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GRAPH OPERATIONS ON THE SYMMETRIC DIVISION DEG INDEX OF GRAPHS

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Abstract. The Symmetric division deg index of a connected graph G, is defined as $SDD(G) = \sum_{uv\in E(G)} \frac{d_u}{d_v} + \frac{d_v}{d_u}$ where d_v is the degree of a vertex v in G. In this paper, we concentrated on the graph operations like lexicographic product, symmetric difference and corona product of graphs related to the symmetric division deg index.

1 Introduction and Preliminaries

Molecular descriptors, being numerical functions of molecular structure, play a fundamental role mathematical chemistry. They are used in QSAR and QSPR studies to relate biological or chemical properties of molecules to specific molecular descriptors [3]. Topological indices, being numerical functions of the underlying molecular graph, represent an important type of molecular descriptors. Some applications related topological indices related to smart polymers are found in [11] and comparative study of topological indices ad molecular weigh of some carbohydrates are in [12]. Recently in [5], C. K. Gupta and et al., established the relations on graph operations on matrix group.

Inspired by the most successful indices of this form, such as second zagreb index,[4], Randic index [10], [15] and others, there was defined a whole family of Adriatic indices [17]. In recent times [16], D. Vukicevic revealed the set of 148 discrete Adriatic indices. They were analyzed on the testing sets provided by the International Academy of Mathematical Chemistry and it had been shown that they have good predictive properties in many cases. There was a vast research regarding various properties of this topological index.

In a new article [17], D. Vukicevic posed the open questions in his end of the paper. Stimulate from this, here we ardent one of the index specifically, SDD. This emphasizes the development of lower and upper bounds for graphs [6]. This acquires some results that are partial answer to the open queries.

Symmetric division deg index is one of the discrete Adriatic indices that is good predictor of total surface area for polychlorobiphenyls. Some results on symmetric division deg index is also found in [1].

In group theory, a nilpotent group is a group that is "almost abelian". This idea is motivated by the fact that nilpotent groups are solvable and for finite nilpotent groups, two elements having relatively prime orders must commute. The multiplicative group of upper unitriangular $n \times n$ matrices over any field F is a nilpotent group of length $n-1$ [14]. Here, we discussed some relations related to cayley graph of nilpotent matrix group of length one related to SDD.

We recall some definitions which are essential.

Definition: 1 The first Zagreb index [9] defined as,

$$M_1(G) = \sum d(u)^2 = \sum_{u,v\in E(G)} [d(u) + d(v)].$$

Definition: 2 The Symmetric division deg index of a connected graph G, is defined as

$$SDD(G) = \sum_{uv\in E(G)} \frac{max(d_u, d_v)}{min(d_u, d_v)} + \frac{min(d_u, d_v)}{max(d_u, d_v)} = \sum_{uv\in E(G)} \frac{d_u}{d_v} + \frac{d_v}{d_u} = \sum_{uv\in E(G)} \frac{{d_u}^2 + {d_v}^2}{d_u d_v}$$

where d_v is the degree of a vertex v in G.

The Composition (also called lexicographic product [7]) $G = G_1[G_2]$ of graph G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$ is the graph with vertex set $V(G_1)\times V(G_2)$ and (u_i, v_j) is adjacent with (u_k, v_l) whenever u_i is adjacent with u_k, or $u_i = u_k$ and v_j is adjacent with v_l.

In [2], the Cartesian product $G_1\times G_2$ of graph G_1 and G_2 has the vertex set $V(G_1\times G_2)$ $=V(G_1)\times V(G_2)$ and $(u_i, v_j)(u_k, v_l)$ is an edge of $G_1\times G_2$ if $u_i = u_k$ and $v_j v_l \in E(G_2)$, or $u_i u_k \in E(G_1)$ and $v_j = v_l$.

For given graph G_1 and G_2 we define their Corona product G_1 o G_2 as the graph obtained by taking $|V(G_1)|$ copies of G_2 and joining each vertex of the i-th copy with vertex $v_i \in V(G_1)$. Obviously, $|V(G_1 o G_2)| = |V(G_1)|(1+|V(G_2)|)$ and $|E(G_1 o G_2)| = |E(G_1)| + |V(G_1)|(|V(G_2)| + |E(G_2)|)$ [4].

