

GENERALIZED SEMI-DERIVATIONS AND GENERALIZED LEFT SEMI-DERIVATIONS OF PRIME RINGS

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Abstract In that paper there is explored the commutativity of a prime ring in which generalized semi-derivations satisfy certain differential identities. Furthermore, we have introduced the notion of generalized left semi-derivations in a noncommutative ring R and the main results state some generalizations of recent results due to Chan, Jun, Jung and Firat.

1 Introduction

In this paper, R will represent an associative ring. For any $x, y \in R$, the symbol $[x, y]$ will denote the commutator $xy - yx$; while the symbol $x \circ y$ will stand for the anti-commutator $xy + yx$. R is said to be 2-torsion free if whenever $2x = 0$, with $x \in R$ implies $x = 0$. Also, R is prime if $aRb = 0$ implies $a = 0$ or $b = 0$. An additive subgroup J of R is said to be a Jordan ideal of R if $u \circ r \in J$, for all $u \in J$ and $r \in R$. A mapping $f: R \rightarrow R$ is said to be centralizing (resp. commuting) on a subset S of R if $[f(x), x] \in Z(R)$ (resp. $[f(x), x] = 0$), for all $x \in S$. A derivation on R is an additive mapping $d: R \rightarrow R$ such that $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. An additive mapping $F: R \rightarrow R$ is called a generalized derivation if there exists a derivation $d: R \rightarrow R$ such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. In this case, F is called the generalized derivation associated with d . An additive mapping d of R into itself is called a semi-derivation (associated with g) if $d(xy) = d(x)y + g(x)d(y) = d(x)g(y) + xd(y)$ and $d(g(x)) = g(d(x))$, for all $x, y \in R$.

Let d be a semi-derivation of R , associated with an endomorphism g . The additive map F on R is a generalized semi-derivation of R if $F(xy) = F(x)y + g(x)d(y) = F(x)g(y) + xd(y)$ and $F(g(x)) = g(F(x))$, for all $x, y \in R$. An additive mapping d is called a left derivation if $d(xy) = yd(x) + xd(y)$, for all $x, y \in R$. An additive mapping $G: R \rightarrow R$ is called a generalized left derivation if there exists a left derivation $d: R \rightarrow R$ such that $G(xy) = xG(y) + yd(x)$ holds for all $x, y \in R$. It is obvious to see that every generalized left derivation on a ring R is a left derivation. But the converse need not be true in general, for more details see [1].

At the end of this paper, we study the concept of left generalized semi-derivations in prime rings. Our aim is to show that zero is the only left generalized semi-derivation of a noncommutative prime ring.

2 Main results

In [6], I. S. Chang, K. W. Jun and Y. S. Jung proved that if there exists a derivation D on a noncommutative 2-torsion-free prime ring R such that the mapping $x \rightsquigarrow [aD(x), x]$ is commuting on R then $a = 0$ or $D = 0$. In [8], A. Firat proved this result to semi-derivation. We give a more generalization of this result for generalized semi-derivation of prime rings as following.

Theorem 2.1. *Let R be a noncommutative 2-torsion-free prime ring, g an onto endomorphism of R , d a semi-derivation associated with g and F is a nonzero generalized semi-derivation associated with d and g . If the mapping $x \rightsquigarrow [aF(x), x]$ is commuting on R , then either $a = 0$ or $aF(x) = \lambda x$, with $\lambda \in C$, the extended centroid of R .*

Proof. Assume that a is a nonzero element of R , then by ([3], Theorem 2), the mapping $x \rightsquigarrow aF(x)$ is commuting on R . Thus we have

$$[aF(x), x] = 0, \quad \text{for all } x \in R. \quad (2.1)$$

By linearization of Equation (2.1), we have

$$[aF(x), y] + [aF(y), x] = 0, \quad \text{for all } x, y \in R. \quad (2.2)$$

Replacing y by yx in (2.2) we get

$$[ag(y)d(x), x] = 0, \quad \text{for all } x, y \in R. \quad (2.3)$$

That is

$$ag(y)[d(x), x] + a[g(y), x]d(x) + [a, x]g(y)d(x) = 0, \quad \text{for all } x, y \in R. \quad (2.4)$$

