Palestine Journal of Mathematics

Vol. 6(1)(2017) , 24–30

A COMMON FIXED POINT THEOREM FOR A SEQUENCE OF MAPPINGS IN COMPLETE G - METRIC SPACES

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Communicated by Hichem Ben-El-Mechaiekh

MSC 2010 Classifications: Primary 54H25; Secondary 47H10.

Keywords and phrases: Complete G - metric space, sequence of mappings, fixed point, implicit relation.

Abstract. In this paper, a common fixed point theorem for a sequence of mappings satisfying implicit relations in complete G - metric spaces, generalizing Theorem 2.1 [8], is proved.

1 Introduction

In [5], [6], Dhage introduced a new class of generalized metric spaces, named D - metric spaces. Mustafa and Sims [9], [10] proved that most of the claims concerning the fundamental topological structures on D - metric spaces are incorrect and introduced an appropriate notion of generalized metric space, named G - metric spaces.

In fact, Mustafa, Sims and other authors [1] - [4], [7], [9] - [15] studied many fixed point results for self mappings in G - metric spaces under certain conditions.

Quite recently, Meena and Nema [8] proved a common fixed point theorem for a sequence of mappigs in G - metric spaces.

Several classical fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit relation in [16], [17] and in other papers. Actually, the method is used in the study of fixed points in metric spaces, symmetric spaces, quasi - metric spaces, b - metric spaces, ultra - metric spaces, convex metric spaces, reflexive spaces, compact metric spaces, paracompact metric spaces, in two and three metric spaces, for single valued mappings, hybrid pairs of mappings and multi - valued mappings. Quite recently, the method is used in the study of fixed points for mappings satisfying contractive conditions of integral type, in fuzzy metric spaces, probabilistic metric spaces, intuitionistic metric spaces, partial metric spaces and ordered metric spaces.

The study of fixed points for mappings satisfying implicit relations in G - metric spaces is initiated in [18] - [23] and in other papers.

The study of fixed points for a sequence of mappings in G - metric spaces is initiated in [8].

2 Preliminaries

Definition 2.1 ([10]). Let X be a nonempty set and $G : X^3 \to \mathbb{R}_+$ be a function satisfying the following conditions:

(G_1) : $G(x, y, z) = 0$ if $x = y = z$,
(G_2) : $0 < G(x, x, y)$ for all $x, y \in X$, with $x \neq y$,
(G_3) : $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$, with $z \neq y$,
(G_4) : $G(x, y, z) = G(x, z, y) = ... = G(y, z, x) = . . .$ (symmetry in all three variables),
(G_5) : $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

The function G is called a G - metric on X and the pair (X, G) is called a G - metric space.

Remark 2.2 ([10]). If $G(x, y, z) = 0$ then $x = y = z$.

Definition 2.3 ([10]). Let (X, G) be a G - metric space. A sequence $\{x_n\}$ in X is said to be

a) G - convergent if for $\varepsilon > 0$, there exists $k \in \mathbb{N}$ and $x \in X$ such that for all $m, n \in \mathbb{N}$, $m, n \geq k$, $G(x, x_n, x_m) < \varepsilon$.

b) G - Cauchy if for each $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that for all $m, n, p \in \mathbb{N}$, $n, m, p \geq k$, $G(x_n, x_m, x_p) < \varepsilon$, that is $G(x_n, x_m, x_p) \to 0$ as $n, m, p \to \infty$.

A G - metric space (X, G) is said to be a G - complete metric space if every G - Cauchy sequence of (X, G) is G - convergent.

Lemma 2.4 ([10]). *Let (X, G) be a G - metric space. Then, the following properties are equivalent:*
1) $\{x_n\}$ is G - convergent to x;
2) $G(x, x_n, x_n) \to 0$ as $n \to \infty$;
3) $G(x_n, x, x) \to 0$ as $n \to \infty$;
4) $G(x_n, x_m, x) \to 0$ as $n, m \to \infty$.

Lemma 2.5 ([10]). *If (X, G) is a G - metric space, then, the following properties are equivalent:*
1) The sequence $\{x_n\}$ is G - Cauchy;
2) For $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x_n, x_n, x_m) < \varepsilon$ for $n, m > k$.

Lemma 2.6 ([10]). *$G(x, y, z)$ is jointly continuous in all three of its variables.*

Note that each G - metric generates a topology τ_G on X, whose base if a family of open G - balls $\{B_G(x, \varepsilon) : x \in X, \varepsilon > 0\}$, $B_G(x, \varepsilon) = \{y \in X : G(x, y, y) < \varepsilon$ for all $x, y \in X$ and $\varepsilon > 0\}$.

