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On a class of quasi-coherent modules

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Abstract Let X be a scheme such that the structural sheaf $\mathcal{O}_X$ have zero higher cohomology group and $\mathcal{M}$ be an $\mathcal{O}_X$-module on X. This paper investigate a sufficient condition for a quasi-coherent module $\mathcal{M}$ to have $H^i(X,\mathcal{M}) = 0$ for all $i > 0$.

1 Introduction

Throughout this papers, X is assumed to be a scheme, A its ring of global sections i.e $A = \mathcal{O}_X(X)$ and $\mathcal{M}$ be an $\mathcal{O}_X$-module on X.

Recall that, an $\mathcal{O}_X$-module $\mathcal{M}$ is quasi-coherent if and only if for any affine open $U = \operatorname{Spec} B \subset X$, there exists a B-module M such that $\mathcal{M}|_U \simeq \widetilde{M}$.

In [12], Serre initiated the study of the Čech cohomology of coherent sheaves on separated algebraic varieties with their Zariski topology. He proved that coherent sheaves on affine varieties have zero higher cohomology group. Using this theorem, he showed that one may compute the cohomology of coherent sheaves by using the more down-to-earth Čech cohomology of any affine open covering of the algebraic variety.

In [5] Grothendieck defined a cohomology theory of sheaves on any topological space by injective resolutions. In this theory , the emphasis is on the long exact sequence of cohomology arising from any short exact sequence of sheaves. In algebraic geometry, Grothendieck greatly extended Serre's results to quasi-coherent sheaves on schemes. His argument appears to be a direct translation of serre's to the more general context and freely employs spectral sequence, see for instance [6].

In [11], A. Neeman proved that a quasi-affine scheme is affine if and only if the structural sheaf have zero higher cohomology group. Accordingly to serre's criterion of affineness, this mean that if X is quasi-affine and $H^i(X,\mathcal{O}_X) = 0$ for all $i > 0$, then for any quasi-coherent module $\mathcal{M}$, and for all $i > 0$, we get $H^i(X,\mathcal{M}) = 0$.

Which allows us, to introduce a new condition on sheaves over a scheme (not necessarly quasi-affine) to get a similar equivalence, as

$$X \text{ affine } \iff H^i(X,\mathcal{O}_X) = 0 \,, \forall i > 0 \,+\, \text{ condition on sheaves}$$

In sense that, this condition will be verified in the case of quasi-affine scheme, and we rediscover Neeman's result.

Here, it is worth to introduce the condition under which $H^i(X,\mathcal{M}) = 0$ for all $i > 0$ for a quasi-coherent module $\mathcal{M}$.

For a scheme X, we say that an $\mathcal{O}_X$-module $\mathcal{M}$ verify the property $(\mathbb{T})$ if:
$(\mathbb{T})$***: For any affine open*** U ***of*** X ***the canonical morphism*** $\mathcal{M}(X) \otimes_A \mathcal{O}_X(U) \to \mathcal{M}(U)$ ***is an isomorphism***.

This paper, contains in addition to the introduction two sections, the first one deals with the study of the local nature of this property, and we show that if a scheme is quasi-affine, any quasi-coherent module verify this property. The second section investigate the vanishing of the higher cohomology group $H^i(X,\mathcal{M})$, where $H^i(X,\mathcal{O}_X) = 0$ for all $i > 0$, and for any quasi-coherent module, that verify the property $(\mathbb{T})$. At this point, we make the following definitions:

Definition 1.1. Let $f : X \to Y$ be a morphism of schemes. f is flat if, for any affine open V of Y, and any affine open U of X such that $f(U) \subset V$, $\mathcal{O}_X(U)$ is a flat $\mathcal{O}_Y(V)$-module.

Definition 1.2. A scheme is called semi-separated if the intersection of any two affine opens is affine.

Definition 1.3. A scheme is quasi-affine if it is isomorphic to a quasi-compact open sub-scheme of an affine scheme.

2 On the property $(\mathbb{T})$

The following proposition examine the property $(\mathbb{T})$ for affine scheme.