Since g is an onto endomorphism of R , then we can writing

$$ay[d(x), x] + a[y, x]d(x) + [a, x]yd(x) = 0, \quad \text{for all } x, y \in R. \quad (2.5)$$

Substituting y by ay in (2.5) we find that

$$[a, x]ayd(x) = 0, \quad \text{for all } x, y \in R. \quad (2.6)$$

The primeness of R forces that $[a, x] = 0$ or $d(x) = 0$ and by Brauer's trick we have $d = 0$ or $a \in Z(R)$ in last case Equation (2.5) becomes

$$ay[d(x), x] + a[y, x]d(x) = 0, \quad \text{for all } x, y \in R. \quad (2.7)$$

That is

$$y[d(x), x] + [y, x]d(x) = 0, \quad \text{for all } x, y \in R. \quad (2.8)$$

Writing zy instead of y in (2.8), we obtain

$$[z, x]yd(x) = 0, \quad \text{for all } x, y, z \in R. \quad (2.9)$$

Since R is a noncommutative prime ring, we obtain $d = 0$, $g = I_R$ and $x \rightsquigarrow G(x) = aF(x)$ is a left multiplier. Thus, Equation (2.2) becomes

$$[G(x), y] + [G(y), x] = 0, \quad \text{for all } x, y \in R. \quad (2.10)$$

Replacing y by yz , we obtain

$$y[G(x), z] + G(y)[z, x] = 0, \quad \text{for all } x, y, z \in R. \quad (2.11)$$

Putting ty instead of y , we arrive at

$$ty[G(x), z] + G(t)y[z, x] = 0, \quad \text{for all } t, x, y, z \in R. \quad (2.12)$$

Left multiplying Equation (2.11) by t , we get

$$ty[G(x), z] + tG(y)[z, x] = 0, \quad \text{for all } t, x, y, z \in R. \quad (2.13)$$

From Eqs (2.12) and (2.13), we conclude that

$$(G(t)y - tG(y))[z, x] = 0, \quad \text{for all } t, x, y, z \in R. \quad (2.14)$$

Replacing z by rz in (2.14), we get

$$(G(t)y - tG(y))R[z, x] = 0, \quad \text{for all } t, x, y, z \in R. \quad (2.15)$$

Since R is a noncommutative prime ring, we result

$$G(t)y - tG(y) = 0, \quad \text{for all } t, y \in R. \quad (2.16)$$

Substituting y by yz , we obtain

$$G(t)yz - tyG(z) = 0, \quad \text{for all } t, y, z \in R. \quad (2.17)$$

Accordingly, ([4], Lemma) forces that $G(x) = \lambda x$, for all $x \in R$, with $\lambda \in C$, the extended centroid of R . This complete the proof. \square

Corollary 2.2. ([8], Theorem 1) *Let R be a noncommutative 2-torsion-free prime ring and f is a semi-derivation of R with $g: R \rightarrow R$ is an onto endomorphism. If the mapping $x \rightsquigarrow [af(x), x]$ is commuting on R , then $a = 0$ or $f = 0$.*

As an application of Theorem 2.1, we get the following theorem for which the proof goes through in the same way as the proof of Theorem 2.1 ones.

Theorem 2.3. *Let R be a noncommutative 2-torsion-free prime ring with nonzero center, g an onto endomorphism of R , d a semi-derivation associated with g and F is a nonzero generalized semi-derivation associated with d and g . If the mapping $x \rightsquigarrow [F(x), x]$ is commuting on R , then $F(x) = \lambda x$, with $\lambda \in C$, the extended centroid of R .*

In [9], K. Kaya, O. Golbasi, N. Aydin proved that if R is a 2-torsion-free prime ring, d is a nonzero derivation of R , then $d(R) \circ a = (0)$, if and only if, $d((R \circ a)) = (0)$. In [8], A. Firat extended this result to semi-derivation. We give a more generalization of this result for generalized semi-derivation of prime rings as following.

Theorem 2.4. *Let R be a prime ring of characteristic different from 2, g an onto endomorphism of R , d a semi-derivation associated with g . If F is a nonzero generalized semi-derivation associated with d , then $F(x) \circ a = 0$, for all $x \in R$ if, and only if, $F(x \circ a) = 0$, for all $x \in R$.*

To prove our result, we need the following lemmas.

