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Some fixed point results for generalized contractions in cone b-metric spaces

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Abstract. In this paper, we establish some fixed point results in cone b-metric spaces satisfying generalized contraction involving rational expressions. Also, as a consequence, some results of integral type for such mapping is obtained. Our results extend and generalize several known results from the existing literature.

1 Introduction and Preliminaries

It is well known that contractive type conditions play an important role in the study of fixed point theory. The Banach contraction mapping [3] is one of the pivotal results of analysis. It is a very popular tool in settling solvability problems in different fields of mathematics. A variety of generalizations of the classical Banach contraction principle are available in the existing literature of metric fixed point theory (see [1, 2, 4, 6, 7, 17] and many others). Many of these generalizations are obtained by improving the underlying contraction condition. The famous Banach contraction principle theorem states as follows.

Theorem 1.1. *([3]) Let (X, d) be a complete metric space and T be a mapping of X into itself satisfying:*

$$d(Tx, Ty) \quad \leq \quad k\, d(x, y),\ \forall\, x,\, y \in X, \tag{1.1}$$

where k is a constant in $(0, 1)$. Then T has a unique fixed point $x^* \in X$.

In 1989, Bakhtin [5] introduced b-metric spaces as a generalization of metric spaces. He proved the contraction mapping principle in b-metric spaces that generalized the famous contraction principle in metric spaces. Czerwik used the concept of b-metric space and generalized the renowned Banach fixed point theorem in b-metric spaces (see, [9, 10]). In 2007, Huang and Zhang [15] introduced the concept of cone metric spaces as a generalization of metric spaces and establish some fixed point theorems for contractive mappings in normal cone metric spaces. In 2008, Rezapour and Hamlbarani [21] omitted the assumption of normality in cone metric space, which is a milestone in developing fixed point theory in cone metric space.

In 2011, Hussain and Shah [16] introduced the concept of cone b-metric space as a generalization of b-metric space and cone metric spaces. They established some topological properties in such spaces and improved some recent results about KKM mappings in the setting of a cone b-metric space. Cone b-metric spaces play a useful role in fixed point theory. In fact there exist mappings with common fixed points which are contraction mappings in a cone b-metric space but are not contraction mappings when defined in a cone metric space. Example 1.11 of this paper illustrates this fact.

It can be seen that many of the generalized metric spaces are not necessarily Hausdorff (see, [13, 19, 23, 24]). Proper examples of non Hausdorff rectangular metric space and rectangular b-metric space can be found in [13, 22, 23] (see, also [12]). Note that spaces with non Hausdorff topology play an important role in Tarskian approach to programming language semantics used in computer science.

In the present work we prove some fixed point results in cone b-metric spaces for mapping satisfying generalized contraction involving rational expressions.

Definition 1.2. ([15]) Let E be a real Banach space. A subset P of E is called a cone whenever the following conditions hold:

$(C1)$ P is closed, nonempty and $P \neq \{0\}$;

$(C2)$ $a, b \in R$, $a, b \geq 0$ and $x, y \in P$ imply $ax + by \in P$;

$(C3)$ $P \cap (-P) = \{0\}$.

Given a cone $P \subset E$, we define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in P^0$, where P^0 stands for the interior of P. If $P^0 \neq \emptyset$ then P is called a solid cone (see [25]).

There exists two kinds of cones: normal cones (with constant K) and non-normal ones ([11]).

Let E be a real Banach space, $P \subset E$ a cone and $\leq$ partial ordering defined by P. Then P is called normal if there is a number $K > 0$ such that for all $x, y \in P$,

$$0 \leq x \leq y \text{ imply } \|x\| \leq K\|y\|, \tag{1.2}$$

or equivalently, if $(\forall n)$ $x_n \leq y_n \leq z_n$ and

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} z_n = x \text{ imply } \lim_{n\to\infty} y_n = x. \tag{1.3}$$

The least positive number K satisfying (1.2) is called the normal constant of P.