Proposition 2.1. *Let X be an affine scheme and U be an affine open of X. Then for any quasi-coherent $\mathcal{O}_X$-module $\mathcal{M}$ on X, the natural morphism $\mathcal{M}(X) \otimes_{\mathcal{O}_X(X)} \mathcal{O}_X(U) \to \mathcal{M}(U)$ is an isomorphism.*

Before, proving the proposition, we establish the following lemma.

Lemma 2.2. *Let A be a ring. If $0 \to M_0 \to M_1 \to \dots \to M_n \to 0$ is an exact sequence of flat A-modules, then, for any A-module M, the sequence $0 \to M_0 \otimes_A M \to M_1 \otimes_A M \to \dots \to M_n \otimes_A M \to 0$ is exact.*

Proof. proposition 2.1 . Note that, since $\mathcal{M}$ is quasi-coherent and X is affine then $\mathcal{M} \simeq \widetilde{M}$, where $M = \mathcal{M}(X)$.

First case: If $U = D(f)$ is a principal affine open, where $f \in A$. We get

$$\mathcal{M}(X) \otimes_{\mathcal{O}_X(X)} \mathcal{O}_X(U) = M \otimes_A A_f \simeq M_f = \mathcal{M}(U)$$

Second case: If $U = \mathrm{Spec}(B)$ is any affine open of X, then it is quasi-compact, and it can be covered by a finite number of principal affine opens, $U_0 = D(f_0), \dots, U_n = D(f_n)$. Since U is affine then, $H^i(U, \mathcal{O}_U) = 0$ for all $i > 0$, thus

$$0 \to \mathcal{O}_X(U) \to C^0(U, \mathcal{O}_X) \to C^1(U, \mathcal{O}_X) \to \dots \to C^n(U, \mathcal{O}_X) \to 0$$

Is an exact sequence of flat B-modules. Since B is a flat A-algebra , then it is an exact sequence of flat A-modules.

By lemma 2.2, the following sequence

$0 \to \mathcal{M}(X) \otimes_A \mathcal{O}_X(U) \to \prod_i \mathcal{M}(X) \otimes_A \mathcal{O}_X(U_i) \to \dots \to \mathcal{M}(X) \otimes_A C^n(U, \mathcal{O}_X) \to 0$ is exact, where $U_{ij} = U_i \cap U_j$. In particular, the sequence

$0 \to \mathcal{M}(X) \otimes_A \mathcal{O}_X(U) \to \prod_i \mathcal{M}(X) \otimes_A \mathcal{O}_X(U_i) \to \prod_{i<j} \mathcal{M}(X) \otimes_A \mathcal{O}_X(U_{ij})$ is exact. Yields that the following commutative diagram

$$\begin{tikzcd}
0 \arrow[r] & \mathcal{M}(X) \otimes_A \mathcal{O}_X(U) \arrow[r] \arrow[d] & \prod\limits_i \mathcal{M}(X) \otimes_A \mathcal{O}_X(U_i) \arrow[r] \arrow[d] & \prod\limits_{i<j} \mathcal{M}(X) \otimes_A \mathcal{O}_X(U_{ij}) \arrow[d] \\
0 \arrow[r] & \mathcal{M}(U) \arrow[r] & \prod\limits_i \mathcal{M}(U_i) \arrow[r] & \prod\limits_{i<j} \mathcal{M}(U_{ij})
\end{tikzcd}$$

with exact lines. The second line is exact by the fact that $H^i(U, \mathcal{M}|_U) = 0$ for all $i > 0$. The second and the third vertical morphisms are isomorphisms, then the first is also an isomorphism (by the five lemma). □

The proposition below establish the relationship between quasi-coherent modules and those verifies the property $(\mathbb{T})$.

Proposition 2.3. *Let X be a scheme, and let $\mathcal{M}$ be an $\mathcal{O}_X$-module.*
If $\mathcal{M}$ verify $(\mathbb{T})$, then $\mathcal{M}$ is quasi-coherent.