Lemma 2.5. *If $F(x) \circ a = 0$, then $d(a) = 0$.*

Proof. We have

$$F(x)a + aF(x) = 0, \quad \text{for all } x \in R. \quad (2.18)$$

Replacing x by xy , we get

$$F(x)ya + g(x)d(y)a + aF(x)y + ag(x)d(y) = 0, \quad \text{for all } x, y \in R. \quad (2.19)$$

That is,

$$F(x)[y, a] + g(x)d(y)a + ag(x)d(y) = 0, \quad \text{for all } x, y \in R. \quad (2.20)$$

Taking $y = a$ in (2.20), we obtain

$$g(x)d(a)a + ag(x)d(a) = 0, \quad \text{for all } x, y \in R. \quad (2.21)$$

Putting ya instead of y in (2.20) and using (2.21), we get

$$[g(x), a]g(y)d(a) = 0, \quad \text{for all } x, y \in R. \quad (2.22)$$

Since R is prime and g is an onto endomorphism, then $d(a) = 0$ or $a \in Z(R)$. In this case, Equation (2.18) becomes $2F(x)a = 0$, thus $a = 0$, so that $d(a) = 0$. \square

Lemma 2.6. *If $F(x \circ a) = 0$, then $d(a) = 0$.*

Proof. We have

$$F(x \circ a) = 0, \quad \text{for all } x \in R. \quad (2.23)$$

Substituting xa instead of x , we get

$$F((x \circ a)a) = 0, \quad \text{for all } x \in R. \quad (2.24)$$

That is,

$$(x \circ a)d(a) = 0, \quad \text{for all } x \in R. \quad (2.25)$$

Replacing x by yx , we get

$$[y, a]Rd(a) = 0, \quad \text{for all } y \in R. \quad (2.26)$$

So that $d(a) = 0$ or $a \in Z(R)$. In this case, we get

$$0 = F((xa)a) = xad(a), \quad \text{for all } x \in R. \quad (2.27)$$

implies $d(a) = 0$, this complete the proof. \square

Proof of Theorem 2.4. We can suppose $a \notin Z(R)$.

If $g = I_R$,
suppose that

$$F(x) \circ a = 0, \quad \text{for all } x \in R. \quad (2.28)$$

Replacing x by xy , we get

$$(F(x)y) \circ a + (xd(y)) \circ a = 0, \quad \text{for all } x, y \in R. \quad (2.29)$$

That is,

$$F(x)[y, a] + x[d(y), a] + (x \circ a)d(y) = 0, \quad \text{for all } x, y \in R. \quad (2.30)$$

Writing ax instead of x , we obtain

$$F(ax)[y, a] + ax[d(y), a] + ((ax) \circ a)d(y) = 0, \quad \text{for all } x, y \in R. \quad (2.31)$$

Multiplying Equation (2.30) by a , we get

$$aF(x)[y, a] + ax[d(y), a] + a(x \circ a)d(y) = 0, \quad \text{for all } x, y \in R. \quad (2.32)$$

Subtracting (2.31) from (2.32), we arrive at

$$(F(ax) - aF(x))[y, a] = 0, \quad \text{for all } x, y \in R. \quad (2.33)$$

Since $a \notin Z(R)$, then

$$F(ax) = aF(x), \quad \text{for all } x \in R. \quad (2.34)$$

On the other hand, we have

$$F(xa) = F(x)a + g(x)d(a) = F(x)a, \quad \text{for all } x \in R. \quad (2.35)$$

So that

$$F(x) \circ a = F(x \circ a) = 0.$$

Suppose that

$$F(x \circ a) = 0, \quad \text{for all } x \in R. \quad (2.36)$$

Replacing x by xy , we get

$$F(x[y, a]) + F((x \circ a)y) = 0, \quad \text{for all } x, y \in R. \quad (2.37)$$

That is,

$$F(x)[y, a] + xd([y, a]) + (x \circ a)d(y) = 0, \quad \text{for all } x, y \in R. \quad (2.38)$$

Putting ax instead of x , we obtain

$$F(ax)[y, a] + axd([y, a]) + ((ax) \circ a)d(y) = 0, \quad \text{for all } x, y \in R. \quad (2.39)$$

Multiplying Equation (2.38) by a , we get

$$aF(x)[y, a] + axd([y, a]) + a(x \circ a)d(y) = 0, \quad \text{for all } x, y \in R. \quad (2.40)$$

Subtracting (2.39) from (2.40), we arrive at

$$(F(ax) - aF(x))[y, a] = 0, \quad \text{for all } x, y \in R. \quad (2.41)$$

Since $a \notin Z(R)$, then $F(ax) = aF(x)$.

