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MULTIPLICATIVE SEMIDERIVATIONS ON IDEALS OF SEMIPRIME RINGS

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Abstract Let R be a semiprime ring and I is a nonzero ideal of R. A mapping $d : R \to R$ is called a multiplicative semiderivation if there exists a function $g : R \to R$ such that (i) $d(xy) = d(x)g(y) + xd(y) = d(x)y + g(x)d(y)$ and (ii) $d(g(x)) = g(d(x))$ hold for all $x, y \in R$. In the present paper, we shall prove that $[x, d(x)] = 0$, for all $x \in I$ if any one of the following holds: i) $d([x, y]) = 0$, ii) $d(xoy) = 0$, iii) $d(xy) \pm xy = 0$, iv) $d(xy) \pm yx = 0$, v) $d(x)d(y) \pm xy = 0$, vi) $d(x)d(y) \pm yx = 0$, vii) $d(xy) = \pm d(x)d(y)$, viii) $d(xy) = \pm d(y)d(x)$, for all $x, y \in I$.

1 Introduction

Let R will be an associative ring with center Z. For any $x, y \in R$ the symbol $[x, y]$ represents the Lie commutator $xy - yx$ and the Jordan product $xoy = xy + yx$. Recall that a ring R is prime if for $x, y \in R$, $xRy = 0$ implies either $x = 0$ or $y = 0$ and R is semiprime if for $x \in R$, $xRx = 0$ implies $x = 0$. It is clear that every prime ring is semiprime ring.

The study of derivations in prime rings was initiated by E. C. Posner in [11]. An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. In [3], J. Bergen has introduced the notion of semiderivation of a ring R which extends the notion of derivations of a ring R. An additive mapping $d : R \to R$ is called a semiderivation if there exists a function $g : R \to R$ such that (i) $d(xy) = d(x)g(y) + xd(y) = d(x)y + g(x)d(y)$ and (ii) $d(g(x)) = g(d(x))$ hold for all $x, y \in R$. In case g is an identity map of R, then all semiderivations associated with g are merely ordinary derivations. On the other hand, if g is a homomorphism of R such that $g \neq 1$, then $f = g - 1$ is a semiderivation which is not a derivation. In case R is prime and $d \neq 0$, it has been shown by Chang [4] that g must necessarily be a ring endomorphism.

Many authors have studied commutativity of prime and semiprime rings admitting derivations, generalized derivations and semiderivations which satisfy appropriate algebraic conditions on suitable subsets of the rings. In [5], the notion of multiplicative derivation was introduced by Daif motivated by Martindale in [10]. $d : R \to R$ is called a multiplicative derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. These maps are not additive. In [9], Goldman and Semrl gave the complete description of these maps. We have $R = C[0, 1]$, the ring of all continuous (real or complex valued) functions and define a mapping $d : R \to R$ such as

$$d(f)(x) = \left\{ \begin{array}{ll} f(x)\log|f(x)|, & f(x) \neq 0 \\ 0, & \text{otherwise} \end{array} \right\}.$$

It is clear that d is multiplicative derivation, but d is not additive. Recently, some well-known results concerning semiprime rings have been proved for multiplicative derivations.

Inspired by the definition multiplicative derivation, we can define the notion of multiplicative semiderivation such as: A mapping $d : R \to R$ is called a multiplicative semiderivation if there exists a function $g : R \to R$ such that (i) $d(xy) = d(x)g(y) + xd(y) = d(x)y + g(x)d(y)$ and (ii) $d(g(x)) = g(d(x))$ hold for all $x, y \in R$. Hence, one may observe that the concept of multiplicative semiderivations includes the concept of derivations and the left multipliers (i.e., $d(xy) = d(x)y$ for all $x, y \in R$). So, it should be interesting to extend some results concerning these notions to multiplicative semiderivations. Every derivation is a multiplicative semiderivation. But the converse is not ture in general.

In [6], Daif and Bell proved that R is semiprime ring, U is a nonzero ideal of R and d is a derivation of R such that $d([x,y]) = \pm[x,y]$, for all $x, y \in U$, then R contains a nonzero central ideal. On the other hand, in [1], Ashraf and Rehman showed that R is prime ring with a nonzero ideal U of R and d is a derivation of R such that $d(xy) \pm xy \in Z$, for all $x, y \in U$, then R is commutative. Also, Bell and Kappe proved that a derivation d of a prime ring R acts as homomorphism or anti-homomorphism on a nonzero right ideal of R, then $d = 0$ on R in [2]. Motivated by these works, we consider similar situations for multiplicative semiderivation on nonzero ideal of semiprime ring R.