Proof. Assume that $\mathcal{M}$ verify the property $(\mathbb{T})$. Let $U = \operatorname{Spec} B$ be an affine open of X and $M = \mathcal{M}(U)$. We show that $\mathcal{M}|_U = \widetilde{M}$.
Let $g \in B$, we have $\mathcal{M}|_U(D(g)) = \mathcal{M}(D(g))$. Since $D(g)$ is affine and $\mathcal{M}$ verify the property $(\mathbb{T})$ then, $\mathcal{M}(D(g)) = \mathcal{M}(X) \otimes_A \mathcal{O}(D(g))$. Thus

$$\begin{aligned}
\mathcal{M}|_U(D(g)) &= \mathcal{M}(X) \otimes_A \mathcal{O}_X(D(g)) = \mathcal{M}(X) \otimes_A B_g \\
&= \mathcal{M}(X) \otimes_A (B \otimes_B B_g) \\
&= (\mathcal{M}(X) \otimes_A B) \otimes_B B_g \\
&= (\mathcal{M}(X) \otimes_A \mathcal{O}_X(U)) \otimes_B B_g \\
&= \mathcal{M}(U) \otimes_B B_g \\
&= M_g = \widetilde{M}(D(g))
\end{aligned}$$

Hence $\mathcal{M}|_U = \widetilde{M}$. □

The following corollary is an immediate consequence of proposition 2.1 and proposition 2.3.

Corollary 2.4. *Let X be an affine scheme, and $\mathcal{M}$ be an $\mathcal{O}_X$-module, then: $\mathcal{M}$ verify the property $(\mathbb{T})$ if and only if $\mathcal{M}$ is quasi-coherent.*

In the following theorem, we study the local nature of the property $(\mathbb{T})$.

Theorem 2.5. *Let X be a quasi-compact and semi-separated scheme, such that the morphism $X \to \operatorname{Spec} A$ is flat , and $\mathcal{M}$ be a quasi-coherent $\mathcal{O}_X$-module on X. The following statements are equivalents:*

(i) There exist a finite covering of X by affine opens , $U_0, \ldots, U_n$, such that, for all $0 \leq i \leq n$; $\mathcal{M}(X) \otimes_{\mathcal{O}_X(X)} \mathcal{O}_X(U_i) = \mathcal{M}(U_i)$.

(ii) $\mathcal{M}$ verify $(\mathbb{T})$.

Proof. $(2) \Rightarrow (1)$: By the quasi-compactness.
$(1) \Rightarrow (2)$: Let $\mathcal{U} = (U_i)_{0 \leq i \leq n}$ be a covering of X, such that, for all $0 \leq i \leq n$; $\mathcal{M}(X) \otimes_{\mathcal{O}_X(X)} \mathcal{O}_X(U_i) = \mathcal{M}(U_i)$, and let V be an affine open of X. So $V = \cup_i V_i$, where $V_i = V \cap U_i$. Since X is semi-separated, then each open V_i is affine.
The following sequence

$$0 \to \mathcal{O}_X(V) \to \prod_i \mathcal{O}_X(V_i) \to \prod_{i<j} \mathcal{O}_X(V_{ij}) \to \ldots \to C^n(V, \mathcal{O}_X) \to 0$$

is an exact sequence of flats A-modules. Which still exact when tonsoring by the A-module $\mathcal{M}(X)$ (by lemma2.2). In particular, the following sequence

$$0 \to \mathcal{M}(X) \otimes_A \mathcal{O}_X(V) \to \prod_i \mathcal{M}(X) \otimes_A \mathcal{O}_X(V_i) \to \prod_{i<j} \mathcal{M}(X) \otimes_A \mathcal{O}_X(V_{ij})$$

is exact. But

$$\begin{aligned}
\mathcal{O}_X(V_i) \otimes_A \mathcal{M}(X) &= \big(\mathcal{O}_X(V_i) \otimes_{\mathcal{O}_X(U_i)} \mathcal{O}_X(U_i)\big) \otimes_A \mathcal{M}(X) \\
&= \mathcal{O}_X(V_i) \otimes_{\mathcal{O}_X(U_i)} (\mathcal{O}_X(U_i) \otimes_A \mathcal{M}(X)) \\
&= \mathcal{O}_X(V_i) \otimes_{\mathcal{O}_X(U_i)} \mathcal{M}(U_i) \\
&= \mathcal{M}(V_i)
\end{aligned}$$

