 0 & 0 & m^2z \\ 0 & 0 & 0 \end{pmatrix}$, $H \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & mx & my \\ 0 & 0 & mz \\ 0 & 0 & 0 \end{pmatrix}$, where $m \in S$. Then it is easy to check that F is a multiplicative (generalized)-derivation on R associated with mapping f on R and H is a multiplicative left multiplier. For all $x, y \in R$, $F(xy) \pm H(x)H(y) = 0$. But we see that $f(R) \neq 0$ and $F(xy) \neq F(x)y$ for all $x, y \in R$.

Example 3.10. In example 3.1 define maps $F, f, H : R \to R$ by $F \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & yz \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $f \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & x^2 & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $H \begin{pmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & y \\ 0 & 0 & z^2 \\ 0 & 0 & 0 \end{pmatrix}$. Then F is a multiplicative (generalized)-derivation on R associated with mapping f on R and H is a multiplicative left multiplier on R satisfying $F(xy) \pm H(y)H(x) = 0$ for all $x, y \in R$. But neither $F(xy) = F(x)y$ for all $x, y \in R$ nor f maps R into $Z(R)$.

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