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Non-degenerate umbilical affine hypersurfaces in recurrent affine manifolds with a semi-symmetric semi-metric connection

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Abstract. We define a semi-symmetric semi-metric connection in a recurrent affine manifold and we consider non-degenerate umbilical affine hypersurface of recurrent affine manifold endowed with a semi-symmetric semi-metric connection. Moreover, we also obtain relation with respect to this induced structure.

1 Introduction

In 1924, A. Friedmann and J. A. Schouten [9] introduced the notion of semi-symmetric linear connection. A linear connection ∇ is said to be a semi-symmetric, if its torsion tensor T is of the form

$$T(X,Y) = \tau(Y)X - \tau(X)Y,$$

where τ is a 1-form. The connection ∇ is symmetric if the torsion tensor T vanishes, otherwise it is non-symmetric. The connection ∇ is metric connection if there is a Riemannian metric g in M such that $\nabla g = 0$, otherwise it is non-metric. It is well know that a linear connection is symmetric and metric if and only if it is the Levi-Civita connection.

H. A. Hayden [10] introduced a semi-symmetric metric connection on a Riemannian manifold. K. Yano [15], proved the theorem: In order that a Riemannian manifold admits a semi-symmetric metric connection whose curvature tensor vanishes, it is necessary and sufficient that the Riemannian manifold to be conformally flat. B. Barua [7] introduced and studied submanifolds of a Riemannian manifold with a semi-symmetric semi-metric connection. Later, semi-symmetric semi-metric connection was further studied in [2, 3, 4, 6]. Y. C. Wong [13, 14] find a recurrent tensor on a connected differentiable manifold and studied a linear connection with tensor and recurrent curvatures.

Y. Ahmet and A. Nihat [1], studied non-degenerate hypersurfaces of semi-Riemannian manifolds with a semi-symmetric metric connection. Z. Olszak [12], studied non-degenerate umbilical affine hypersurfaces in recurrent affine manifolds. Motivated by the studies [1, 2, 3, 4, 5, 6,7] , in the present paper, we define a semi-symmetric semi-metric connection on non-degenerate umbilical affine hypersurfaces in recurrent affine manifolds similar to the hypersurfaces in a Riemannian manifold [11].

2 Preliminaries

Let $\bar{M}$ be an $(n+1)$-dimensional affine manifold, that is, a connected differentiable manifold endowed with an affine connection $\bar{\nabla}$.
and let M be an n-dimensional connected differentiable manifold immersed into $\bar{M}$ and assume that there exists a transversal vector field ξ along the hypersurface M (which is not tangent to M in general), by $\bar{X}^T$ and $\bar{X}^\perp$ we indicate its tangential and transversal parts repectively.

We denote ∇, the affine connection induced on M by assuming $\nabla_X Y = (\bar{\nabla}_X Y)^T$ for all vector fields X, Y tangent to M. In the sequel, M will be called an affine hypersurface of the manifold $\bar{M}$. Thus, we have the Gauss equation

$$\bar{\bar{\nabla}}_X Y = \bar{\nabla}_X Y + h(X,Y)\xi \tag{2.1}$$

for all X, Y tangent to M, where h is a symmetric $(0,2)$-tensor field, which is called the affine fundamental form of M or the affine metric corresponding to ξ.

The affine hypersurface M is said to be non-degenerate if the affine metric h is non-degenerate. In this case, h is Riemannian or pseudo-Riemannian metric on M. It should be mentioned that there is no relation between the affine metric h and the induced connection ∇.

For the affine hypersurface M, we also have the Weingarten equation

$$\bar{\bar{\nabla}}_X \xi = -AX + \tau(X)\xi, \tag{2.2}$$

where A is a $(1,1)$-tensor field and τ is a 1-form on M. A and τ are called, the shape operator and the transversal connection form of M respectively.