A sum $G_1 + G_2$ of two graph G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ is the graph on the vertex set $V(G_1) \cup V(G_2)$ and the edge set
$E(G_1) \cup E(G_2) \cup \{uv \mid u \in V(G_1), v \in V(G_2)\}$. Hence, the sum of two graph is obtained by connecting each vertex of one graph to each vertex of the other graph, while keeping all edges of both graph.[18].

The Symmetric difference [8] $G_1 \oplus G_2$ of two graph G_1 and G_2 is the graph with vertex set $V(G_1) \times V(G_2)$ and
$E(G_1 \oplus G_2) = \{(u_1, u_2)(v_1, v_2) | u_1 v_1 \in E(G_1) \ \ or \ \ u_2 v_2 \in E(G_2) \ \ but \ \ not \ \ both\}$.

Obviously,
$|E(G_1 \oplus G_2)| = |E(G_1)||V(G_2)|^2 + |E(G_2)||V(G_1)|^2 - 4|E(G_1)||E(G_2)|$
$d_{G_1\oplus G_2}(u, v) = |V(G_2)| d_{G_1}(u) + |V(G_1)| d_{G_2}(v) - 2 d_{G_1}(u) d_{G_2}(v)$.

Some graph operations on Harmonic index are found in [13]. Motivated from this, in this paper, we concentrate on graph operations like join, corona product, cartesian product, composition and symmetric difference of graph are established.

2 Main Results

In this section, we established the graph operations for SDD index.

Theorem 2.1. *Let G_1 and G_2 be two connected graph with order n_1, n_2, size m_1, m_2, maximum degree Δ_1, Δ_2 and minimum degree δ_1, δ_2 respectively. Then*

$$\begin{aligned} SDD(G_1[G_2]) \leq{} & \frac{n_1}{n_2\delta_1 + \delta_2}(2m_2 n_2 \Delta_1 + M_1(G_2)) \\ & + \frac{{n_2}^2}{n_2\delta_1 + \delta_2}(2m_1 \Delta_2 + n_2 M_1(G_1)) \end{aligned}$$

Equality hold only if graph is regular.

Proof. Let $V(G_1) = \{u_1, u_2, ...u_{n_1}\}$ and $V(G_2) = \{v_1, v_2, ...v_{n_2}\}$ be a set of vertex for G_1 and G_2 respectively. By the Definition of the composition of two graph one can see that,

$$|E(G_1[G_2])| = |E(G_1)||V(G_2)|^2 + |E(G_2)||V(G_1)|$$
$$d_{G_1[G_2]}(u, v) = |V(G_2)| d_{G_1}(u) + d_{G_2}(v)$$