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2 Results

Throughout the paper, R will be semiprime ring and I be a nonzero ideal of R and d a multiplicative semiderivation of R with associated a nonzero epimorphism g of R.

Also, we will make some extensive use of the basic commutator identities:

$$
\begin{aligned}
[x, yz] &= y[x, z] + [x, y]z \\
[xy, z] &= [x, z]y + x[y, z] \\
xo(yz) &= (xoy)z - y[x, z] = y(xoz) + [x, y]z \\
(xy)oz &= x(yoz) - [x, z]y = (xoz)y + x[y, z].
\end{aligned}
$$

Lemma 2.1. *[12, Lemma 2.1]Let R be a semiprime ring, I be a nonzero ideal of R and $a \in R$ such that $axa = 0$, for all $x \in I$, then $a = 0$.*

Theorem 2.2. *Let R be a semiprime ring, I be a nonzero ideal of R and d be a multiplicative semiderivation associated with a nonzero epimorphism g of R. If $d([x, y]) = 0$, for all $x, y \in I$, then $[x, d(x)] = 0$, for all $x \in I$.*

Proof. By the hypothesis, we have

$$d([x, y]) = 0, \text{ for all } x, y \in I. \tag{2.1}$$

Replacing y by yx in (2.1) and using this, we get

$$[x, y]d(x) = 0, \text{ for all } x, y \in I. \tag{2.2}$$

Writting $ry, r \in R$ for y in (2.2) and using (2.2), we obtain that

$$[x, r]yd(x) = 0, \text{ for all } x, y \in I, r \in R. \tag{2.3}$$

Taking yx by y in (2.3), we have

$$[x, r]yxd(x) = 0, \text{ for all } x, y \in I, r \in R.$$

Right multipliying (2.3) with x, we get

$$[x, r]yd(x)x = 0, \text{ for all } x, y \in I, r \in R.$$

Subtracting the last two equations, we arrive at

$$[x, r]y[x, d(x)] = 0, \text{ for all } x, y \in I, r \in R.$$

Replacing $d(x)$ by r in this equation, we have

$$[x, d(x)]y[x, d(x)] = 0, \text{ for all } x, y \in I.$$

By Lemma 1, we get $[x, d(x)] = 0$, for all $x \in I$. The proof is completed. □

Theorem 2.3. *Let R be a semiprime ring, I be a nonzero ideal of R and d be a multiplicative semiderivation associated with a nonzero epimorphism g of R. If $d(xoy) = 0$, for all $x, y \in I$, then $[x, d(x)] = 0$, for all $x \in I$.*

Proof. By our hypothesis, we get

$$d(xoy) = 0, \text{ for all } x, y \in I. \tag{2.4}$$

Writting yx for y in (2.4) and using (2.4), we obtain that

$$(xoy)d(x) = 0, \text{ for all } x, y \in I. \tag{2.5}$$

Substituting $ry, r \in R$ for y in this equation and using this, we arrive at

$$[x, r]yd(x) = 0, \text{ for all } x, y \in I, r \in R.$$

Using the same arguments after (2.3) in the proof of Theorem 1, we get the required result. □

Theorem 2.4. *Let R be a semiprime ring, I be a nonzero ideal of R and d be a multiplicative semiderivation associated with a nonzero epimorphism g of R. If $d(xy)\pm xy = 0$, for all $x, y \in I$, then $[x, d(x)] = 0$, for all $x \in I$.*

Proof. If $d = 0$, then we get $xy = 0$, for all $x, y \in I$, and so $x \in I \cap ann(I) = (0)$, for all $x \in I$. Since I is a nonzero ideal of R, we assume that $d \neq 0$.

By our hypothesis, we get

$$d(xy) \pm xy = 0, \text{ for all } x, y \in I. \tag{2.6}$$

Replacing y by yz in (2.6), we get

$$(d(xy) \pm xy)z + g(xy)d(z) = 0, \tag{2.7}$$

and so

$$g(xy)d(z) = 0.$$

That is

$$g(x)g(y)d(z) = 0, \text{ for all } x, y \in I. \tag{2.8}$$

Taking $d(r)y, r \in R$ instead of y in this equation and using $dg = gd$, it reduces to

$$g(x)d(g(r))g(y)d(z) = 0.$$

Since g is an epimorphism of R, we have

$$g(x)d(r)g(y)d(z) = 0, \text{ for all } x, y, z \in I.$$

This implies that

$$g(x)d(z)g(y)d(z) = 0, \text{ for all } x, y, z \in I.$$

Writting $ty, t \in R$ for y in this equation and using g is surjective, we obtain that