A nonempty set $A \in (X, G)$ is G - closed if $A = Cl\,(A)$.

Lemma 2.7 ([7]). *Let (X, G) be a G - metric space and A a subset of X. A is G - closed if for any G - convergent sequence $\{x_n\}$ in A with $\lim_{n\to\infty} x_n = x$, then $x \in A$.*

The following theorem is recently published in [8].

Theorem 2.8. *Let S be a closed subset of complete G – metric space (X, G) and $\{T_n\}_{n\in\mathbb{N}} : S \to S$ be a sequence of mappings such that for all $x, y, z \in S$:*

$$G(T_i x, T_j y, T_k z) \leq aG(x, y, z) + bG(x, T_i x, T_i x) + G(y, T_j y, T_j y) + dG(z, T_k z, T_k z),$$

where a, b, c, d are positive constants such that $a + b + c + d < 1$. Then $\{T_n\}_{n\in\mathbb{N}}$ has an unique common fixed point.

The purpose of this paper is to prove a general fixed point theorem for a sequence of mappings satisfying an implicit relation in G - complete metric spaces generalizing Theorem 2.8 and to obtain other general new results.

3 Implicit relations

Definition 3.1. Let $\mathfrak{F}_S$ be the set of all continuous functions $F(t_1, ..., t_5) : \mathbb{R}_+^5 \to \mathbb{R}$ satisfying the following conditions:
(F_1) : F is non - increasing in variables t_2, t_3, t_4, t_5,
(F_2) : There exists $h \in [0, 1)$ such that for all $u, v \geq 0$, $F(u, v, v, u, u) \leq 0$ implies $u \leq hv$,
(F_3) : There exists $k \in [0, 1)$ such that for all $t, t' > 0$, $F(t, t, t, t, t') \leq 0$ implies $t \leq kt'$.

In the following examples, property (F_1) is obviously.

Example 3.2. $F(t_1, ..., t_5) = t_1 - at_2 - bt_3 - ct_4 - dt_5$, where $a, d > 0, b, c \geq 0$ and $a+b+c+d < 1$.
(F_2) : Let $u, v \geq 0$ and $F(u, v, v, u, u) = u - av - bv - cu - du \leq 0$. Then $u \leq hv$, where $0 \leq h = \frac{a+b}{1-(c+d)} < 1$.
(F_3) : Let $t, t' > 0$ and $F(t, t, t, t, t') = t - at - bt - ct - dt' \leq 0$. Then $t \leq kt'$, where $0 < k = \frac{d}{1-(a+b+c)} < 1$.

Example 3.3. $F(t_1, ..., t_5) = t_1 - c\max\{t_2, t_3, t_4, t_5\}$, where $c \in [0, 1)$.
(F_2) : Let $u, v \geq 0$ and $F(u, v, v, u, u) = u - c\max\{u, v\} \leq 0$. If $u > v$ then $u(1 - c) \leq 0$, a contradiction. Hence $u \leq v$ which implies $u \leq hv$, where $0 \leq h = c < 1$.
(F_3) : Let $t, t' > 0$ and $F(t, t, t, t, t') = t - c\max\{t, t'\} \leq 0$. If $t > t'$, then $t(1 - c) \leq 0$, a contradiction. Hence, $t \leq t'$, which implies $t \leq kt'$, where $0 < k = c < 1$.

Example 3.4. $F(t_1,...,t_5) = t_1 - c\max\left\{t_2, t_3, \frac{t_4+t_5}{2}\right\}$, where $c \in [0,1)$.

The proof is similar to the proof of Example 3.3.

Example 3.5. $F(t_1,...,t_5) = t_1^2 - at_2t_3 - bt_3t_4 - ct_4t_5$, where $c > 0, a, b \geq 0$ and $a+b+c < 1$.

(F_2) : Let $u, v \geq 0$ and $F(u,v,v,u,u) = u^2 - av^2 - buv - cu^2 \leq 0$. If $u > v$, then $u^2[1-(a+b+c)] \leq 0$, a contradiction. Hence $u \leq v$, which implies $u \leq hv$, where $0 \leq h = \sqrt{a+b+c} < 1$.

(F_3) : Let $t, t' > 0$ and $F(t,t,t,t,t') = t^2 - at^2 - bt^2 - ctt' \leq 0$. Hence $t - at - bt - ct' \leq 0$ which implies $t \leq kt'$, where $0 < k = \frac{c}{1-(a+b)} < 1$.