On the other hand, we have $F(xa) = F(x)a + g(x)d(a) = F(x)a$, by Lemma 2.6. Thus,

$$F(x \circ a) = F(x) \circ a = 0.$$

If $g \neq I_R$, then $F(x) = \alpha x + d(x)$, where $\alpha \in C$, the extended centroid of R (by ([7], Theorem 17)), so that

$$\begin{aligned} F(x \circ a) &= \alpha(x \circ a) + d(x \circ a) = \\ &= (\alpha x) \circ a + d(x) \circ a = \\ &= (\alpha x + d(x)) \circ a = \\ &= F(x) \circ a, \quad \text{for all } x \in R. \end{aligned}$$

This complete the proof of Theorem 2.4.

Corollary 2.7. ([8], Theorem 2) *Let R be a 2-torsion-free prime ring and $a \in R$. If f is a nonzero semi-derivation of R , with associated endomorphism g , then $f(x) \circ a = 0$, for all $x \in R$ if, and only if, $f(x \circ a) = 0$, for all $x \in R$.*

Theorem 2.8. *Let R be a prime ring of characteristic different from 2, g an onto endomorphism of R , d a semi-derivation associated with g and F is a nonzero generalized semi-derivation associated with d and g . If $[d(x), F(y)] = 0$, for all $x, y \in R$, then R is commutative.*

Proof. Suppose that

$$[d(x), F(y)] = 0, \quad \text{for all } x, y \in R. \quad (2.42)$$

Replacing x by xz in (2.42), we get

$$d(x)[z, F(y)] + [g(x), F(y)]d(z) = 0, \quad \text{for all } x, y, z \in R. \quad (2.43)$$

Putting $d(z)$ instead of z in (2.43), we arrive at

$$[g(x), F(y)]d^2(z) = 0, \quad \text{for all } x, y, z \in R. \quad (2.44)$$

Substituting xt instead of x in (2.44), we obtain

$$[g(x), F(y)]g(t)d^2(z) = 0, \quad \text{for all } x, y, z \in R. \quad (2.45)$$

Since g is an onto endomorphism, then the primeness of R forces that $F(y) \in Z(R)$, for all $y \in R$, or $d^2(x) = 0$. In this case, $d = 0$.

If $F(y) \in Z(R)$, for all $y \in R$, then

$$[F(x), r] = 0, \quad \text{for all } x, r \in R. \quad (2.46)$$

Replacing x by xy , we get

$$F(x)[y, r] + g(x)[d(y), r] + [g(x), r]d(y) = 0, \quad \text{for all } x, y, r \in R. \quad (2.47)$$

Writing $F(x)$ instead of x in (2.47), we obtain

$$F^2(x)[y, r] + F(g(x))[d(y), r] = 0, \quad \text{for all } x, y, r \in R. \quad (2.48)$$

Taking $r = y$ in (2.48), we result

$$F(g(x))[d(y), y] = 0, \quad \text{for all } x, y, r \in R. \quad (2.49)$$

Putting $d(y)$ instead of r and $g(x)$ instead of x in (2.47) and using (2.49), we arrive at

$$[g^2(x), d(y)]d(y) = 0, \quad \text{for all } x, y \in R. \quad (2.50)$$

Since g is an onto, we can write

$$[x, d(y)]d(y) = 0, \quad \text{for all } x, y \in R. \quad (2.51)$$

It follows that $d(y) \in Z(R)$, for all $y \in R$. This forces R to be commutative. \square

Corollary 2.9. *Let R be a prime ring of characteristic different from 2 and F is a nonzero generalized derivation associated with d . If $[d(x), F(y)] = 0$, for all $x, y \in R$, then R is commutative.*

