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# Hemi-slant Riemannian maps from almost contact metric manifolds

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**Abstract** In this paper, we introduce hemi-slant Riemannian maps from almost contact metric manifolds into Riemannian manifolds as a generalization of anti-invariant Riemannian maps, semi-invariant Riemannian maps and slant Riemannian maps from almost contact metric manifolds into Riemannian manifolds. Further, we obtain necessary and sufficient conditions for integrability of distributions which are involved in the definition of hemi-slant Riemannian maps and study the geometry of leaves. After it, we investigate the necessary and sufficient condition for hemi-slant Riemannian maps to be totally geodesic and harmonic. Finally, we obtain a characterization theorem for the proper hemi-slant Riemannian maps from Sasakian manifolds to Riemannian manifolds with totally umbilical fibers and also we provide some examples of such maps.

## 1 Introduction

To compare the geometric structure defined on two Riemannian manifolds, we need suitable types of maps between them. Some maps like an isometric immersion and Riemannian submersion are used widely to compare the geometric structures between two Riemannian manifolds in differential geometry. Firstly B. O' Neill [3] and A. Gray [2] studied the map $F$ to be a Riemannian submersion. Riemannian submersions between Riemannian manifolds equipped with differentiable structure were studied by Watson in [4]. Watson also showed that the base manifold and each fiber have the same kind of structure as the total space, in most cases [4] and [5]. As a generalization of an isometric immersion, Riemannian submersion and an isometry Riemannian maps are introduced by A. E. Fischer [1] and harmonic maps between Riemannian manifolds in [14] and [19]. After that, there are lots of papers on this topic. Moreover, B. Sahin defined slant Riemannian maps [6], Hemi-slant Riemannian maps [10], conformal Riemannian maps [7], Conformal slant Riemannian maps [18], semi-invariant Riemannian maps [8], hemi-slant submersion [12], studied the geometry of the total manifolds and the base manifolds by showing the existence of such maps. Given a $C^\infty$ -map $F$ from a Riemannian manifold $(M, g_m)$ to a Riemannian manifold $(N, g_n)$, according to the conditions on the map $F$, we call $F$ a harmonic map [1], a totally geodesic map [1], an isometric immersion [4], a semi-slant Riemannian submersion [17], a Riemannian map [11], etc. Recently, K.S. Park studied many important results on semi-slant Riemannian maps [15], almost h-semi-slant Riemannian maps [16]. R. Prasad and S. Pandey obtained some interesting results on slant Riemannian map from an almost contact manifold [20]. B. Sahin [9] studied and investigated the application of Riemannian submersions and Riemannian maps on Hermitian manifolds . Motivated by above study, we are interested to study hemi-slant Riemannian maps from almost contact metric manifolds to Riemannian manifolds.

## 2 Preliminaries

Let $(M, g_m)$ and $(N, g_n)$ be Riemannian manifolds, where $M, N$ are $C^\infty$ manifolds and $g_m, g_n$ are Riemannian metrics on $M, N$ respectively. Let $F : (M, g_m) \to (N, g_n)$ be a $C^\infty$-map. We call the map $F$ a $C^\infty$-submersion if $F$ is surjective and the differential $(F_*)_x$ has a maximal

rank for any $x \in M$. The map $F$ is said to be a Riemannian submersion if $F$ is a $C^\infty$ submersion and $(F_*)_x : (ker(F_*)_x)^\perp, (g_m)_x) \to (T_{F(x)}N, (g_n)_{F(x)})$ is a linear isometry for each $x \in M$, where $(ker(F_*)_x)^\perp$ is the orthogonal complement of the space $ker(F_*)_x$ in the tangent space $T_xM$ of $M$ at $x$. We say $F$ be a Riemannian map, if $(F_*)_x : (ker(F_*)_x)^\perp, (g_m)_x \to ((rangeF_*)_{F(x)}, (g_n)_F(x))$ is a linear isometry for each $x \in M$, where $(rangeF_*)_{F(x)} = (F_*)_x)(ker(F_*)_x)^\perp$ for $x \in M$. Let $F : (M, g_m) \to (N, g_n)$ be a $C^\infty$-map, we call the map $F$ a slant Riemannian map, if $F$ is a Riemannian map and the angle $\theta = \theta(X)$ between $JX$ and the space $ker(F_*)_x$ is constant for nonzero $X \in ker(F_*)_x$ and $x \in M$. We call the angle $\theta$ a slant angle.
The map $F$ is said to be anti-invariant Riemannian map if $kerF_*$ is anti-invariant with respect to $J$ such that $J(kerF_*) \subset kerF_*^\perp$.
The map $F$ is said to be a semi-invariant Riemannian map if $F$ is a Riemannian map and there is a distribution $D_1 \subset kerF_*$ such that