By the same argument, we have $\mathcal{O}_X(V_{ij}) \otimes_A \mathcal{M}(X) = \mathcal{M}(V_{ij})$. So we get the following commutative diagram:

$$\begin{tikzcd}
0 \arrow[r] & \mathcal{O}_X(V)\otimes\mathcal{M}(X) \arrow[r] \arrow[d] & \prod_i \mathcal{O}_X(V_i)\otimes\mathcal{M}(X) \arrow[r] \arrow[d] & \prod_{i<j}\mathcal{O}_X(V_{ij})\otimes\mathcal{M}(X) \arrow[d] \\
0 \arrow[r] & \mathcal{M}(V) \arrow[r] & \prod_i \mathcal{M}(V_i) \arrow[r] & \prod_{i<j}\mathcal{M}_X(V_{ij})
\end{tikzcd}$$

Where the horizontal lines are exact, and the second and the third vertical morphisms are isomorphisms. It follows that the first vertical arrow is an isomorphism (by the five lemma). □

The next result study the property $(\mathbb{T})$ for quasi-affine scheme.

Theorem 2.6. *Let X be a quasi-affine scheme and $\mathcal{M}$ be a quasi-coherent $\mathcal{O}_X$-module. Then*

(i) The morphism $X \to \operatorname{Spec} A$ is flat.

(ii) $\mathcal{M}$ verify the property $(\mathbb{T})$.

The proof of this theorem requires the following lemma.

Lemma 2.7 ([9] Proposition 5.1.6). *Let $\mathcal{M}$ be a quasi-coherent $\mathcal{O}_X$-module on a scheme X. Let us suppose X is Noetherian or separated and quasi-compact. Then for any $f \in \mathcal{O}_X(X)$ the canonical homomorphism*

$$\mathcal{M}(X)_f = \mathcal{M}(X)\otimes_{\mathcal{O}_X(X)}\mathcal{O}_X(X)_f \to \mathcal{M}(X_f)$$

where $X_f := \{x \in X\ ,\ f_x \in \mathcal{O}^*_{X,x}\}$, is an isomorphism.

Proof. of theorem 2.6 (1) Let X be a quasi-compact open sub-scheme of an affine scheme $Y = \operatorname{Spec} B$, hence it is separated. Since X is quasi-compact, then there exist $f_1, \ldots, f_n \in B$ such that $X = \cup_i U_i$, where $U_i = D(f_i)$. Set $A = \mathcal{O}_X(X)$ and $g_i = f_i|_X \in A$. Let $\mathcal{M}$ be a quasi-coherent $\mathcal{O}_X$-module on X. By lemma 1, the morphism

$$A_{g_i} = \mathcal{O}_X(X)_{g_i} \to \mathcal{O}_X(X_{g_i})$$

is an isomorphism. But

$$\begin{aligned} X_{g_i} &= \{x\in X\ ,\ g_{ix}\in\mathcal{O}^*_{X,x}\} \\ &= \{x\in X\ ,\ f_{ix}\in\mathcal{O}^*_{X,x}\} \\ &= Y_{f_i}\cap X \\ &= D(f_i)\cap X \\ &= D(f_i) \end{aligned}$$

Hence $A_{g_i} \simeq \mathcal{O}_X(X_{g_i}) = \mathcal{O}_Y(D(f_i)) = B_{f_i}$. It follows that $B_{f_i} = \mathcal{O}_X(U_i)$ is a flat A-module. Which complete the proof of (1).

(2) According to the lemma 2.7, the morphism

$$\mathcal{M}(X)_{g_i} = \mathcal{M}(X)\otimes_{\mathcal{O}_X(X)}\mathcal{O}_X(X)_{g_i} \to \mathcal{M}(X_{g_i})$$

is an isomorphism.