Example 1.3. ([25]) Let $E = C^1_{\mathbb{R}}[0, 1]$ with $\|x\| = \|x\|_\infty + \|x'\|_\infty$ on $P = \{x \in E : x(t) \geq 0\}$. This cone is not normal. Consider, for example, $x_n(t) = \frac{t^n}{n}$ and $y_n(t) = \frac{1}{n}$. Then $0 \leq x_n \leq y_n$, and $\lim_{n\to\infty} y_n = 0$, but $\|x_n\| = \max_{t\in[0,1]} |\frac{t^n}{n}| + \max_{t\in[0,1]} |t^{n-1}| = \frac{1}{n} + 1 > 1$; hence x_n does not converge to zero. It follows by (1.2) that P is a non-normal cone.

Definition 1.4. ([15, 26]) Let X be a nonempty set. Suppose that the mapping $d\colon X \times X \to E$ satisfies:

$(CM1)$ $0 \leq d(x, y)$ for all $x, y \in X$ with $x \neq y$ and $d(x, y) = 0 \;\Leftrightarrow\; x = y$;

$(CM2)$ $d(x, y) = d(y, x)$ for all $x, y \in X$;

$(CM3)$ $d(x, y) \leq d(x, z) + d(z, y)$ $x, y, z \in X$.

Then d is called a cone metric [15] on X and (X, d) is called a cone metric space (CMS) [15].

The concept of a cone metric space is more general than that of a metric space, because each metric space is a cone metric space where $E = \mathbb{R}$ and $P = [0, +\infty)$.

Example 1.5. ([15]) Let $E = \mathbb{R}^2$, $P = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0\}$, $X = \mathbb{R}$ and $d\colon X \times X \to E$ defined by $d(x, y) = (|x - y|, \alpha|x - y|)$, where $\alpha \geq 0$ is a constant. Then (X, d) is a cone metric space with normal cone P where $K = 1$.

Example 1.6. ([20]) Let $E = \ell^2$, $P = \{\{x_n\}_{n\geq1} \in E : x_n \geq 0, \text{for all } n\}$, (X, ρ) a metric space, and $d\colon X \times X \to E$ defined by $d(x, y) = \{\rho(x, y)/2^n\}_{n\geq1}$. Then (X, d) is a cone metric space.

Clearly, the above examples show that the class of cone metric spaces contains the class of metric spaces.

Definition 1.7. ([16]) Let X be a nonempty set and $s \geq 1$ be a given real number. A mapping $d\colon X \times X \to E$ is said to be cone b-metric if and only if, for all x, y, $z \in X$, the following conditions are satisfied:

$(CbM1)$ $0 \leq d(x,y)$ with $x \neq y$ and $d(x,y) = 0 \Leftrightarrow x = y$;

$(CbM2)$ $d(x,y) = d(y,x)$;

$(CbM3)$ $d(x,y) \leq s[d(x,z) + d(z,y)]$.

The pair (X,d) is called a cone b-metric space (CbMS).

Remark 1.8. The class of cone b-metric spaces is larger than the class of cone metric space since any cone metric space must be a cone b-metric space. Therefore, it is obvious that cone b-metric spaces generalize b-metric spaces and cone metric spaces.

We give some examples, which show that introducing a cone b-metric space instead of a cone metric space is meaningful since there exist cone b-metric space which are not cone metric space.

Example 1.9. ([14]) Let $E = \mathbb{R}^2$, $P = \{(x,y) \in E : x \geq 0, y \geq 0\} \subset E$, $X = \mathbb{R}$ and $d\colon X \times X \to E$ defined by $d(x,y) = (|x-y|^p, \alpha|x-y|^p)$, where $\alpha \geq 0$ and $p > 1$ are two constants. Then (X,d) is a cone b-metric space with the coefficient $s = 2^p > 1$, but not a cone metric space.