$$kerF_* = D_1 \oplus D_2,\ J(D_1) = D_1,\ J(D_2) \subset (kerF_*)^\perp,$$

where $D_2$ is the orthogonal complement of $D_1$ in $kerF_*$.
The map $F$ is said to be a semi-slant Riemannian map if $F$ is a Riemannian map and there is a distribution $D_1 \subset kerF_*$ such that

$$kerF_* = D_1 \oplus D_2,\ J(D_1) = D_1,$$

and the angle $\theta = \theta(X)$ between $JX$ and the space $(D_2)_x$ is constant for nonzero $X \in (D_2)_x$ and $x \in M$.
We call the angle $\theta$ a slant angle. where $D_2$ is the orthogonal complement of $D_1$ in $kerF_*$.
Let $(M, g_m)$ and $(N, g_n)$ be Riemannian manifolds, where $M, N$ are $C^\infty$ manifolds and $g_m, g_n$ are Riemannian metrics on $M, N$ respectively. Now, we recall a useful results which are related to the second fundamental form and the tension field of Riemannian map. Let $(M, g_m)$ and $(N, g_n)$ are Riemannian manifolds and suppose that $F : (M, g_m) \to (N, g_n)$ is a smooth map between them. Then the differential $F_*$ of $F$ can be viewed a section of bundle $Hom(TM, F^{-1}TN) \to M$, where $F^{-1}TN$ is the pullback bundle which has fibers $(F^{-1}TN)_x = T_{F(x)}N, x \in M$. $Hom(TM, F^{-1}TN)$ has a connection $\nabla$ induced from the Levi-Civita connection $\nabla^M$ and the pullback connection. The second fundamental form of $F$ is given by

$$(\nabla F_*)(X, Y) = \nabla^F_X F_*(Y) - F_*(\nabla^M_X Y), \tag{2.1}$$

for $X, Y \in \Gamma(TM)$.
Denote the range of $F_*$ by $(rangeF_*)$ as a subset of the pullback bundle $F^{-1}TN$ with its orthogonal complement $(rangeF_*)^\perp$, we have the following decomposition

$$F^{-1}TN = rangeF_* \oplus (rangeF_*)^\perp.$$

Moreover, we have

$$TM = kerF_* \oplus (kerF_*)^\perp.$$

It is known that the second fundamental form is symmetric and note that the second fundamental form $(\nabla F_*)(X, Y), \forall X, Y \in \Gamma(kerF_*)^\perp$, of a Riemannian map has no component in $rangeF_*$. More precisely we have the following.

**Lemma 2.1.** *Let $F$ be a Riemannian map from a Riemannian manifold $(M, g_m)$ to a Riemannian manifold $(N, g_n)$. Then*

$$g_n((\nabla F_*)(X, Y), F_*(Z)) = 0, \forall X, Y, Z \in \Gamma((kerF_*)^\perp). \tag{2.2}$$

As a result of Lemma (2.1), we obtain

$$(\nabla F_*)(X, Y) \in \Gamma((rangeF_*)^\perp), \forall X, Y, \in \Gamma((kerF_*)^\perp). \tag{2.3}$$

For the tension field of a Riemannian map between Riemannian manifolds, we get the following

**Lemma 2.2.** *Let $F : (M, g_m) \longrightarrow (N, g_n)$ be a Riemannian map between Riemannian manifolds. Then the tension field $\tau$ of $F$ is*

$$\tau = -m_1 F_*(H) + m_2 H_2, \tag{2.4}$$

*where $m_1 = dim(kerF_*), m_2 = rankF$, $H$ and $H_2$ are the mean curvature vector fields of the distributions $kerF_*$ and $rangeF_*$, respectively.*