Since $\mathcal{O}_X(U_i) = \mathcal{O}_X(D(f_i)) = \mathcal{O}_Y(D(f_i)) = B_{f_i} = \mathcal{O}_X(X)_{g_i}$, then
$\mathcal{M}(X)\otimes_{\mathcal{O}_X(X)}\mathcal{O}_X(D(f_i)) = \mathcal{M}(X)\otimes_{\mathcal{O}_X(X)}\mathcal{O}_X(X)_{g_i} \simeq \mathcal{M}(X_{g_i}) = \mathcal{M}(D(f_i)) = \mathcal{M}(U_i)$. □

3 Vanishing theorem for a class of quasi-coherent modules

The main result of this section examine the vanishing of the higher cohomology group of an $\mathcal{O}_X$-module that verify the property $(\mathbb{T})$.

Theorem 3.1. *Let X be a quasi-compact and semi-separated scheme such that the morphism $X \to \operatorname{Spec} A$ is flat, and $H^i(X, \mathcal{O}_X) = 0$ for all $i > 0$. If $\mathcal{M}$ is an $\mathcal{O}_X$-module that verify $(\mathbb{T})$, then $H^i(X, \mathcal{M}) = 0$ for all $i > 0$.*

Proof. Let $\mathcal{U} = (U_i)_{0 \leq i \leq n}$ be an affine open covering of X, and $\mathcal{M}$ be a quasi-coherent $\mathcal{O}_X$-module that verify $(\mathbb{T})$.

Since $H^i(X, \mathcal{O}_X) = 0$ for all $i > 0$, then the sequence

$$0 \to \mathcal{O}_X(X) \to C^0(\mathcal{U}, \mathcal{O}_X) \to \ldots \to C^n(\mathcal{U}, \mathcal{O}_X) \to 0$$

is exact.

For $p \in \{0, \ldots, n\}$, by definition

$$C^p(\mathcal{U}, \mathcal{O}_X) = \prod_{i_0 < \ldots < i_p} \mathcal{O}_X(U_{i_0,\ldots,i_p})$$

Where $U_{i_0,\ldots,i_p} = U_{i_0} \cap \ldots \cap U_{i_p}$.
Since X is semi-separated, then $U_{i_0,\ldots,i_p}$ is an affine open of X. $\mathcal{O}_X(U_{i_0,\ldots,i_p})$ is a flat A-module, by the fact that the morphism $X \to \operatorname{Spec} A$ is flat. Now $C^p(\mathcal{U}, \mathcal{O}_X)$ is a finite product of flats A-modules , then it is a flat A-module. Moreover, A is flat module over itself, then the last sequence is an exact sequence of flats A-modules. By lemma 2.2, the sequence
$0 \to A \otimes_A \mathcal{M}(X) \to C^0(\mathcal{U}, \mathcal{O}_X) \otimes_A \mathcal{M}(X) \to \ldots \to C^n(\mathcal{U}, \mathcal{O}_X) \otimes_A \mathcal{M}(X) \to 0$ is exact.
Now, it remains to show that $C^p(\mathcal{U}, \mathcal{O}_X) \otimes_A \mathcal{M}(X) = C^p(\mathcal{U}, \mathcal{M})$.
Since $C^p(\mathcal{U}, \mathcal{O}_X)$ is a finite product of A-modules, and the tensor product commute with finite product, we have

$$C^p(\mathcal{U}, \mathcal{O}_X) \otimes_A \mathcal{M}(X) = \prod_{i_0 < \ldots < i_p} \left(\mathcal{O}_X(U_{i_0,\ldots,i_p}) \otimes_A \mathcal{M}(X)\right)$$

Let $i_0 < \ldots < i_p$, since $U_{i_0,\ldots,i_p}$ is affine and the $\mathcal{O}_X$-module $\mathcal{M}$ verify $(\mathbb{T})$, then

$$\mathcal{O}_X(U_{i_0,\ldots,i_p}) \otimes \mathcal{M}(X) = \mathcal{M}(U_{i_0,\ldots,i_p})$$

It follow that $C^p(\mathcal{U}, \mathcal{O}_X) \otimes \mathcal{M}(X) = C^p(\mathcal{U}, \mathcal{M})$, hence the sequence

$$0 \to \mathcal{M}(X) \to C^0(\mathcal{U}, \mathcal{M}) \to \ldots \to C^n(\mathcal{U}, \mathcal{M}) \to 0$$

is exact, consequently $H^i(X, \mathcal{M}) = 0$ for all $i \geq 0$. □

The corollary below is an immediate consequence of theorem 3.1.