$$g(x)d(z)Rg(y)d(z) = (0), \text{ for all } x, y, z \in I.$$

In particulary, we can write

$$g(x)d(z)Rg(x)d(z) = (0), \text{ for all } x, z \in I, r \in R$$

and so

$$g(x)d(z) = 0, \text{ for all } x, z \in I.$$

Using this in the following equation, we have $d(xz) = d(x)z + g(x)d(z) = d(x)z$, and so

$$d(xz) = d(x)z, \text{ for all } x, z \in I.$$

Returning our hypothesis and using this, we find that

$$(d(x) \pm x)y = 0, \text{ for all } x, y \in I. \tag{2.9}$$

and so

$$y(d(x) \pm x)Ry(d(x) \pm x) = (0), \text{ for all } x, y \in I.$$

Since R is semiprime ring, we get

$$y(d(x) \pm x) = 0, \text{ for all } x, y \in I. \tag{2.10}$$

Comparing (2.9) and (2.10), we arrive at

$$[(d(x) \pm x), y] = 0,$$

and so

$$[(d(x) \pm x), x] = 0.$$

It reduces to

$$[d(x), x] = 0, \text{ for all } x \in I.$$

This completes the proof. □

Theorem 2.5. *Let R be a semiprime ring, I be a nonzero ideal of R and d be a multiplicative semiderivation associated with a nonzero epimorphism g of R. If $d(xy) \pm yx = 0$, for all $x, y \in I$, then $[x, d(x)] = 0$, for all $x \in I$.*

Proof. If $d = 0$, then we get $yx = 0$, for all $x, y \in I$ and so $x \in I \cap ann(I) = (0)$, for all $x \in I$. Since I is a nonzero ideal of R, we assume that $d \neq 0$.

Assume that

$$d(xy) + yx = 0, \text{ for all } x, y \in I. \tag{2.11}$$

Taking yz instead of y in this equation, we have

$$d(xy)z + g(xy)d(z) + yzx = 0, \text{ for all } x, y, z \in I.$$

For all $x, y, z \in I$, we can write this equation

$$d(xy)z + g(xy)d(z) + yzx + yxz - yxz = 0, \text{ for all } x, y, z \in I$$

and so

$$(d(xy) + yx)z + g(xy)d(z) + y[z, x] = 0, \text{ for all } x, y, z \in I.$$

Using the hypothesis, we arrive at

$$g(xy)d(z) + y[x, z] = 0, \text{ for all } x, y, z \in I. \tag{2.12}$$

Replacing z by x in (2.12) and using this, we get

$$g(xy)d(x) = 0, \text{ for all } x, y \in I.$$

Writing $d(t)ry, t, r \in R$ for y in this equation and using g is surjective, we obtain that

$$g(x)g(d(t))Rg(y)d(x) = (0).$$

Using $dg = gd$, we have

$$g(x)d(g(t))Rg(y)d(x) = (0).$$

We can write this equation using g is surjective such as

$$g(x)d(r)Rg(y)d(x) = (0),$$

and so

$$g(x)d(x)Rg(x)d(x) = (0).$$

Hence we find that

$$g(x)d(x) = 0, \text{ for all } x \in I.$$

Now, let return (2.12). Writing z by y in this equation and using $g(z)d(z) = 0$, we arrive at

$$z[x, z] = 0, \text{ for all } x, z \in I. \tag{2.13}$$

Replacing x by yx in this equation and using this, we get

$$zy[x, z] = 0, \text{ for all } x, y, z \in I,$$

and so

$$zyw[x, z] = 0, \text{ for all } x, y, z, w \in I. \tag{2.14}$$

Similarly, (2.13) gives that

$$yzw[x, z] = 0, \text{ for all } x, y, z, w \in I. \tag{2.15}$$

Subtracting (2.15) from (2.14), we arrive at

$$[y, z]w[x, z] = 0, \text{ for all } x, y, z, w \in I,$$

and so

$$[x, z]I[x, z] = (0), \text{ for all } x, z \in I.$$

By Lemma 1, we get

$$[x, z] = (0), \text{ for all } x, z \in I$$

Replacing z by $d(x)z$ in this equation and using this, we get

$$[x, d(x)]z = 0, \text{ for all } x, z \in I$$

and so

$$[x, d(x)]I[x, d(x)] = (0), \text{ for all } x \in I.$$

Again using Lemma 1, we get the required result.