Consider,

$$
\begin{aligned}
SDD(G_1[G_2]) &= \sum_{(u_i,v_j),(u_k,v_l)\in E(G_1[G_2]),(u_i,v_j)\neq(u_k,v_l)} \frac{d_{G_1[G_2]}(u_i,v_j)}{d_{G_1[G_2]}(u_k,v_l)} + \frac{d_{G_1[G_2]}(u_k,v_l)}{d_{G_1[G_2]}(u_i,v_j)} \\
&= \sum_{(u_i,v_j),(u_i,v_l)\in E(G_1[G_2]),j\neq l} \frac{d_{G_1[G_2]}(u_i,v_j)}{d_{G_1[G_2]}(u_i,v_l)} + \frac{d_{G_1[G_2]}(u_i,v_l)}{d_{G_1[G_2]}(u_i,v_j)} \\
&\quad + \sum_{(u_i,v_j),(u_k,v_l)\in E(G_1[G_2]),i\neq k} \frac{d_{G_1[G_2]}(u_i,v_j)}{d_{G_1[G_2]}(u_k,v_l)} + \frac{d_{G_1[G_2]}(u_k,v_l)}{d_{G_1[G_2]}(u_i,v_j)} \\
&= \sum_{u_i\in V(G_1)} \sum_{v_j,v_l\in E(G_2)} \frac{|V(G_2)|d_{G_1}(u_i)+d_{G_2}(v_j)}{|V(G_2)|d_{G_1}(u_i)+d_{G_2}(v_l)} + \frac{|V(G_2)|d_{G_1}(u_i)+d_{G_2}(v_l)}{|V(G_2)|d_{G_1}(u_i)+d_{G_2}(v_j)} \\
&\quad + \sum_{u_i,u_k\in E(G_1)} \sum_{v_j\in V(G_2)} \sum_{v_l\in V(G_2)} \frac{|V(G_2)|d_{G_1}(u_i)+d_{G_2}(v_j)}{|V(G_2)|d_{G_1}(u_k)+d_{G_2}(v_l)} + \frac{|V(G_2)|d_{G_1}(u_k)+d_{G_2}(v_l)}{|V(G_2)|d_{G_1}(u_i)+d_{G_2}(v_j)} \\
&= n_1 \sum_{v_j,v_l\in E(G_2)} \frac{n_2 d_{G_1}(u_i)+d_{G_2}(v_j)}{n_2 d_{G_1}(u_i)+d_{G_2}(v_l)} + \frac{n_2 d_{G_1}(u_i)+d_{G_2}(v_l)}{n_2 d_{G_1}(u_i)+d_{G_2}(v_j)} \\
&\quad + {n_2}^2 \sum_{u_i,u_k\in E(G_1)} \frac{n_2 d_{G_1}(u_i)+d_{G_2}(v_j)}{n_2 d_{G_1}(u_k)+d_{G_2}(v_l)} + \frac{n_2 d_{G_1}(u_k)+d_{G_2}(v_l)}{n_2 d_{G_1}(u_i)+d_{G_2}(v_j)} \\
&\leq n_1 \sum_{v_j,v_l\in E(G_2)} \frac{n_2\Delta_1+d_{G_2}(v_j)}{n_2\delta_1+\delta_2} + \frac{n_2\Delta_1+d_{G_2}(v_l)}{n_2\delta_1+\delta_2} \\
&\quad + {n_2}^2 \sum_{u_i,u_k\in E(G_1)} \frac{n_2 d_{G_1}(u_i)+\Delta_2}{n_2\delta_1+\delta_2} + \frac{n_2 d_{G_1}(u_k)+\Delta_2}{n_2\delta_1+\delta_2} \\
&= \frac{n_1}{n_2\delta_1+\delta_2} \sum_{v_j,v_l\in E(G_2)} (2n_2\Delta_1 + d_{G_2}(v_j) + d_{G_2}(v_l)) \\
&\quad + \frac{{n_2}^2}{n_2\delta_1+\delta_2} \sum_{u_i,u_k\in E(G_1)} (2\Delta_2 + n_2(d_{G_1}(u_i)+d_{G_1}(u_k))) \\
&= \frac{n_1}{n_2\delta_1+\delta_2}(2m_2 n_2\Delta_1 + M_1(G_2)) + \frac{{n_2}^2}{n_2\delta_1+\delta_2}(2m_1\Delta_2 + n_2 M_1(G_1)).
\end{aligned}
$$

□

Corollary 2.2. *Let G_i for $i \in \{1,2\}$, be a cayley graph of nilpotent matrix group of length one. Then*

$$SDD(G_1[G_2]) = 4n^2(n+1) \;\; for \;\; n \geq 7.$$

Proof. Result is directly from the theorem 2.1. Since For $n \geq 7$, cayley graph of nilpotent matrix group of length one with n vertices has $2n$ edges and degree of each vertices is 4.
Also $M_1(G_i) = 16n$. □

Theorem 2.3. *Let G_1 and G_2 be two connected graph with order n_1, n_2 and size m_1, m_2 respectively. Then*

$$SDD(G_1 \times G_2) \leq \frac{2(\Delta_1+\Delta_2)}{\delta_1+\delta_2}(n_1 m_2 + n_2 m_1).$$

Equality hold only if graph is regular.