Let $F$ be a Riemannian map from a Riemannian manifold $(M, g_m)$ to a Riemannian manifold $(N, g_n)$. Then we define $\mathcal{T}$ and $\mathcal{A}$ as

$$\mathcal{A}_E F = \mathcal{H}\nabla_{\mathcal{H}E}\mathcal{V}F + \mathcal{V}\nabla_{\mathcal{H}E}\mathcal{H}F \tag{2.5}$$

$$\mathcal{T}_E F = \mathcal{H}\nabla_{\mathcal{V}E}\mathcal{V}F + \mathcal{V}\nabla_{\mathcal{V}E}\mathcal{H}F \tag{2.6}$$

for vector fields $E$,$F$ on $M$, where $\nabla$ is the Levi-Civita connection of $g_m$. In fact one can see that these tensor fields are O'Neill's tensor fields which are defined for Riemannian submersions. For any $E \in \Gamma(TM)$, $\mathcal{T}_E$ and $\mathcal{A}_E$ are skew-symmetric on $(\Gamma(TM), g_m)$ reversing the horizontal and vertical distributions. It is also easy to see that $\mathcal{T}$ is vertical, $\mathcal{T}_E = \mathcal{T}_{\mathcal{V}E}$ and $\mathcal{A}$ is horizontal, $\mathcal{A} = \mathcal{A}_{\mathcal{H}E}$. We note that the tensor field $\mathcal{T}$ satisfies

$$\mathcal{T}_U W = \mathcal{T}_W U \tag{2.7}$$

$$\mathcal{A}_X Y = -\mathcal{A}_X Y = \frac{1}{2}\mathcal{V}[X, Y]$$

for $U, W \in \Gamma(kerF_*)$ and $X, Y \in \Gamma(kerF_*)^{\perp}$.
On the other hand, from (2.5) and (2.6), we obtain

$$\nabla_V W = \mathcal{T}_V W + \hat{\nabla}_V W \tag{2.8}$$

$$\nabla_V X = \mathcal{H}\nabla_V X + \mathcal{T}_V X \tag{2.9}$$

$$\nabla_X V = \mathcal{A}_X V + \mathcal{V}\nabla_X V \tag{2.10}$$

$$\nabla_X Y = \mathcal{H}\nabla_X Y + \mathcal{A}_X Y \tag{2.11}$$

for $X, Y \in \Gamma((kerF_*)^{\perp})$ and $V, W \in \Gamma(kerF_*)$, where $\hat{\nabla}_V W = \mathcal{V}\nabla_V W$.
Recall that $F$ is said to be harmonic if we have the tension field

$$\tau(F) = trace(\nabla F_*) = 0, \tag{2.12}$$

and we call the tension field a totally geodesic map if

$$(\nabla F_*)(X, Y) = 0, \tag{2.13}$$

for $X, Y \in \Gamma(TM)$.

**Lemma 2.3.** *Let $F$ be a Riemannian map from a Riemannian manifold $(M, g_m)$ to a Riemannian manifold $(N, g_n)$. Then the map $F$ satisfies a generalized eikonal equation*

$$2e(F) = ||F_*||^2 = rankF.$$

As we know, $||F_*||^2$ is a continuous function on $M$ and rank $F$ is integer valued so that rank $F$ is locally constant. Hence, if $M$ is connected, then rank $F$ is a constant function.

### 2.1 Almost contact metric manifolds

An odd dimensional differentiable manifold $M$ is said to have an almost contact structure $(M, J, \xi, \eta)$ if it carries a tensor field $J$ of type $(1, 1)$, a vector field $\xi$ and $1-$form $\eta$ on $M$ respectively such that

$$J^2 = -I + \eta \otimes \xi, \ J\xi = 0, \ \eta \circ J = 0, \ \eta(\xi) = 1, \tag{2.14}$$

where $I$ denotes identity tensor. An almost contact structure is said to be normal if $N + d\eta \otimes \xi = 0$, where $N$ is the Nijenhuis tensor of $J$. Suppose that a Riemannian metric tensor $g_m$ is given in $M$ and satisfies the condition

$$g_m(JX, JY) = g_m(X, Y) - \eta(X)\eta(Y), \ g_m(X, \xi) = \eta(X). \tag{2.15}$$