Let $\bar{\bar{R}}$ and $\bar{R}$ be the curvature tensor fields of connection $\bar{\bar{\nabla}}$ and the induced connection $\bar{\nabla}$. Thus

$$\bar{\bar{R}}(X,Y) = [\bar{\bar{\nabla}}_X, \bar{\bar{\nabla}}_Y] - \bar{\bar{\nabla}}_{[X,Y]}$$

for any vector fields X, Y on M and

$$\bar{R}(X,Y) = [\bar{\nabla}_X, \bar{\nabla}_Y] - \bar{\nabla}_{[X,Y]}$$

for any vector fields X, Y on M.
As the integrability conditions of (2.1) and (2.2), we have the Gauss and Codazzi equations [11]

$$\begin{aligned}\bar{\bar{R}}(X,Y)Z &= \bar{R}(X,Y)Z - h(Y,Z)AX + h(X,Z)AY \\ &+ ((\nabla_X h)(Y,Z) + \tau(X)h(Y,Z) - (\nabla_Y h)(X,Z) + \tau(Y)h(X,Z))\,\xi,\end{aligned} \tag{2.3}$$

$$\begin{aligned}\bar{\bar{R}}(X,Y)\xi &= -(\nabla_X A)Y + \tau(X)AY + (\nabla_Y A)X - \tau(Y)AX \\ &+ ((-h(X,AY) + h(Y,AX) + 2d\tau(X,Y))\,\xi.\end{aligned} \tag{2.4}$$

In the above formulas and in the sequel, symbols X, Y, Z denotes arbitrary vector fields tangent to M.

3 Structure equations with semi-symmetric semi-metric connection

Let $\bar{M}$ be an $(n+1)$-dimensional differentiable manifold of class C^∞ and $\bar{\nabla}$ be a linear connection in $\bar{M}$. The torsion tensor $\bar{T}$ of $\bar{\nabla}$ is given by

$$\bar{T}(\bar{X},\bar{Y}) = \bar{\nabla}_{\bar{X}}\bar{Y} - \bar{\nabla}_{\bar{Y}}\bar{X} - [\bar{X},\bar{Y}], \tag{3.1}$$

for every $\bar{X}, \bar{Y} \in \chi(\bar{M})$ and is of the type $(1,2)$. If the torsion tensor $\bar{T}$ satisfies

$$\bar{T}(\bar{X},\bar{Y}) = \bar{\tau}(\bar{Y})\bar{X} - \bar{\tau}(\bar{X})\bar{Y}$$

for a 1-form $\bar{\tau}$, the connection $\bar{\nabla}$ is said to be semi-symmetric [5].

Let there be a given pseudo Riemannian metric $\bar{g}$ in $\bar{M}$ and $\bar{\nabla}$ satisfies

$$\bar{\nabla} g = 0,$$

such a linear connection is called metric connection. Now, owing due to existence of one form τ on affine manifold $\bar{M}$, we define a semi-symmetric semi-metric connection by

$$\bar{\nabla}_{\bar{X}}\bar{Y} = \bar{\bar{\nabla}}_{\bar{X}}\bar{Y} - \bar{\tau}(\bar{X})\bar{Y} + \bar{g}(\bar{X},\bar{Y})\bar{P} \tag{3.2}$$

for arbitrary vector fields $\bar{X}$, $\bar{Y}$ of $\bar{M}$, where $\bar{\bar{\nabla}}$ denotes the Levi-Civita connection with respect to the semi-Riemannian (pseudo-Riemannian) metric $\bar{g}$, $\bar{\tau}$ is a 1-form and $\bar{P}$ the vector field defined by

$$\bar{g}(\bar{P}, \bar{X}) = \bar{\tau}(\bar{X}).$$

Denoting by $\dot{\nabla}$ the Levi-Civita connection induced on the non-degenerate affine umbilical hypersurfaces from $\dot{\bar{\nabla}}$ with respect to the unit normal vector field ξ, we have

$$\dot{\bar{\nabla}}_X Y = \dot{\nabla}_X Y + \dot{h}(X, Y)\xi \tag{3.3}$$

for arbitrary vector fields $\bar{X}$, $\bar{Y}$ of $\bar{M}$, where $\dot{h}$ is the second fundamental form of the non-degenerate umbilical affine hypersurface M. Denoting by ∇ the connection induced on the non-degenerate umbilical affine hypersurface from $\bar{\nabla}$ with respect to the unit normal vector field ξ, we have