Proof. Let $V(G_1) = \{u_1, u_2, ...u_{n_1}\}$ and $V(G_2) = \{v_1, v_2, ...v_{n_2}\}$ be a set of vertex for G_1 and G_2 respectively. By the Definition of the cartesian product of two graph one can see that,

$$
\begin{aligned}
&|E(G_1\times G_2)| = |E(G_1)||V(G_2)| + |E(G_2)||V(G_1)| \\
&d_{G_1\times G_2}(u,v) = d_{G_1}(u) + d_{G_2}(v).
\end{aligned}
$$

Consider,

$$
\begin{aligned}
SDD(G_1 \times G_2) &= \sum_{(u_i,v_j),(u_k,v_l)\in E(G_1\times G_2),(u_i,v_j)\neq(u_k,v_l)} \Big[\frac{d_{G_1\times G_2}(u_i,v_j)}{d_{G_1\times G_2}(u_k,v_l)} + \frac{d_{G_1\times G_2}(u_k,v_l)}{d_{G_1\times G_2}(u_i,v_j)}\Big] \\
&= \sum_{(u_i,v_j),(u_i,v_l)\in E(G_1\times G_2),v_jv_l\in E(G_2)} \Big[\frac{d_{G_1\times G_2}(u_i,v_j)}{d_{G_1\times G_2}(u_i,v_l)} + \frac{d_{G_1\times G_2}(u_i,v_l)}{d_{G_1\times G_2}(u_i,v_j)}\Big] \\
&+ \sum_{(u_i,v_j),(u_k,v_j)\in E(G_1\times G_2),u_iu_k\in E(G_1)} \Big[\frac{d_{G_1\times G_2}(u_i,v_j)}{d_{G_1\times G_2}(u_k,v_j)} + \frac{d_{G_1\times G_2}(u_k,v_j)}{d_{G_1\times G_2}(u_i,v_j)}\Big] \\
&= \sum_{u_i\in V(G_1)} \sum_{v_j,v_l\in E(G_2)} \Big[\frac{d_{G_1}(u_i)+d_{G_2}(v_j)}{d_{G_1}(u_i)+d_{G_2}(v_l)} + \frac{d_{G_1}(u_i)+d_{G_2}(v_l)}{d_{G_1}(u_i)+d_{G_2}(v_j)}\Big] \\
&+ \sum_{v_j\in V(G_2)} \sum_{u_i,u_k\in E(G_1)} \Big[\frac{d_{G_1}(u_i)+d_{G_2}(v_j)}{d_{G_1}(u_k)+d_{G_2}(v_j)} + \frac{d_{G_1}(u_k)+d_{G_2}(v_j)}{d_{G_1}(u_i)+d_{G_2}(v_j)}\Big] \\
&\le n_1 \sum_{v_j,v_l\in E(G_2)} \Big[\frac{\Delta_1+\Delta_2}{\delta_1+\delta_2} + \frac{\Delta_1+\Delta_2}{\delta_1+\delta_2}\Big] \\
&+ n_2 \sum_{u_i,u_k\in E(G_1)} \Big[\frac{\Delta_1+\Delta_2}{\delta_1+\delta_2} + \frac{\Delta_1+\Delta_2}{\delta_1+\delta_2}\Big] \\
&= 2n_1m_2\Big[\frac{\Delta_1+\Delta_2}{\delta_1+\delta_2}\Big] + 2n_2m_1\Big[\frac{\Delta_1+\Delta_2}{\delta_1+\delta_2}\Big] \\
&= 2\Big[\frac{\Delta_1+\Delta_2}{\delta_1+\delta_2}\Big](n_1m_2+n_2m_1).
\end{aligned}
$$

Equality hold only if graph is regular. □

Corollary 2.4. *Let G_i for $i \in \{1,2\}$, be a cayley graph of nilpotent matrix group of length one. Then*

$$SDD(G_1 \times G_2) = 8n^2 \ \ for \ \ n \ge 7.$$

Theorem 2.5. *For $i \in \{1,2\}$, let G_i be a graph of minimum degree δ_i, maximum degree Δ_i,order n_i and size m_i. Then*

$$SDD(G_1oG_2) \le 2m_1\Big(\frac{\Delta_1+n_2}{\delta_1+n_2}\Big) + 2m_2n_1\Big(\frac{\Delta_2+1}{\delta_2+1}\Big) + n_1n_2\Big(\frac{\Delta_1+n_2}{\delta_2+1} + \frac{\Delta_2+1}{\delta_1+n_2}\Big).$$

Equality hold only if graph is regular.