Example 3.6. $F(t_1,...,t_5) = t_1 - at_2 - b\max\{2t_3, t_4+t_5\}$, where $a \geq 0, b > 0$ and $a+2b < 1$.

(F_2) : Let $u, v \geq 0$ and $F(u,v,v,u,u) = u - av - b\max\{2v, 2u\} \leq 0$. If $u > v$, then $u[1-(a+2b)] \leq 0$, a contradiction. Hence $u \leq v$, which implies $u \leq hv$, where $0 \leq h = a+2b < 1$.

(F_3) : Let $t, t' > 0$ and $F(t,t,t,t,t') = t - at - b\max\{2t, t+t'\} \leq 0$. If $t > t'$, then $t[1-(a+2b)] \leq 0$, a contradiction. Hence, $t \leq t'$, which implies $t \leq kt'$, where $0 < k = a+2b < 1$.

Example 3.7. $F(t_1,...,t_5) = t_1 - at_2 - b\max\{t_3+t_4, 2t_5\}$, where $a, b \geq 0$ and $0 < a+2b < 1$.

The proof is similar to the proof of Example 3.6.

Example 3.8. $F(t_1,...,t_5) = t_1^2 - at_2^2 - bt_3^2 - ct_4t_5$, where $c > 0, a, b \geq 0$ and $a+b+c < 1$.

(F_2) : Let $u, v \geq 0$ be and $F(u,v,v,u,u) = u^2 - av^2 - bv^2 - cu^2 \leq 0$, which implies $u \leq hv$, where $0 \leq h = \sqrt{\frac{a+b}{1-c}} < 1$.

(F_3) : Let $t, t' > 0$ be and $F(t,t,t,t,t') = t^2 - at^2 - bt^2 - ctt' \leq 0$, which implies $t - at - bt - ct' \leq 0$. Hence $t \leq kt'$, where $0 < k = \frac{c}{1-(a+b)} < 1$.

Example 3.9. $F(t_1,...,t_5) = t_1 - c\max\{t_2, t_3, \sqrt{t_4t_5}\}$, where $c \in (0,1)$.

(F_2) : Let $u, v \geq 0$ be and $F(u,v,v,u,u) = u - c\max\{v,u\} \leq 0$. If $u > v$, then $u(1-c) \leq 0$, a contradiction. Hence $u \leq v$, which implies $u \leq hv$, where $0 \leq h = c < 1$.

(F_3) : Let $t, t' > 0$ and $F(t,t,t,t,t') = t - c\max\{t, \sqrt{tt'}\} \leq 0$. If $t > t'$, then $t(1-c) \leq 0$, a contradiction. Hence, $t \leq t'$, which implies $t \leq kt'$, where $0 < k = c < 1$.

Example 3.10. $F(t_1,...,t_5) = t_1 - c\max\left\{t_2, t_3, \frac{2t_4+t_5}{3}, \frac{2t_5+t_4}{3}\right\}$, where $c \in (0,1)$.

(F_2) : Let $u, v \geq 0$ be and $F(u,v,v,u,u) = u - c\max\{u,v\} \leq 0$. If $u > v$ then $u(1-c) \leq 0$, a contradiction. Hence $u \leq v$ which implies $u \leq hv$, where $0 \leq h = c < 1$.

(F_3) : Let $t, t' > 0$ be and $F(t,t,t,t,t') = t - c\max\left\{t, \frac{2t+t'}{3}, \frac{2t'+t}{3}\right\} \leq 0$. If $t > t'$, then $t(1-c) \leq 0$, a contradiction. Hence $t \leq t'$ which implies $t \leq kt'$, where $0 < k = c < 1$.

Example 3.11. $F(t_1,...,t_5) = t_1 - a\max\{t_2, t_3\} - b\max\{t_4, t_5\}$, where $a, b \geq 0$ and $0 < a+b < 1$.

(F_2) : Let $u, v \geq 0$ be and $F(u,v,v,u,u) = u - av - bu \leq 0$, which implies $u \leq hv$, where $0 \leq h = \frac{a}{1-b} < 1$.

(F_3) : Let $t, t' > 0$ be and $F(t,t,t,t,t') = t - at - b\max\{t,t'\} \leq 0$. If $t > t'$, then $t[1-(a+b)] \leq 0$, a contradiction. Hence $t \leq t'$ which implies $t \leq kt'$, where $0 < k = a+b < 1$.