Theorem 2.10. *Let R be a prime ring of characteristic different from 2, g an onto endomorphism of R , d a semi-derivation associated with g and F is a nonzero generalized semi-derivation associated with d and g . If $d(x) \circ F(y) = 0$, for all $x, y \in R$, then $d = 0$ and $g = I_R$ (i.e. F is a left multiplier).*

Proof. Suppose that

$$d(x) \circ F(y) = 0, \quad \text{for all } x, y \in R. \quad (2.52)$$

Replacing y by yz in (2.52), we get

$$d(x) \circ (F(y)z) + d(x) \circ (g(y)d(z)) = 0, \quad \text{for all } x, y, z \in R. \quad (2.53)$$

That is,

$$-F(y)[d(x), z] + [d(x), g(y)]d(z) + g(y)(d(x) \circ d(z)) = 0, \quad \text{for all } x, y, z \in R. \quad (2.54)$$

Putting $d(x)$ instead of z in (2.54), we arrive at

$$[d(x), g(y)]d^2(x) + g(y)(d(x) \circ d^2(x)) = 0, \quad \text{for all } x, y \in R. \quad (2.55)$$

Replacing y by yz , we obtain

$$[d(x), g(z)]g(y)d^2(x) = 0, \quad \text{for all } x, y \in R. \quad (2.56)$$

Since g is an onto endomorphism and R is a prime ring, then $d(x) \in Z(R)$ or $d^2(x) = 0$, for all $x \in R$ and by Brauer's trick we have $d(x) \in Z(R)$, for all $x \in R$ or $d^2(x) = 0$, for all $x \in R$.

If $d^2(x) = 0$, for all $x \in R$, then replacing x by xy , we get $d(x)g(y) + xd(y) = 0$. Once again replacing y by yz and using last expression, we obtain $xyd(z) = 0$, for all $x, y, z \in R$. This implies that $d = 0$.

If $d(x) \in Z(R)$, for all $x \in R$, then our hypothesis becomes $2d(x)F(y) = 0$, for all $x, y \in R$. It follows that $d = 0$.

Finally, we conclude that

$$\begin{aligned} F(xy) &= F(x)g(y) + xd(y) = \\ &= F(x)g(y), \quad \text{for all } x, y \in R \end{aligned}$$

On the other hand,

$$\begin{aligned} F(xy) &= F(x)y + g(x)d(y) = \\ &= F(x)y, \quad \text{for all } x, y \in R \end{aligned}$$

From this two last expression, it follows that $F(x)(g(y) - y) = 0$. Replace now x by xz to get $F(x)z(g(y) - y) = 0$, that is, $F(x)R(g(y) - y) = 0$, and the primeness of R forces that $g = I_R$. This completes the proof. \square

Corollary 2.11. *Let R be a prime ring of characteristic different from 2 and F is a nonzero generalized derivation associated with d . If $d(x) \circ F(y) = 0$, for all $x, y \in R$, then F is a left multiplier.*

Motivated by the concepts of left derivations and left generalized derivations one hand and the semi-derivations and generalized semi-derivations secondly, [1] [7], we initiate the concepts of left semi-derivations and left generalized semi-derivation, as follows:

Definition 2.12. Let R be a ring and g an endomorphism of R . The additive mapping d is called a left semi-derivation associated with g , if

$$d(xy) = yd(x) + g(x)d(y) = g(y)d(x) + xd(y) \text{ and } d(g(x)) = g(d(x)), \text{ for all } x, y \in R.$$

Definition 2.13. Let R be a ring and g an endomorphism of R . The additive mapping F is called a left generalized semi-derivation associated with d and g , if

$$F(xy) = yF(x) + g(x)d(y) = g(y)F(x) + xd(y) \text{ and } F(g(x)) = g(F(x)), \text{ for all } x, y \in R.$$

It is well-known that zero is the only left derivation on a noncommutative prime ring. In the following theorem we prove a similar result for left generalized semi-derivation, in particular for a left semi-derivation.

Theorem 2.14. *Let R be a prime ring and g be an endomorphism. If R is noncommutative, then there exists no nonzero left generalized semi-derivation F associated with a left semi-derivation d and g .*

To prove our result, we need the following lemma.