Proof. The edges of G_1oG_2 are partitioned into three subsets E_1, E_2 and E_3 as follows

$$
\begin{aligned}
&E_1 = \{e \in E(G_1oG_2), e \in E(G_1)\} \\
&E_2 = \{e \in E(G_1oG_2), e \in E(G_{2i}), i = 1,2...|V(G_1)|\} \\
&E_3 = \{e \in E(G_1oG_2), e = uv, u \in V(G_{2i}), i = 1,2...|V(G_1)| \ and \ v \in V(G_1)\}.
\end{aligned}
$$

and if u is a vertex of G_1oG_2, then

$$d_{G_1oG_2}(u) = \begin{cases} d_{G_1}(u) + |V(G_2)| & if \ \ u \in V(G_1) \\ d_{G_2}(u) + 1 & if \ \ u \in V(G_2) \end{cases}$$

Let $G_1 = (V_i, E_i)$, $i \in \{1,2\}$ and let $G_1oG_2 = (V, E)$
we have

$$
\begin{aligned}
SDD(G_1oG_2) &= \sum_{uv\in E(G_1oG_2)} \Big(\frac{d_{G_1oG_2}(u)}{d_{G_1oG_2}(v)} + \frac{d_{G_1oG_2}(v)}{d_{G_1oG_2}(u)}\Big) \\
&= Q_1 + Q_2 + Q_3.
\end{aligned}
$$

where

$$\begin{aligned} Q_1 &= \sum_{uv\in E_1}\Big(\frac{d_{G_1}(u)+n_2}{d_{G_1}(v)+n_2}+\frac{d_{G_1}(v)+n_2}{d_{G_1}(u)+n_2}\Big)\\ &\le \sum_{uv\in E_1}\Big(\frac{\Delta_1+n_2}{\delta_1+n_2}+\frac{\Delta_1+n_2}{\delta_1+n_2}\Big)=2m_1\Big(\frac{\Delta_1+n_2}{\delta_1+n_2}\Big)\\ Q_2 &= n_1\sum_{uv\in E_2}\Big(\frac{d_{G_2}(u)+1}{d_{G_1}(v)+1}+\frac{d_{G_1}(v)+1}{d_{G_1}(u)+1}\Big)\\ &\le n_1m_2\Big(\frac{\Delta_2+1}{\delta_2+1}+\frac{\Delta_2+1}{\delta_2+1}\Big)=2n_1m_2\Big(\frac{\Delta_2+1}{\delta_2+1}\Big)\\ Q_3 &= \sum_{uv\in E_3, u\in V_1 and v\in V_2}\Big(\frac{d_{G_1}(u)+n_2}{d_{G_2}(v)+1}+\frac{d_{G_2}(v)+1}{d_{G_1}(u)+n_2}\Big)\\ &\le n_1n_2\Big(\frac{\Delta_1+n_2}{\delta_2+1}+\frac{\Delta_2+1}{\delta_1+n_2}\Big). \end{aligned}$$

Using Q_1 to Q_3 in $SDD(G_1oG_2)$, we get

$$SDD(G_1oG_2)\le 2m_1\Big(\frac{\Delta_1+n_2}{\delta_1+n_2}\Big)+2m_2n_1\Big(\frac{\Delta_2+1}{\delta_2+1}\Big)+n_1n_2\Big(\frac{\Delta_1+n_2}{\delta_2+1}+\frac{\Delta_2+1}{\delta_1+n_2}\Big).$$