4 Main results

Theorem 4.1. *Let S be a closed subset of a complete G - metric space (X, G) and $\{T_n\}_{n\in\mathbb{N}} : S \to S$ be a sequence of mappings such that for all $x, y, z \in S$ and $i, j, k \in \mathbb{N}$:*

$$\begin{aligned} F(G(T_ix, T_jy, T_kz), G(x,y,z), G(x, T_ix, T_jy), \\ G(y, T_jy, T_kz), G(z, T_ix, T_kz)) \leq 0 \end{aligned} \tag{4.1}$$

where $F \in \mathfrak{F}_S$. Then, $\{T_n\}_{n\in\mathbb{N}}$ has an unique common fixed point.

Proof. Let $x_0 \in S$ be any arbitrary point. We define a sequence $\{x_n\}$ in S such that $x_{n+1} = T_{n+1}x_n$, for $n = 0, 1, 2, ...$. By (4.1) we have successively

$$F(G(T_n x_{n-1}, T_{n+1}x_n, T_{n+2}x_{n+1}), G(x_{n-1}, x_n, x_{n+1}), G(x_{n-1}, T_n x_{n-1}, T_{n+1}x_n), G(x_n, T_{n+1}x_n, T_{n+2}x_{n+1}), G(x_{n+1}, T_{n+2}x_{n+1}, T_n x_{n-1})) \leq 0$$

$$F(G(x_n, x_{n+1}, x_{n+2}), G(x_{n-1}, x_n, x_{n+1}), G(x_{n-1}, x_n, x_{n+1}), G(x_n, x_{n+1}, x_{n+2}), G(x_n, x_{n+1}, x_{n+2})) \leq 0.$$

By (F_2),

$$G(x_n, x_{n+1}, x_{n+2}) \leq hG(x_{n-1}, x_n, x_n)$$

for $n = 1, 2, ...$ which implies that

$$G(x_n, x_{n+1}, x_{n+2}) \leq h^n G(x_0, x_1, x_2).$$

Now for any positive integers $k \geq m > n \geq 1$ we obtain by (G_4) that

$$\begin{aligned} G(x_n, x_m, x_k) &\leq G(x_n, x_{n+1}, x_{n+2}) + G(x_{n+1}, x_{n+2}, x_{n+3}) + ... + G(x_{k-2}, x_{k-1}, x_k) \\ &\leq h^n \left(1 + h + ... + h^{k-n}\right) G(x_0, x_1, x_2) \\ &= \frac{h^n}{1-h} G(x_0, x_1, x_2) \to 0 \text{ as } n \to \infty. \end{aligned}$$

Hence, $\{x_n\}$ is a G - Cauchy sequence. Since (X, G) is G - complete, there exists $u \in X$ such that $\{x_n\}$ is G - convergent. Since $\{x_n\} \in S$ and S is closed, then $u \in S$.

Now we prove that u is a common fixed point of $\{T_n\}_{n\in\mathbb{N}}$.

By (4.1) we have successively

$$F(G(T_n x_{n-1}, T_j u, T_k u), G(x_{n-1}, u, u), G(x_{n-1}, T_n x_{n-1}, T_j u), G(u, T_j u, T_k u), G(u, T_k u, T_n x_{n-1})) \leq 0$$

$$F(G(x_n, T_j u, T_k u), G(x_{n-1}, u, u), G(x_{n-1}, x_n, T_j u), G(u, T_j u, T_k u), G(u, T_k u, x_n)) \leq 0.$$

Letting n tends to infinity we obtain

$$F(G(u, T_j u, T_k u), 0, G(u, u, T_j u), G(u, T_j u, T_k u), G(u, u, T_k u)) \leq 0. \tag{4.2}$$

By (G_3) we have

$$G(u, u, T_j u) \leq G(u, T_j u, T_k u)$$

and

$$G(u, u, T_k u) \leq G(u, T_j u, T_k u)$$

and by (4.2) and (F_1) we obtain

$$F(G(u, T_j u, T_k u), G(u, T_j u, T_k u), G(u, T_j u, T_k u), G(u, T_j u, T_k u), G(u, T_j u, T_k u)) \leq 0$$

which implies by (F_2) that

$$G(u, T_j u, T_k u) \leq hG(u, T_j u, T_k u)$$

and

$$G(u, T_j u, T_k u)(1 - h) \leq 0,$$

i.e. $u = T_j u = T_k u$ for any $j, k \in \mathbb{N}$. Hence u is a common fixed point of $\{T_n\}_{n\in\mathbb{N}}$.