Example 1.10. ([14]) Let $X = \ell^p$ with $0 < p < 1$, where $\ell^p = \{\{x_n\} \subset \mathbb{R} : \sum_{n=1}^{\infty} |x_n|^p < \infty\}$. Let $d\colon X \times X \to \mathbb{R}_+$ defined by $d(x,y) = \left(\sum_{n=1}^{\infty} |x_n - y_n|^p\right)^{\frac{1}{p}}$, where $x = \{x_n\}$, $y = \{y_n\} \in \ell^p$. Then (X,d) is a cone b-metric space with the coefficient $s = 2^p > 1$, but not a cone metric space.

Example 1.11. ([14]) Let $X = \{1,2,3,4\}$, $E = \mathbb{R}^2$, $P = \{(x,y) \in E : x \geq 0, y \geq 0\}$. Define $d\colon X \times X \to E$ by

$$d(x,y) = \begin{cases} (|x-y|^{-1}, |x-y|^{-1}) & \text{if } x \neq y, \\ 0, & \text{if } x = y. \end{cases}$$

Then (X,d) is a cone b-metric space with the coefficient $s = \frac{6}{5} > 1$. But it is not a cone metric space since the triangle inequality is not satisfied,

$$d(1,2) > d(1,4) + d(4,2), \quad d(3,4) > d(3,1) + d(1,4).$$

Definition 1.12. ([16]) Let (X,d) be a cone b-metric space, $x \in X$ and $\{x_n\}$ be a sequence in X. Then

- $\{x_n\}$ is a Cauchy sequence whenever, if for every $c \in E$ with $0 \ll c$, then there is a natural number N such that for all $n, m \geq N$, $d(x_n, x_m) \ll c$;

- $\{x_n\}$ converges to x whenever, for every $c \in E$ with $0 \ll c$, then there is a natural number N such that for all $n \geq N$, $d(x_n, x) \ll c$. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$.

- (X,d) is a complete cone b-metric space if every Cauchy sequence is convergent.

In the following (X,d) will stands for a cone b-metric space with respect to a cone P with $P^0 \neq \emptyset$ in a real Banach space E and $\leq$ is partial ordering in E with respect to P.

The following lemmas are often used (in particular when dealing with cone metric spaces in which the cone need not be normal).

Lemma 1.13. *([18]) Let P be a cone and $\{a_n\}$ be a sequence in E. If $c \in$ int P and $0 \leq a_n \to 0$ as $n \to \infty$, then there exists N such that for all $n > N$, we have $a_n \ll c$.*

Lemma 1.14. *([18]) Let $x,\ y,\ z \in E$, if $x \leq y$ and $y \ll z$, then $x \ll z$.*

Lemma 1.15. *([16]) Let P be a cone and $0 \leq u \ll c$ for each $c \in int\, P$, then $u = 0$.*

Lemma 1.16. *([8]) Let P be a cone, if $u \in P$ and $u \leq k\,u$ for some $0 \leq k < 1$, then $u = 0$.*

Lemma 1.17. *([18]) Let P be a cone and $a \leq b + c$ for each $c \in int\, P$, then $a \leq b$.*

2 Main Results

In this section we shall prove some fixed point theorems for generalized contractions in the framework of cone b-metric spaces.

Theorem 2.1. *Let (X, d) be a complete cone b-metric space (CCbMS) with the coefficient $s \geq 1$. Suppose that the mapping $T\colon X \to X$ satisfies:*

$$\begin{aligned} d(Tx, Ty) \quad &\leq \quad a\, d(x, y) + b\, \frac{d(x, Tx)\, d(y, Ty)}{1 + d(x, y)} \\ &\qquad + c\, \frac{d(x, Tx)\, d(y, Ty)}{1 + d(Tx, Ty)} \end{aligned} \tag{2.1}$$

for all $x, y \in X$, where a, b, c are nonnegative reals with $sa + sb + sc < 1$. Then T has a unique fixed point in X.