Corollary 3.2. *Let X be a quasi-compact and semi-separated scheme. Then, X is affine if and only if the following statements holds,*

(i) The structural sheaf have zero higher cohomology group .

(ii) The morphism $X \to \operatorname{Spec} A$ is flat.

(iii) Any quasi-coherent $\mathcal{O}_X$-module on X verify $(\mathbb{T})$.

Proof. If X is affine, then by Serrs's vanishing theorem, $H^i(X, \mathcal{O}_X) = 0$ for all $i > 0$, and by proposition 2.1, any quasi-coherent module verify $(\mathbb{T})$.
Conversely, by theorem 3.1, for any quasi-coherent module $\mathcal{M}$ on X, $H^i(X, \mathcal{M}) = 0$, for all $i > 0$. Hence X is affine. □

In the following corollary we rediscover a known result of A. Neeman [11].

Corollary 3.3. *Let X be a quasi-affine scheme. Then,*
X is affine if and only if for all $i > 0$, $H^i(X, \mathcal{O}_X) = 0$

Proof. The direct implication is straightforward. For the converse, since X is quasi-affine, then by theorem 3.1, the morphism $X \mapsto \operatorname{Spec} A$ is flat, and any quasi-coherent module $\mathcal{M}$ verify $(\mathbb{T})$. Hence $H^i(X, \mathcal{M}) = 0$ for all $i > 0$, and for any quasi-coherent module $\mathcal{M}$. Consequently X is affine. □

In the following, we give an example of scheme in which every $\mathcal{O}_X$-module that verify the property $(\mathbb{T})$ has zero higher cohomology group.

Example 3.4. Let B be a ring and $X = \mathbb{P}^1_B = \operatorname{Proj}(B[T_0, T_1])$. If $\mathcal{M}$ is a quasi-coherent module on X that verify $(\mathbb{T})$, then $H^i(X, \mathcal{M}) = 0$, for all $i > 0$.

Proof. Note that $\mathcal{O}_X(X) = B$.
Consider the covering $\mathcal{U} = \{U_0, U_1\}$ of X, where $U_0 = D_+(T_0)$ and $U_1 = D_+(T_1)$. We have $\mathcal{O}_X(U_0) = B[T_1/T_0]$ and $\mathcal{O}_X(U_1) = B[T_0/T_1]$. Since $\mathcal{O}_X(U_i)$ for $i = 0, 1$ are flats B-modules, then the natural morphism $\mathbb{P}^1_B \to \operatorname{Spec} B$ is flat.

The Čech complex is

$$0 \longrightarrow \mathcal{O}_X(X) \longrightarrow \mathcal{O}_X(U_0) \times \mathcal{O}_X(U_1) \longrightarrow \mathcal{O}_X(U_{01}) \longrightarrow 0$$

Set $T = T_0/T_1$, then $\mathcal{O}_X(U_{01}) = B[T, 1/T]$ and the last complex become

$$0 \longrightarrow B \longrightarrow B[1/T] \times B[T] \xrightarrow{d} B[T, 1/T] \longrightarrow 0$$

Where $d(P, Q) = P - Q$. The morphism d is surjective, which implies that $H^i(X, \mathcal{O}_X) = 0$ for all $i > 0$.
Now, if $\mathcal{M}$ is a quasi-coherent $\mathcal{O}_X$-module that verify $(\mathbb{T})$, then by theorem 3.1, $H^i(X, \mathcal{M}) = 0$ for all $i > 0$. □

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