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y)\xi \tag{3.4}$$

for arbitrary vector fields X, Y of M, where h is the second fundamental form of the non-degenerate umbilical affine hypersurface M and we call (3.4) the equation of Gauss with respect to the semi-symmetric semi-metric connection .
From (3.1), we obatin

$$\bar{\nabla}_X Y = \dot{\bar{\nabla}}_X Y - \bar{\tau}(X)Y + \bar{g}(X, Y)\bar{P} \tag{3.5}$$

and hence, using (3.3) and (3.4) we have

$$\nabla_X Y + h(X, Y)\xi = \dot{\nabla}_X Y + \dot{h}(X, Y)\xi - \bar{\tau}(X)Y + \bar{g}(X, Y)\bar{P}.$$

Substituting (3.2) into (3.5), we get

$$\nabla_X Y + h(X, Y)\xi = (\dot{\nabla}_X Y - \bar{\tau}(X)Y + \bar{g}(X, Y)\bar{P}) + \dot{h}(X, Y)\xi,$$

from which we have

$$\nabla_X Y = \dot{\nabla}_X Y - \bar{\tau}(X)Y + \bar{g}(X, Y)\bar{P},$$

where $\tau(X) = \bar{\tau}(X)$ and

$$h(X, Y) = \dot{h}(X, Y). \tag{3.6}$$

Taking account of (3.6), we find

$$\nabla_X(g(Y, Z)) = (\nabla_X g)(Y, Z) + \dot{\nabla}_X(g(Y, Z))$$

from which we have

$$(\nabla_X g)(Y, Z) = 0. \tag{3.7}$$

We also have from (3.6)

$$T(X, Y) = \tau(Y)X - \tau(X)Y. \tag{3.8}$$

From (3.7) and (3.8), we have the following theorem:

Theorem 3.1. *The connection induced on a non-degenerate umbilical affine hypersurfaces of recurrent affine manifold with a semi-symmetric semi-metric connection with respect to the unit normal vector field is also a semi-symmetric semi-metric connection.*

Now, the Weingarten equation with respect to the Levi-Civita connection $\dot{\bar{\nabla}}$ is

$$\dot{\bar{\nabla}}_X \xi = -\dot{A}X + \tau(X)\xi \tag{3.9}$$

for any vector field X in M, where A is a tensor field of type $(1, 1)$ of M. On the other hand, using (3.1), we get

$$\bar{\nabla}_{\bar{X}} \bar{Y} = \dot{\bar{\nabla}}_{\bar{X}} \bar{Y} - \bar{\tau}(\bar{X})\bar{Y} + \bar{g}(\bar{X}, \bar{Y})\bar{P},$$

$$\bar{\nabla}_{\bar{X}} \xi = \dot{\bar{\nabla}}_{\bar{X}} \xi - \bar{\tau}(\bar{X})\xi + \bar{g}(\bar{X}, \xi)\bar{P}. \tag{3.10}$$

Since $\bar{g}(\bar{X}, \xi) = 0$. Thus from (3.9) and (3.10), we get

$$\bar{\nabla}_X \xi = -\dot{A}X, \tag{3.11}$$

which is the Weingarten equation with respect to the semi-symmeyric semi-metric connection, where $\dot{A}$ is a (1,1)-tensor field on M and A is called the shape operator of M.

Let $\bar{R}$ and R be the curvature tensor fields of semi-symmetric semi-metric connection $\bar{\nabla}$ and the induced connection ∇. Thus

As the integrability conditions of (3.4) and (3.11), we have the Gauss and Codazzi equations with respect to the semi-symmetric semi-metric connection using (2.3), (2.4), (3.4) and (3.11).

$$\bar{R}(X,Y)Z = R(X,Y)Z - h(Y,Z)AX + h(X,Z)AY \tag{3.12}$$

$$+ ((\nabla_X h)(Y,Z) - (\nabla_Y h)(X,Z)) - h\,(\tau(Y)X - \tau(X)Y, Z)\,\xi,$$

$$\bar{R}(X,Y)\xi = -(\nabla_X A)Y + (\nabla_Y A)X - \tau(Y)AX \tag{3.13}$$

$$-\,((h(X,AY) - h(Y,AX))\,\xi - A\,(\tau(Y)X - \tau(X)Y)\,,$$

where X, Y, Z are arbitrary vector fields tangent to M.