Proof. Choose $x_0 \in X$. We construct the iterative sequence $\{x_n\}$, where $x_n = Tx_{n-1}$, $n \geq 1$, that is, $x_{n+1} = Tx_n = T^{n+1}x_0$. From (2.1), we have

$$\begin{aligned} d(x_n, x_{n+1}) \quad &= \quad d(Tx_{n-1}, Tx_n) \\ &\leq \quad a\, d(x_{n-1}, x_n) + b\, \frac{d(x_{n-1}, Tx_{n-1})\, d(x_n, Tx_n)}{1 + d(x_{n-1}, x_n)} \\ &\qquad + c\, \frac{d(x_{n-1}, Tx_{n-1})\, d(x_n, Tx_n)}{1 + d(Tx_{n-1}, Tx_n)} \\ &= \quad a\, d(x_{n-1}, x_n) + b\, \frac{d(x_{n-1}, x_n)\, d(x_n, x_{n+1})}{1 + d(x_{n-1}, x_n)} \\ &\qquad + c\, \frac{d(x_{n-1}, x_n)\, d(x_n, x_{n+1})}{1 + d(x_n, x_{n+1})} \\ &\leq \quad (a + b + c)\, d(x_{n-1}, x_n) \\ &= \quad k\, d(x_{n-1}, x_n), \end{aligned} \tag{2.2}$$

where $k = a + b + c$. As $sa + sb + sc < 1$, it follows that $0 < k < \frac{1}{s}$.

By induction, we have

$$\begin{aligned} d(x_{n+1}, x_n) \quad &\leq \quad k\, d(x_{n-1}, x_n) \leq k^2\, d(x_{n-2}, x_{n-1}) \leq \dots \\ &\qquad \leq k^n\, d(x_0, x_1). \end{aligned} \tag{2.3}$$

Let $m, n \geq 1$ and $m > n$, we have

$$\begin{aligned}
d(x_n, x_m) &\leq s[d(x_n, x_{n+1}) + d(x_{n+1}, x_m)] \\
&= sd(x_n, x_{n+1}) + sd(x_{n+1}, x_m) \\
&\leq sd(x_n, x_{n+1}) + s^2[d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_m)] \\
&= sd(x_n, x_{n+1}) + s^2 d(x_{n+1}, x_{n+2}) + s^2 d(x_{n+2}, x_m) \\
&\leq sd(x_n, x_{n+1}) + s^2 d(x_{n+1}, x_{n+2}) + s^3 d(x_{n+2}, x_{n+3}) \\
&\quad + \cdots + s^{n+m-1} d(x_{n+m-1}, x_m) \\
&\leq sk^n d(x_1, x_0) + s^2 k^{n+1} d(x_1, x_0) + s^3 k^{n+2} d(x_1, x_0) \\
&\quad + \cdots + s^m k^{n+m-1} d(x_1, x_0) \\
&= sk^n [1 + sk + s^2k^2 + s^3k^3 + \cdots + (sk)^{m-1}] d(x_1, x_0) \\
&\leq \Big[\frac{sk^n}{1 - sk}\Big] d(x_1, x_0).
\end{aligned}$$

Let $0 \ll r$ be given. Notice that $\frac{sk^n}{1-sk} d(x_1, x_0) \to 0$ as $n \to \infty$ since $0 < sk < 1$. Making full use of ([14], Lemma 1.8), we find $n_0 \in \mathbb{N}$ such that

$$\Big(\frac{sk^n}{1 - sk}\Big) d(x_1, x_0) \ll r$$

for each $n > n_0$. Thus,

$$d(x_n, x_m) \leq \Big(\frac{sk^n}{1 - sk}\Big) d(x_1, x_0) \ll r$$

for all $n, m \geq 1$. So, by ([14], Lemma 1.9), $\{x_n\}$ is a Cauchy sequence in (X, d). Since (X, d) is a complete cone b-metric space, there exists $z \in X$ such that $x_n \to z$ as $n \to \infty$. Take $n_1 \in \mathbb{N}$ such that $d(x_n, z) \ll \frac{r}{s(a+1)}$ for all $n > n_1$. Hence,