Lemma 2.15. *Let R be a prime ring and g be an endomorphism. If R is noncommutative, then there exists no nonzero left semi-derivation d associated with g .*

Proof. Assume that $d \neq 0$. We have both

$$\begin{aligned} d(xyz) &= xyd(z) + g(z)d(xy) = \\ &= xyd(z) + g(z)g(y)d(x) + g(z)xd(y). \end{aligned}$$

On the other hand,

$$\begin{aligned} d(xyz) &= xd(yz) + g(yz)d(x) = \\ &= xyd(z) + xg(z)d(y) + g(y)g(z)d(x). \end{aligned}$$

From two later expressions, we get

$$[g(y), g(z)]d(x) + [x, g(z)]d(y) = 0, \quad \text{for all } x, y, z \in R. \quad (2.57)$$

In particular, for $y = z$, we obtain

$$[x, g(y)]d(y) = 0, \quad \text{for all } x, y \in R. \quad (2.58)$$

Replacing x by xt , we obtain

$$[x, g(y)]Rd(y) = 0, \quad \text{for all } x, y \in R. \quad (2.59)$$

Since d is nonzero, then $g(R) \subset Z(R)$.

In this case, we have

$$d(xy) = xd(y) + g(y)d(x) = g(x)d(y) + yd(x), \quad \text{for all } x, y \in R. \quad (2.60)$$

That is,

$$(x - g(x))d(y) = (y - g(y))d(x), \quad \text{for all } x, y \in R. \quad (2.61)$$

Replacing x by xz , we get, for all $x, y, z \in R$,

$$(xz - xg(z))d(y) + (xg(z) - g(x)g(z))d(y) = (y - g(y))g(z)d(x) + (y - g(y))xd(z). \quad (2.62)$$

That is,

$$x(z - g(z))d(y) = (y - g(y))xd(z), \quad \text{for all } x, y, z \in R, \quad (2.63)$$

so that

$$x(y - g(y))d(z) = (y - g(y))xd(z), \quad \text{for all } x, y, z \in R \quad (2.64)$$

implies that

$$[x, z - g(z)]d(z) = [x, z]d(z) = 0, \quad \text{for all } x, z \in R. \quad (2.65)$$

Replacing z by zy , we get

$$[x, z]Rd(z) = 0, \quad \text{for all } x, z \in R. \quad (2.66)$$

Since $d \neq 0$, then R is commutative which contradicts our hypothesis. So that $d = 0$. \square

Proof of Theorem 2.14. Assume that $F \neq 0$. Since R is noncommutative, then Lemma 2.15 forces that $d = 0$, so that $F(xy) = yF(x) = g(y)F(x)$. That is, $(y - g(y))F(x) = 0$. Replacing x by xz , we obtain $(y - g(y))RF(z) = 0$. Since $F \neq 0$, then $g(y) = y$, for all $y \in R$. In this case, we have $F(xyz) = zF(xy) = zyF(x) = yzF(x)$, that is, $[y, z]F(x) = 0$, which implies $[y, z]RF(x) = 0$. Since $F \neq 0$, then R is commutative, which contradicts our hypothesis. Thus, $F = 0$.

As an application of Theorem 2.14, we obtain the following corollary, whose is a more generalization of ([10], Theorem 1.1).

Corollary 2.16. *Let R be a prime ring. If R is noncommutative, then there exists no nonzero left generalized derivation F associated with a left derivation d .*

The following example proves that the primeness hypothesis in Theorems 2.1, 2.3 and 2.8 is not superfluous.

Example 2.17. Let $R = \mathbb{Q}[X] \times T$, where T is a noncommutative 2-torsion-free ring and set $F(P, t) = d(P, t) = (P', 0)$. It is obvious that R is a noncommutative ring and d is a derivation of R such that $[d(r), s] = 0$, for all $r, s \in R$.

It is easy to verify that F is a generalized derivation of R which satisfies the conditions of Theorem 2.1, 2.3 and 2.8; however R is a noncommutative ring and $F(x) \neq \lambda x$, for any $\lambda \in C$, the extended centroid of R .

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