□

Corollary 2.6. *Let G_i for $i\in\{1,2\}$, be a cayley graph of nilpotent matrix group of length one. Then*

$$SDD(G_1oG_2)=4n^2+4n+n^2\Big(\frac{n^2+8n+41}{5n+20}\Big)\ \ for\ \ n\ge 7.$$

Theorem 2.7. *Let G_1 and G_2 be two connected graph with order n_1, n_2 and size m_1, m_2 respectively. Then*

$$SDD(G_1+G_2)\le 2m_1\Big(\frac{\Delta_1+n_2}{\delta_1+n_2}\Big)+2m_2\Big(\frac{\Delta_2+n_1}{\delta_2+n_1}\Big)+n_1n_2\Big(\frac{\Delta_1+n_2}{\delta_2+n_1}+\frac{\Delta_2+n_1}{\delta_1+n_2}\Big).$$

Proof. Let $V(G_1)=\{u_1,u_2,...u_{n_1}\}$ and $V(G_2)=\{v_1,v_2,...v_{n_2}\}$ be a set of vertex for G_1 and G_2 respectively. By the Definition of the join of two graph one can see that, if u is a vertex of G_1+G_2, then

$$d_{G_1+G_2}(u)=\begin{cases} d_{G_1}(u)+|V(G_2)| & if\ \ u\in V(G_1)\\ d_{G_2}(u)+|V(G_1)| & if\ \ u\in V(G_2)\end{cases}$$

Therefore,

$$\begin{aligned} SDD(G_1+G_2) &= \sum_{uv\in E(G_1+G_2)}\Big(\frac{d_{G_1+G_2}(u)}{d_{G_1+G_2}(v)}+\frac{d_{G_1+G_2}(v)}{d_{G_1+G_2}(u)}\Big)\\ &= \sum_{uv\in E(G_1)}\Big(\frac{d_{G_1}(u)+n_2}{d_{G_1}(v)+n_2}+\frac{d_{G_1}(v)+n_2}{d_{G_1}(u)+n_2}\Big)+\sum_{uv\in E(G_2)}\Big(\frac{d_{G_2}(u)+n_1}{d_{G_2}(v)+n_1}+\frac{d_{G_2}(v)+n_1}{d_{G_2}(u)+n_1}\Big)\\ &\quad+\sum_{u\in V(G_1), v\in V(G_2)}\Big(\frac{d_{G_1}(u)+n_2}{d_{G_2}(v)+n_1}+\frac{d_{G_2}(v)+n_1}{d_{G_1}(u)+n_2}\Big)\\ &\le 2m_1\Big(\frac{\Delta_1+n_2}{\delta_1+n_2}\Big)+2m_2\Big(\frac{\Delta_2+n_1}{\delta_2+n_1}\Big)+n_1n_2\Big(\frac{\Delta_1+n_2}{\delta_2+n_1}+\frac{\Delta_2+n_1}{\delta_1+n_2}\Big). \end{aligned}$$

□

Corollary 2.8. *Let G_i for $i\in\{1,2\}$, be a cayley graph of nilpotent matrix group of length one. Then*

$$SDD(G_1+G_2)=2n^2+8n\ \ for\ \ n\ge 7.$$

Theorem 2.9. *For $i \in \{1,2\}$, let G_i be a graph of maximum degree Δ_i, minimum degree δ_i order n_i and size m_i. Then*

$$SDD(G_1 \oplus G_2) \leq 2(n_2^2 m_1 + n_1^2 m_2 - 4m_1 m_2)\frac{n_2\Delta_1 + n_1\Delta_2 - 2\delta_1\delta_2}{n_2\delta_1 + n_1\delta_2 - 2\Delta_1\Delta_2} \quad .$$

Proof. Let $V(G_1) = \{u_1, u_2, ...u_{n_1}\}$ and $V(G_2) = \{v_1, v_2, ..., v_{n_2}\}$ be a set of vertex for G_1 and G_2 respectively.