$$\begin{aligned}
d(z, Tz) &\leq s[d(z, Tx_n) + d(Tx_n, Tz)] \\
&= sd(z, x_{n+1}) + sd(Tx_n, Tz) \\
&\leq s\Big[a\, d(x_n, z) + b\, \frac{d(x_n, Tx_n)\, d(z, Tz)}{1 + d(x_n, z)} + c\, \frac{d(x_n, Tx_n)\, d(z, Tz)}{1 + d(Tx_n, Tz)}\Big] \\
&\quad + sd(z, x_{n+1}) \\
&= s\Big[a\, d(x_n, z) + b\, \frac{d(x_n, x_{n+1})\, d(z, Tz)}{1 + d(x_n, z)} + c\, \frac{d(x_n, x_{n+1})\, d(z, Tz)}{1 + d(x_{n+1}, Tz)}\Big] \\
&\quad + sd(z, x_{n+1}) \\
&\leq s(a + 1)\, d(x_n, z).
\end{aligned}$$

This implies that

$$d(z, Tz) \ll r,$$

for each $n > n_1$. Then, by Lemma 1.15, we deduce that $d(z, Tz) = 0$, that is, $Tz = z$. Thus z is a fixed point of T.

Uniqueness

Let z^* be another fixed point T, that is, $Tz^* = z^*$ such that $z \neq z^*$. Then from (2.1), we have

$$\begin{aligned}
d(z, z^*) &= d(Tz, Tz^*) \\
&\leq a\, d(z, z^*) + b\, \frac{d(z, Tz)\, d(z^*, Tz^*)}{1 + d(z, z^*)} + c\, \frac{d(z, Tz)\, d(z^*, Tz^*)}{1 + d(Tz, Tz^*)} \\
&= a\, d(z, z^*) + b\, \frac{d(z, z)\, d(z^*, z^*)}{1 + d(z, z^*)} + c\, \frac{d(z, z)\, d(z^*, z^*)}{1 + d(z, z^*)} \\
&\leq a\, d(z, z^*).
\end{aligned}$$

By Lemma 1.16, $d(z, z^*) = 0$ and so $z = z^*$. This shows that z is a unique fixed point of T. This completes the proof. □

Theorem 2.2. *Let (X, d) be a complete cone b-metric space (CCbMS) with the coefficient $s \geq 1$. Suppose that the mapping $T\colon X \to X$ satisfies (for some n):*

$$\begin{aligned} d(T^n x, T^n y) &\leq a\, d(x, y) + b\, \frac{d(x, T^n x)\, d(y, T^n y)}{1 + d(x, y)} \\ &\quad + c\, \frac{d(x, T^n x)\, d(y, T^n y)}{1 + d(T^n x, T^n y)} \end{aligned} \tag{2.4}$$

for all $x, y \in X$, where a, b, c are nonnegative reals with $sa + sb + sc < 1$. Then T has a unique fixed point in X.

Proof. By Theorem 2.1 there exists $z \in X$ such that $T^n z = z$. Then

$$\begin{aligned} d(Tz, z) &= d(TT^n z, T^n z) = d(T^n Tz, T^n z) \\ &\leq a\, d(Tz, z) + b\, \frac{d(Tz, T^n Tz)\, d(z, T^n z)}{1 + d(Tz, z)} \\ &\quad + c\, \frac{d(Tz, T^n Tz)\, d(z, T^n z)}{1 + d(T^n Tz, T^n z)} \\ &\leq a\, d(Tz, z) + b\, \frac{d(Tz, TT^n z)\, d(z, T^n z)}{1 + d(Tz, z)} \\ &\quad + c\, \frac{d(Tz, TT^n z)\, d(z, T^n z)}{1 + d(TT^n z, T^n z)} \\ &= a\, d(Tz, z) + b\, \frac{d(Tz, Tz)\, d(z, z)}{1 + d(Tz, z)} \\ &\quad + c\, \frac{d(Tz, Tz)\, d(z, z)}{1 + d(Tz, z)} \\ &\leq a\, d(Tz, z). \end{aligned}$$

By Lemma 1.16, $d(Tz, z) = 0$ and so $Tz = z$. This shows that T has a unique fixed point in X. This completes the proof. □

Putting $a = k$, $b = c = 0$ in Theorem 2.1, then we have the following result.