We prove that u is the unique common fixed point of $\{T_n\}_{n\in\mathbb{N}}$. Suppose that there exists another common fixed point v of $\{T_n\}_{n\in\mathbb{N}}$. Then by (4.1) we have successively

$$\begin{gathered}F(G(T_iu,T_ju,T_kv),G(u,u,v),G(u,T_iu,T_ju),\\ G(u,T_ju,T_kv),G(v,T_iu,T_kv))\leq 0,\end{gathered}$$

$$F(G(u,u,v),G(u,u,v),0,G(u,u,v),G(u,v,v))\leq 0.$$

By (F_1) we have

$$F(G(u,u,v),G(u,u,v),G(u,u,v),G(u,u,v),G(u,v,v))\leq 0$$

which implies by (F_3) that

$$G(u,u,v)\leq kG(u,v,v).$$

Similarly,

$$G(u,v,v)\leq kG(u,u,v),$$

which implies

$$G(u,u,v)\left(1-k^2\right)\leq 0.$$

Hence $G(u,u,v)=0$, i.e. $u=v$. □

Corollary 4.2. *Let S be a closed subset of a complete G - metric space (X,G) and $\{T_n\}_{n\in\mathbb{N}}: S\to S$ be a sequence of mappings such that for all $x,y,z\in S$ and $i,j,k\in\mathbb{N}$:*

(a) $F(G(T_ix,T_jy,T_kz),G(x,y,z),G(x,T_ix,T_ix),G(y,T_jy,T_jy),G(z,T_kz,T_kz))\leq 0$

or

(b) $F(G(T_ix,T_jy,T_kz),G(x,y,z),G(x,T_jy,T_jy),G(y,T_kz,T_kz),G(z,T_ix,T_ix))\leq 0$

or

(c) $F(G(T_ix,T_jy,T_kz),G(x,y,z),G(x,x,T_ix),G(y,y,T_jy),G(z,z,T_kz))\leq 0$

or

(d) $F(G(T_ix,T_jy,T_kz),G(x,y,z),G(x,x,T_jy),G(y,y,T_kz),G(z,z,T_ix))\leq 0$,

where $F\in\mathfrak{F}_S$. Then, $\{T_n\}_{n\in\mathbb{N}}$ has an unique common fixed point.

Proof. We prove this theorem in the case when $\{T_n\}_{n\in\mathbb{N}}$ satisfy (a).

By (G_3)

$$\begin{aligned}G(x,T_ix,T_ix)&\leq G(x,T_ix,T_jy),\\ G(y,T_jy,T_jy)&\leq G(y,T_jy,T_kz),\\ G(z,T_kz,T_kz)&\leq G(z,T_ix,T_kz).\end{aligned}$$

By (a) and (F_1) we obtain

$$\begin{gathered}F(G(T_ix,T_jy,T_kz),G(x,y,z),G(x,T_ix,T_jy),\\ G(y,T_jy,T_kz),G(z,T_ix,T_kz))\leq 0,\end{gathered}$$

which is inequality (4.1). Hence by Theorem 4.1, $\{T_n\}_{n\in\mathbb{N}}$ has an unique common fixed point.

If $\{T_n\}_{n\in\mathbb{N}}$ satisfy one of the inequalities (b), (c), (d), the proofs are similarly. □

Remark 4.3. 1) By Example 3.2 and Corollary 4.2 (a) we obtain Theorem 2.8.

2) By Examples 3.3 - 3.11, Theorem 4.1 and Corollary 4.2 we obtain new particular results.

Example 4.4. Let $X=[0,1]$ and $G(x,y,z)=\begin{cases}0, & x=y=z\\ \max\{x,y,z\}, & \text{otherwise.}\end{cases}$

Then (X,G) is a complete G - metric space. Let $h\in(0,1)$ and $\{T_n\}_{n\in\mathbb{N}}:X\to X$ defined by $T_nx=h^nx$ for all $x\in X$. Then

$$G(x,y,z)=\max\{x,y,z\}$$

and

$$G\left(T_i x, T_j y, T_k z\right) = \max\{h^i x, h^j y, h^k z\} \leq hG\left(x, y, z\right),$$

which implies

$$G\left(x, y, z\right) \leq h \max\{G\left(x, y, z\right), G\left(x, T_i x, T_j y\right), G\left(y, T_i x, T_j y\right), G\left(z, T_i x, T_k z\right)\}.$$

By Theorem 4.1 and Example 3.3, $\{T_n\}_{n\in\mathbb{N}}$ has an unique common fixed point $x = 0$.

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Received: October 23, 2015.

Accepted: January 10, 2016.