Now, we get

$$d(xy) - yx = 0, \text{ for all } x, y \in I.$$

For all $x, y, z \in I$, we can write

$$d(x(yz)) - (yz)x = 0, \text{ for all } x, y, z \in I$$

and

$$d((xy)z) - z(xy) = 0, \text{ for all } x, y, z \in I.$$

Subtracting these two equations, we find that

$$zxy - yzx = 0, \text{ for all } x, y, z \in I.$$

That is $[y, zx] = 0$, for all $x, y, z \in I$. Writing z by y in this equation and using this equation, we have

$$z[x, z] = 0, \text{ for all } x, z \in I.$$

This is the same as (2.13) above. Using the same arguments after this equation, we get the required result. □

Theorem 2.6. *Let R be a semiprime ring, I be a nonzero ideal of R and d be a multiplicative semiderivation associated with a nonzero epimorphism g of R. If $d(x)d(y) \pm xy = 0$, for all $x, y \in I$, then $[x, d(x)] = 0$, for all $x \in I$.*

Proof. If $d = 0$, then we get $xy = 0$, for all $x, y \in I$. We had done in the proof of Theorem 3. So, we have $d \neq 0$.

By our hypothesis, we get

$$d(x)d(y) \pm xy = 0, \text{ for all } x, y \in I. \tag{2.16}$$

Replacing y by yz in this equation and using the hypothesis, we get

$$d(x)d(y)z + d(x)g(y)d(z) \pm xyz = 0,$$

and so

$$d(x)g(y)d(z) = 0, \text{ for all } x, y, z \in I.$$

Taking $ry, r \in R$ instead of y in this equation and using g is an epimorphism, we have

$$d(x)Rg(y)d(z) = (0),$$

and so

$$g(y)d(x)Rg(y)d(x) = (0), \text{ for all } x, y \in I.$$

By the semiprimeness of R, we obtain that

$$g(y)d(x) = 0, \text{ for all } x, y \in I. \tag{2.17}$$

Hence we get $d(xy) = d(x)y + g(x)d(y) = d(x)y$,and so

$$d(xy) = d(x)y, \text{ for all } x, y \in I. \tag{2.18}$$

On the other hand, right multiplying our hypothesis with y, we get

$$d(x)d(y)y \pm xy^2 = 0, \text{ for all } x, y \in I. \tag{2.19}$$

Now, writing xy in place of x in the hypothesis and using (2.18), we find that

$$d(x)yd(y) \pm xy^2 = 0, \text{ for all } x, y \in I. \tag{2.20}$$

Subtracting (2.19) from (2.20), we obtain that

$$d(x)[d(y), y] = 0, \text{ for all } x, y \in I.$$

Replacing x by xz in this equation and using this, we get

$$d(x)z[d(y), y] = 0, \text{ for all } x, y, z \in I.$$

It follows that

$$[d(y), y]z[d(y), y] = 0, \text{ for all } x, y, z \in I.$$

By Lemma 1, we get $[d(y), y] = 0$, for all $y \in I$. □

Theorem 2.7. *Let R be a semiprime ring, I be a nonzero ideal of R and d be a multiplicative semiderivation associated with a nonzero epimorphism g of R. If $d(x)d(y) \pm yx = 0$, for all $x, y \in I$, then $[x, d(x)] = 0$, for all $x \in I$.*

Proof. Using the same arguments begining of the proof of Theorem 3, we must have $d \neq 0$.

By our hypothesis, we get

$$d(x)d(y) \pm yx = 0, \text{ for all } x, y \in I. \tag{2.21}$$

Replacing y by yx in this equation and using this, we get

$$d(x)g(y)d(x) = 0, \text{ for all } x, y \in I.$$

Writing $ry, r \in R$ instead of y in this equation and using g is an epimorphism, we have

$$d(x)Rg(y)d(x) = (0).$$

In particulary, we get

$$g(y)d(x)Rg(y)d(x) = (0),$$

and so

$$g(y)d(x) = 0, \text{ for all } x, y \in I.$$

Hence we have $d(xy) = d(x)y + g(x)d(y) = d(x)y$,and so $d(xy) = d(x)y$, for all $x, y \in I$.