Consider,

$$\begin{aligned} SDD(G_1 \oplus G_2) &= \sum_{(u_i,v_j),(u_k,v_l)\in E(G_1\oplus G_2)} \frac{d_{G_1\oplus G_2}(u_i,v_j)}{d_{G_1\oplus G_2}(u_k,v_l)} + \frac{d_{G_1\oplus G_2}(u_k,v_l)}{d_{G_1\oplus G_2}(u_i,v_j)} \\ &= \sum_{v_j\in V(G_2)} \sum_{v_l\in V(G_2)} \sum_{u_i,u_k\in E(G_1)} \frac{d_{G_1\oplus G_2}(u_i,v_j)}{d_{G_1\oplus G_2}(u_k,v_l)} + \frac{d_{G_1\oplus G_2}(u_k,v_l)}{d_{G_1\oplus G_2}(u_i,v_j)} \\ &\quad + \sum_{u_i\in V(G_1)} \sum_{u_k\in V(G_1)} \sum_{v_j,v_l\in E(G_2)} \frac{d_{G_1\oplus G_2}(u_i,v_j)}{d_{G_1\oplus G_2}(u_k,v_l)} + \frac{d_{G_1\oplus G_2}(u_k,v_l)}{d_{G_1\oplus G_2}(u_i,v_j)} \\ &\quad - \sum_{u_i,u_k\in E(G_1)} \sum_{v_j,v_l\in E(G_1)} \frac{d_{G_1\oplus G_2}(u_i,v_j)}{d_{G_1\oplus G_2}(u_k,v_l)} + \frac{d_{G_1\oplus G_2}(u_k,v_l)}{d_{G_1\oplus G_2}(u_i,v_j)} ......(1) \end{aligned}$$

As per the definition of the Symmetric difference of a graph

$$\begin{aligned} \frac{d_{G_1\oplus G_2}(u_i,v_j)}{d_{G_1\oplus G_2}(u_k,v_l)} + \frac{d_{G_1\oplus G_2}(u_k,v_l)}{d_{G_1\oplus G_2}(u_i,v_j)} &= \frac{n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - 2d_{G_1}(u_i)d_{G_2}(v_j)}{n_2 d_{G_1}(u_k) + n_1 d_{G_2}(v_l) - 2d_{G_1}(u_k)d_{G_2}(v_l)} \\ &\quad + \frac{n_2 d_{G_1}(u_k) + n_1 d_{G_2}(v_l) - 2d_{G_1}(u_k)d_{G_2}(v_l)}{n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - 2d_{G_1}(u_i)d_{G_2}(v_j)} \\ &\leq \frac{n_2\Delta_1 + n_1\Delta_2 - 2\delta_1\delta_2}{n_2\delta_1 + n_1\delta_2 - 2\Delta_1\Delta_2} + \frac{n_2\Delta_1 + n_1\Delta_2 - 2\delta_1\delta_2}{n_2\delta_1 + n_1\delta_2 - 2\Delta_1\Delta_2} \\ &= 2\frac{n_2\Delta_1 + n_1\Delta_2 - 2\delta_1\delta_2}{n_2\delta_1 + n_1\delta_2 - 2\Delta_1\Delta_2} ......(2) \end{aligned}$$

From equation (1) and (2) we get

$$\begin{aligned} SDD(G_1 \oplus G_2) \leq n_2^2 m_1 &\frac{2(n_2\Delta_1 + n_1\Delta_2 - 2\delta_1\delta_2)}{n_2\delta_1 + n_1\delta_2 - 2\Delta_1\Delta_2} \\ &+ n_1^2 m_2 \frac{2(n_2\Delta_1 + n_1\Delta_2 - 2\delta_1\delta_2)}{n_2\delta_1 + n_1\delta_2 - 2\Delta_1\Delta_2} \\ &- 4m_1 m_2 \frac{2(n_2\Delta_1 + n_1\Delta_2 - 2\delta_1\delta_2)}{n_2\delta_1 + n_1\delta_2 - 2\Delta_1\Delta_2} \\ = 2(n_2^2 m_1 + n_1^2 m_2 - 4m_1 m_2)&\frac{n_2\Delta_1 + n_1\Delta_2 - 2\delta_1\delta_2}{n_2\delta_1 + n_1\delta_2 - 2\Delta_1\Delta_2}. \end{aligned}$$

□

Corollary 2.10. *Let G_i for $i \in \{1,2\}$, be a cayley graph of nilpotent matrix group of length one. Then*

$$SDD(G_1 + G_2) = 8n^2(n-4) \quad for \quad n \geq 7.$$

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