Corollary 2.3. *Let (X, d) be a complete cone b-metric space (CCbMS) with the coefficient $s \geq 1$. Suppose that the mapping $T\colon X \to X$ satisfies:*

$$d(Tx, Ty) \leq k\, d(x, y)$$

for all $x, y \in X$, where $k \in (0, 1)$ is a constant with $sk < 1$. Then T has a unique fixed point in X.

Remark 2.4. Corollary 2.3 extends well known Banach contraction principle from complete metric space to that setting of complete cone b-metric space considered in this paper.

Other consequences of our results for the mapping involving contractions of integral type are the following.

Denote Λ the set of functions $\varphi\colon [0, \infty) \to [0, \infty)$ satisfying the following hypothesis:

$(h1)$ φ is a Lebesgue-integrable mapping on each compact subset of $[0, \infty)$;

$(h2)$ for any $\varepsilon > 0$ we have $\int_0^\varepsilon \varphi(t)\, dt > 0$.

Theorem 2.5. *Let (X,d) be a complete cone b-metric space (CCbMS) with the coefficient $s \geq 1$. Suppose that the mapping $T\colon X \to X$ satisfies:*

$$\int_0^{d(Tx,Ty)} \psi(t)\,dt \leq a \int_0^{d(x,y)} \psi(t)\,dt + b \int_0^{\frac{d(x,Tx)\,d(y,Ty)}{1+d(x,y)}} \psi(t)\,dt + c \int_0^{\frac{d(x,Tx)\,d(y,Ty)}{1+d(Tx,Ty)}} \psi(t)\,dt$$

for all $x, y \in X$, where a, b, c are nonnegative reals with $sa + sb + sc < 1$ and $\psi \in \Lambda$. Then T has a unique fixed point in X.

Putting $a = k$, $b = c = 0$ in Theorem 2.5, we have the following result.

Theorem 2.6. *Let (X,d) be a complete cone b-metric space (CCbMS) with the coefficient $s \geq 1$. Suppose that the mapping $T\colon X \to X$ satisfies:*

$$\int_0^{d(Tx,Ty)} \psi(t)\,dt \leq k \int_0^{d(x,y)} \psi(t)\,dt$$

for all $x, y \in X$, where k is a nonnegative real with $0 < sk < 1$ and $\psi \in \Lambda$. Then T has a unique fixed point in X.

Remark 2.7. Theorem 2.6 extends Theorem 2.1 of Branciari [4] from complete metric space to that setting of complete cone b-metric space considered in this paper.

Example 2.8. Let $E = C_{\mathbb{R}}[0,1]$, $P = \{f \in E : f \geq 0\} \subset E$, $X = [0,\infty)$ and $d(x,y) = |x-y|^2\, e^t$. Then (X,d) is a cone b-metric space with the coefficient $s = 2$. But it is not a cone metric space. We consider the mappings $T\colon X \to X$ defined by $T(x) = \frac{2x+3}{5}$. Hence

$$\begin{aligned} d(Tx,Ty) &= \Big|\Big(\frac{2x+3}{5}\Big) - \Big(\frac{2y+3}{5}\Big)\Big|^2 e^t \\ &= \frac{4}{25}\,|x-y|^2\, e^t \\ &\leq \frac{1}{2}\,|x-y|^2\, e^t \\ &= \frac{1}{2}\, d(x,y). \end{aligned}$$

Clearly $1 \in X$ is the unique fixed point of T.

3 Conclusion

In this paper, we establish some fixed point results for generalized contractions in the setting of cone b-metric spaces. Also, as a consequence, we obtain some results of integral type for such mapping. Our results extend and generalize several results from the existing literature.

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