4 Umbilical affine hypersurfaces

An affine hypersurface [12] M is said to be umbilical ([12],[13],[14]) if its shape operator A is proportional to the identity tensor field at every point of the hypersurface, that is, we have $A = \rho I_d$, where I_d is the identity tensor field and ρ is a certain function on M. Consequently, for such a hypersurface, we also have

$$\nabla A = d\rho \otimes I_d, \tag{4.1}$$

where d indicates the exterior derivative.
For an umbilical affine hypersurface, the Gauss and Codazzi equations (3.12) and (3.13) with respect to the semi-symmetric semi-metric connection (3.12) and (3.13) takes the form

$$\bar{R}(X,Y)Z = R(X,Y)Z - \rho h(Y,Z)AX + \rho h(X,Z)AY \tag{4.2}$$

$$+ ((\nabla_X h)(Y,Z) - (\nabla_Y h)(X,Z)) - h\,(\tau(Y)X - \tau(X)Y, Z)\,\xi,$$

$$\bar{R}(X,Y)\xi = (\rho\tau - d\rho)(Y)X - (\rho\tau - d\rho)(X)Y \tag{4.3}$$

respectively.

5 Main Results

Proposition 5.1. *For an umbilical affine hypersurface in an affine manifold $\bar{M}$ with a semi-symmetric semi-metric connection, we have*

$$\left((\bar{\nabla}_Z \bar{R})(X,Y)\xi\right)^T = \rho R(X,Y)Z - \rho^2\,(h(Y,Z)X - h(X,Z)Y) \tag{5.1}$$

$$+\rho h\,(\tau(Y)X - \tau(X)Y, Z)\xi)^T - ((\nabla_Z(\rho\tau - d\rho))\,(Y)X + ((\nabla_Z(\rho\tau - d\rho))\,(X)Y$$

$$+h(Y,Z)(\bar{R}(\xi,X)\xi)^T - h(X,Z)(\bar{R}(\xi,Y)\xi)^T.$$

Proof. Using equations (3.4), (3.11) and $A = \rho I_d$ into the general formula

$$(\bar{\nabla}_Z \bar{R})(X,Y)\xi = \bar{\nabla}_Z \bar{R}(X,Y)\xi - \bar{R}(\bar{\nabla}_Z X, Y)\xi$$

$$-\bar{R}(X, \bar{\nabla}_Z Y)\xi - \bar{R}(X,Y)\bar{\nabla}_Z \xi,$$

we find

$$(\bar{\nabla}_Z\bar{R})(X,Y)\xi = \bar{\nabla}_Z\bar{R}(X,Y)\xi - \bar{R}(\bar{\nabla}_Z X,Y)\xi - \bar{R}(X,\bar{\nabla}_Z Y)\xi \quad (5.2)$$
$$-h(Z,X)\bar{R}(\xi,Y) + h(Z,Y)\bar{R}(\xi,X)\xi + \rho\bar{R}(X,Y)Z.$$

On the other hand, using the equations (4.3), (3.11) and (3.4), we find

$$(\bar{\nabla}_Z\bar{R})(X,Y)\xi - \bar{\nabla}_Z\bar{R}(X,Y)\xi - \bar{R}(\bar{\nabla}_Z X,Y)\xi - (\bar{R}(X,\bar{\nabla}_Z Y)\xi)^T \quad (5.3)$$
$$= (\nabla_Z(\rho\tau - d\rho))(X)Y - (\nabla_Z(\rho\tau - d\rho))(Y)X.$$

Moreover, (4.2) and (4.3) implies

$$(\bar{R}(X,Y)Z)^T = R(X,Y)Z - \rho h(Y,Z)X + \rho h(X,Z)Y, \quad (5.4)$$

$$(\bar{R}(X,Y)\xi)^T = (\rho\tau - d\rho)(X)Y - (\rho\tau - d\rho)(Y)X. \quad (5.5)$$

Using (5.3)-(5.5) in (5.2) and comparing tangential parts, we get (5.1). □

Now, we will study the case when the ambient affine manifold $\bar{M}$ is a recurrent affine manifold, that is, the curvature tensor field $\bar{R}$ of $\bar{M}$ is non-zero and its covariant derivative $\bar{\nabla}\bar{R}$ satisfies the condition ([13])

$$\bar{\nabla}\bar{R} = \psi \otimes \bar{R} \quad (5.6)$$

for certain 1-form ψ.
We will prove the following result:

Proposition 5.2. *Let M be an umbilical affine hypersurface in a recurrent affine manifold $\bar{M}$ with a semi-symmetric semi-metric connection. Then the curvature tensor R is given by*

$$\rho R(X,Y)Z = \rho^2(h(Y,Z)X - h(X,Z)Y) + \rho h(\tau(Y)X - \tau(X)Y, Z)\xi)^T \quad (5.7)$$
$$-\psi(Z)\left((\rho\tau - d\rho)(Y)X - (\rho\tau - d\rho)(X)Y\right)$$
$$-\left((\nabla_Z(\rho\tau - d\rho)(Y))X - (\nabla_Z(\rho\tau - d\rho)(X))\,Y\right.$$
$$-h(Y,Z)(\bar{R}(\xi,X)\xi)^T + h(X,Z)(\bar{R}(\xi,Y)\xi)^T.$$

Proof. At first, note that (5.6) and (4.3) enable us to find

$$(\bar{\nabla}_Z\bar{R})(X,Y)\xi = \psi(Z)\left((\rho\tau - d\rho)(Y)X - (\rho\tau - d\rho)(X)Y\right).$$

Then, applying the above in (5.1), we obtain (5.7). □

6 A special class of Semi-symmetric semi-metric connection

In this section, a geometric situation occurs in which a pseudo-Riemannian manifold (M,g) admits a semi-symmetric semi-metric connection $\bar{\nabla}$ which is related to the Levi-Civita connection $\dot{\bar{\nabla}}$ of the metric g by the formula

$$\bar{\nabla}_X Y = \dot{\bar{\nabla}}_X Y - \eta(X)Y + g(X,Y)E, \quad (6.1)$$

where η is a 1-form and E a vector field on M.

Proposition 6.1. *Let $\bar{\nabla}$ be a semi-symmetric semi-metric connection on a pseudo-Riemannian manifold (M,g), which is related to the Levi-Civita connection $\dot{\bar{\nabla}}$ of g by the formula (6.1). Then for the curvature tensor fields R and R^* of $\bar{\nabla}$ and $\dot{\bar{\nabla}}$, respectively it holds*

$$R^*(X,Y)Z = R(X,Y)Z - ((\dot{\bar{\nabla}}_Y\eta)Z)X - ((\dot{\bar{\nabla}}_X\eta)Z)Y \quad (6.2)$$
$$-g(Y,Z)(\dot{\bar{\nabla}}_X E + g(X,E)E) + g(X,Z)(\dot{\bar{\nabla}}_Y E + g(Y,E)E).$$

Proof. Let $\bar{\nabla}^2$ and $\dot{\bar{\nabla}}^2$ denotes the second covariant derivatives with respect to $\bar{\nabla}$ and $\dot{\bar{\nabla}}$, respectively,

$$\bar{\nabla}^2_{XY}Z = \bar{\nabla}_X\bar{\nabla}_Y Z - \bar{\nabla}_{\bar{\nabla}_X Y}Z,$$

and

$$\dot{\bar{\nabla}}^2_{XY}Z = \dot{\bar{\nabla}}_X\dot{\bar{\nabla}}_Y Z - \dot{\bar{\nabla}}_{\dot{\bar{\nabla}}_X Y}Z.$$

Then, obviously

$$R(X,Y) = \bar{\nabla}^2_{XY} - \bar{\nabla}^2_{YX} \quad and \quad R^*(X,Y) = \dot{\bar{\nabla}}^2_{XY} - \dot{\bar{\nabla}}^2_{YX}. \tag{6.3}$$

At first, using (6.1), we find the following relation for the second covariant derivative

$$\begin{aligned}\dot{\bar{\nabla}}^2_{XY}Z = \bar{\nabla}^2_{XY}Z - ((\dot{\bar{\nabla}}_Y\eta)Z)X - ((\dot{\bar{\nabla}}_X\eta)Z)Y \\ -g(Y,Z)((\dot{\bar{\nabla}}_Y E + g(X,E)E) + SP(X,Y)Z,\end{aligned} \tag{6.4}$$

where, $SP(X,Y)Z$ indicates an expression which is symmetric with respect to X and Y. By using (6.3) and (6.4) and the following expression for exterior derivative [16]

$$d\eta(X,Y) = \frac{1}{2}\Big(((\dot{\bar{\nabla}}_X\eta)Y) - (\dot{\bar{\nabla}}_Y\eta)X)\Big).$$