Now, right multiplying our hypothesis with y, we get

$$d(x)d(y)y \pm yxy = 0, \text{ for all } x, y \in I.$$

Taking xy in place of x in the hypothesis and using $d(xy) = d(x)y$, we have

$$d(x)yd(y) \pm yxy = 0, \text{ for all } x, y \in I.$$

Comparing the last two equations, we obtain that

$$d(x)[d(y), y] = 0, \text{ for all } x, y \in I.$$

Applying the same arguments as used the end of the proof of Theorem 5, we arrive at $[x, d(x)] = 0$, for all $x \in I$. □

Theorem 2.8. *Let R be a semiprime ring, I be a nonzero ideal of R and d be a multiplicative semiderivation associated with a nonzero epimorphism g of R. If $d(xy) = \pm d(x)d(y)$, for all $x, y \in I$, then $[x, d(x)] = 0$, for all $x \in I$.*

Proof. Assume that

$$d(xy) = d(x)y + g(x)d(y) = \pm d(x)d(y), \text{ for all } x, y \in I. \tag{2.22}$$

Replacing y by yz in (2.22) and using the hypothesis, we have

$$d(x)yz + g(x)d(yz) = \pm d(xy)d(z).$$

Since d is multiplicative semiderivation of R, we get

$$d(x)yz + g(x)d(yz) = \pm(d(x)yd(z) + g(x)d(y)d(z))$$

and so

$$d(x)yz + g(x)d(yz) = \pm d(x)yd(z) + g(x)d(yz).$$

That is

$$d(x)y(z \mp d(z)) = 0, \text{ for all } x, y, z \in I. \tag{2.23}$$

It follows that

$$d(x)yd(w)(z \mp d(z)) = 0, \text{ for all } x, y, z, w \in I. \tag{2.24}$$

Returning (2.22), we can write

$$d(x)y + g(x)d(y) = \pm d(x)d(y).$$

That is

$$d(x)(y \mp d(y)) = -g(x)d(y), \text{ for all } x, y, z \in I. \tag{2.25}$$

We can write from (2.24) using (2.25)

$$d(x)yg(w)d(z) = 0$$

and so

$$g(w)d(x)yRg(w)d(x)y = (0), \text{ for all } x, y, w \in I.$$

Since R is semiprime, we conclude that $g(w)d(x)I = 0$, for all $x, w \in I$.

Now, right multiplying (2.25) with y and using $g(x)d(y)y = 0$, we get

$$d(x)(y \mp d(y))y = 0, \text{ for all } x, y \in I.$$

By (2.23), we can write

$$d(x)y(y \mp d(y)) = 0, \text{ for all } x, y, z \in I.$$

Subtracting the last two equations, we get

$$d(x)[d(y), y] = 0, \text{ for all } x, y \in I. \tag{2.26}$$

Replacing x by xz in (2.26) and using this, we obtain

$$d(x)z[d(y), y] = 0, \text{ for all } x, y \in I \tag{2.27}$$

which yields that

$$xd(x)z[d(y), y] = 0, \text{ for all } x, y \in I.$$

Taking xz instead of z in (2.27), we get

$$d(x)xz[d(y), y] = 0, \text{ for all } x, y \in I.$$

Subtracting the last two equations, we find that

$$[d(x), x]z[d(y), y] = (0), \text{ for all } x, y, z \in I.$$

In particular,

$$[d(x), x]z[d(x), x] = (0), \text{ for all } x, z \in I.$$

By Lemma 1, we have $[d(x), x] = (0)$, for all $x \in I$. □

Theorem 2.9. *Let R be a semiprime ring, I be a nonzero ideal of R and d be a multiplicative semiderivation associated with a nonzero epimorphism g of R. If $d(xy) = \pm d(x)d(y)$, for all $x, y \in I$, then $[x, d(x)] = 0$, for all $x \in I$.*

Proof. We have

$$d(xy) = d(x)y + g(x)d(y) = \pm d(y)d(x), \text{ for all } x, y \in I. \tag{2.28}$$

Taking xy in place of y in this equation, we get

$$d(x)xy + g(x)d(xy) = \pm d(xy)d(x).$$

Since d is a multiplicative semiderivation of R, we have

$$d(x)xy + g(x)d(xy) = \pm d(x)yd(x) \pm g(x)d(y)d(x).$$

Using the hypothesis, we arrive at

$$d(x)xy = d(x)yd(x), \text{ for all } x, y \in I. \tag{2.29}$$

Replacing y by yx in (2.29) and using this, we obtain

$$d(x)y[d(x), x] = 0, \text{ for all } x, y \in I. \tag{2.30}$$

Left multiplying this equation by x, we get

$$xd(x)y[d(x), x] = 0, \text{ for all } x, y \in I.$$

Writing y by xy in (2.30), we have

$$d(x)xy[d(x), x] = 0, \text{ for all } x, y \in I.$$

Subtracting the last two equations, we find that

$$[d(x), x]y[d(x), x] = 0, \text{ for all } x, y \in I.$$

By Lemma 1, we have $[d(x), x] = (0)$, for all $x \in I$. This completes the proof. □

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