(6.2) follows. □

Now, we have the following theorem:

Theorem 6.2. *Let $\bar{M}$ be a recurrent affine manifold with dimension $M \geq 5$. And let M be a non-degenerate umbilical affine hypersurface in $\bar{M}$ with a semi-symmetric semi-metric connection, whose shape operator A does not vanish at every point of M. Then the induced affine metric h is conformally flat.*

Proof. By considering proposition (6.1) and using (5.7), (6.2) with $g = h$, we have

$$\begin{aligned}\rho h\left(R^*(X,Y)Z,W\right) = \alpha(h(Y,Z)h(X,W) - h(X,Z)h(Y,W)) \\ + (h(Y,Z)w_1(X,W) - h(X,Z)w_1(Y,W)) \\ + (w_2(Y,Z)h(X,W) - w_2(X,Z)h(Y,W)) \\ +\beta(h(X,W)) - \gamma(h(Z,W)),\end{aligned} \tag{6.5}$$

where α, β and γ are the scalar functions and $w_{i's}$ are the $(0,2)$ tensor fields defined by

$$\alpha = \rho^2,$$

$$\beta = \rho\tau(Y),$$

$$\gamma = \rho\tau(X),$$

$$w_1 = \rho(\dot{\bar{\nabla}}_X E) + \rho h(X,E)E + (\bar{R}(\xi,X)\xi)^T, Y),$$

$$w_2 = -\psi(\rho\tau - d\rho)(X) - (\nabla_Z(\rho\tau - d\rho))(Y).$$

The anti-symmetrization of (6.5) with respect to Z and W, we get

$$\begin{aligned}\rho h\left(R^*(X,Y)Z,W\right) = \alpha(h(Y,Z)h(X,W) - h(X,Z)h(Y,W)) \\ + (h(Y,Z)w(X,W) - h(X,Z)w(Y,W)) \\ + (w(Y,Z)h(X,W) - w(X,Z)h(Y,W)) \\ +\beta(h(X,W)) - \gamma(h(Z,W)),\end{aligned} \tag{6.6}$$

where $w = \frac{1}{2}(w_1 + w_2)$.
From (6.6), for the Ricci tensor S^* and the scalar curvature r^* of $\dot{\bar{\nabla}}$, we find

$$\rho S^*(Y,Z) = (n-2)w(Y,Z) + ((n-1)\alpha + trh(w))h(Y,Z), \tag{6.7}$$

$$n(\beta\tau(Y) - \beta\tau(Z)).\rho r^* = 2(n-1)trh(w) + n(n-1)\alpha + n\beta\tau(Y), \tag{6.8}$$

where, $trh(w)$ indicates the trace of w with respect to the metric h.
Next, from (6.7) and (6.8), we get

$$w(Y,Z) = \frac{1}{n-2}\rho S^*(Y,Z) - \frac{1}{2}\left(\frac{1}{(n-1)(n-2)}\rho r^* + \alpha\right)$$
$$-\frac{1}{n-2}\left(n\beta\tau(Y) - \beta\tau(Z)\right).$$

which by using in (6.6) gives

$$\rho(h(R^*(X,Y)Z,W) - \frac{1}{n-2}(S^*(Y,Z)h(X,W) \tag{6.9}$$
$$-S^*(X,Z)h(Y,W) + h(Y,Z)S^*(X,W) - h(X,Z)S^*(Y,W))$$
$$+\frac{r^*}{(n-1)(n-2)}\left(h(Y,Z)h(X,W) - h(X,Z)h(Y,W)\right)$$
$$+\frac{1}{n-2}\left(\tau(Y)h(X,W) - \tau(Z)h(Z,W)\right) = 0,$$

that is, $\rho C^* = 0$, where C^* is the Weyl-Conformal curvature tensor of the metric h. This implies the assertion, since $n = dimM \geq 4$ and ρ is a non-zero everywhere on M. □

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