Then $(J, \xi, \eta, g_m)$ structure is called an almost contact metric structure. A manifold $M$ with an almost contact metric structure $(J, \xi, \eta, g_m)$ is called an almost contact metric manifold and is denoted by $(M, J, \xi, \eta, g_m)$.
Define a tensor field $\Phi$ of type $(0, 2)$ by $\Phi(X, Y) = g(X, JY)$. If $d\eta = \Phi$ then an almost contact metric structure is said to be normal contact metric structure. A normal contact metric structure is called a Sasakian structure, which satisfies

$$(\nabla_X J)Y = g_m(X, Y)\xi - \eta(Y)X, \tag{2.16}$$

$$\nabla_X \xi = -JX, \tag{2.17}$$

where $\nabla$ denotes the Levi-Civita connection of $g_m$. For a Sasakian manifold $(M, J, \xi, \eta, g_m)$, it is known that

$$R(\xi, X)Y = g_m(X, Y)\xi - \eta(Y)X, \tag{2.18}$$

for all $X, Y \in \Gamma(M)$

## 3 Hemi-slant Riemannian maps from an almost contact metric manifold into a Riemannian manifold

In this section, we introduce and study hemi-slant Riemannian maps from an almost contact metric manifold into a Riemannian manifold. We give definition and we obtain necessary and sufficient conditions for integrability of distributions and study the geometry of leaves for hemi-slant Riemannian map.

**Definition 3.1.** Let $(M, J, \xi, \eta, g_m)$ be an almost contact metric manifold and $(N, g_n)$ be a Riemannian manifold. A Riemannian map $F : (M, J, \xi, \eta, g_m) \to (N, g_n)$ is said to be a hemi-slant Riemannian map if there is a distributions $D_1$ and $D_2$ of $kerF_*$ of $F$ such that

$$kerF_* = D_1 \oplus D_2 \oplus < \xi >, \tag{3.1}$$

where orthogonal complementry distributions $D_1$ is slant and $D_2$ is anti-invariant. The angle $\theta = \theta(X)$ between $JX$ and the space $(D_1)_x$ is constant for nonzero $X \in (D_1)_x$ and $x \in M$, where $< \xi >$ is one dimensional vector space orthogonal to distributions $D_1$ and $D_2$ in $kerF_*$. We call the angle $\theta$ is a hemi-slant angle.

We can easily observe the notion of hemi-slant Riemannian map is natural generalization of both the notions of anti-invariant Riemannian maps, semi-invariant Riemannian maps and slant Riemannian maps. More precisely, if we denote the dimension of $D_1$ and $D_2$ by $p$ and $q$ , respectively, then we have the following:
(a) If $p = 0$, then M is an anti-invariant Riemannian map.
(b) If $q = 0$ and $\theta = 0$, then M is an invariant Riemannian map.
(c) If $q = 0$ and $\theta \neq 0, \frac{\pi}{2}$ , then M is a proper slant submersion with slant angle $\theta$.
(d) If $\theta = \frac{\pi}{2}$ , then M is an anti-invariant Riemannian map.
We say that the hemi-slant Riemannian map $F : (M, J, \xi, \eta, g_m) \to (N, g_n)$ is proper if $D_2 \neq 0$

and $\theta \neq 0, \frac{\pi}{2}$.
For any $X \in \Gamma(kerF_*)$, we have

$$X = PX + QX + \eta(X)\xi, \tag{3.2}$$

where $PX \in D_1$ and $QX \in D_2$ and we put

$$JX = \phi X + \omega X, \tag{3.3}$$

where $\phi X \in \Gamma(kerF_*)$ and $\omega X \in \Gamma(kerF_*)^\perp$.
Also for any $Z \in \Gamma(kerF_*)^\perp$, we get

$$JZ = BZ + CZ, \tag{3.4}$$

where $BZ \in \Gamma(kerF_*)$ and $CZ \in \Gamma(kerF_*)^\perp$.
Then the horizontal distribution $\mathcal{H} = (kerF_*)^\perp$ is decomposed as

$$(kerF_*)^\perp = \omega D_1 \oplus JD_2 \oplus \mu, \tag{3.5}$$

where $\mu$ is orthogonal complementry distribution $\omega D_1 \oplus JD_2$ and it is invariant distribution of $(kerF_*)^\perp$ with respect to $J$. As we have seen from above argument, anti- invariant Riemannian map, semi-invariant Riemannian map and slant Riemannian map are all examples of hemi-slant Riemannian map. Now, using equations (3.3), (3.4) and (3.5), we have following lemma:

**Lemma 3.2.** *Let $F$ be a hemi-slant Riemannian map from an almost contact metric manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$. Then, we get*
*(i) $\phi D_1 = D_1$*
*(ii) $\phi D_2 = \{0\}$*
*(iii) $B\omega D_1 \subseteq D_1$*
*(iv) $BJD_2 = D_2$.*

**Lemma 3.3.** *Let $F$ be a hemi-slant Riemannian map from an almost contact metric manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$. Then, we have*
*(i) $\phi^2 + B\omega = -I + \eta \otimes \xi$ on $kerF_*$,*
*(ii) $C^2 + \omega B = -I$ on $kerF_*^\perp$,*
*(iii) $\phi B + BC = 0$ on $kerF_*$,*
*(iv) $\omega\phi + C\omega = 0$ on $kerF_*^\perp$.*

Now, we define
(i) $(\nabla_X \phi)Y = \hat{\nabla}_X \phi Y - \phi \hat{\nabla}_X Y$
and
(ii) $(\nabla_X \omega)Y = \hat{\nabla}_X \omega Y - \omega \hat{\nabla}_X Y$ for $X, Y \in \Gamma(kerF_*)$.
Then from $(i)$ and $(ii)$, we get following lemma.

**Lemma 3.4.** *Let $F$ be a hemi-slant Riemannian map from almost contact manifolds $(M, J, \xi, \eta, g_m)$ into Riemannian manifolds $(N, g_n)$. Then, we have*
*$\phi$ is parallel i.e.*

$$\nabla\phi \equiv 0 \Leftrightarrow \mathcal{T}_X \omega Y = B\mathcal{T}_X Y,$$

*$\omega$ is parallel i.e.*

$$\nabla\omega \equiv 0 \Leftrightarrow \mathcal{T}_X \phi Y = B\mathcal{T}_X Y,$$

*for $X, Y \in \Gamma(kerF_*)$.*

**Theorem 3.5.** *Let $F$ be a hemi-slant Riemannian map from an almost contact metric manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$ with the hemi-slant angle $\theta$. Then, $F$ is a hemi-slant Riemannian map if and only if there exist a constant $\lambda \in [0, 1]$ and a distribution $D$ on $(kerF_*)$ such that*
*(1) $D = \{X \in kerF_* : \phi^2 = -\lambda(I - \eta \otimes \xi)\}$,*
*(2) for $X \in kerF_*$ orthogonal to $D$, we get $\phi X = 0$.*
*In this case $\lambda = \cos^2\theta$ and $\theta$ is a hemi-slant angle of $F$.*

*Proof.* Let $F$ be a hemi-slant Riemannian map from an almost contact metric manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$ with the hemi-slant angle $\theta$. Then for $X \in \Gamma(D_1)$, we get

$$cos\theta = \frac{|\phi X|}{|JX|}, \tag{3.6}$$

If $X \in \Gamma(D) = \Gamma(D_1 \oplus < \xi >)$ and $X$ is not parallel to $\xi$, then we have

$$cos\theta = \frac{|\phi X|}{|JX|},$$

$$cos\theta = \frac{g_m(JX, \phi X)}{|JX||\phi X|},$$

Using (3.3), we obtain

$$cos\theta = \frac{g_m(\phi X, \phi X)}{|JX||\phi X|},$$

$$cos\theta = -\frac{g_m(X, \phi^2 X)}{|JX||\phi X|}, \tag{3.7}$$

Now, using (2.14), (3.6) and (3.7), we get

$$\phi^2 X = -\cos^2\theta(X - \eta(X)\xi), \tag{3.8}$$

for $X \in \Gamma(D)$. If $\lambda = \cos^2\theta$, then

$$\phi^2 X = -\lambda(X - \eta(X)\xi), \tag{3.9}$$

for $X \in \Gamma(D)$.
Conversely, Let there exist a constant $\lambda$ for $X \in \Gamma(kerF_*)$ such that $\phi^2 = -\lambda(I - \eta \otimes \xi)$ is satisfied. Then $X \in \Gamma(D)$, we have

$$cos\theta = \frac{g_m(JX, \phi X)}{|JX||\phi X|},$$

$$cos\theta = \frac{g_m(\phi X, \phi X)}{|JX||\phi X|},$$

So, we obtain

$$cos\theta = \frac{|\phi X|}{|JX|},$$

since $cos\theta = \frac{|\phi X|}{|JX|}$, then we get from above $\lambda = cos^2\theta$, which implies that $\theta$ is constant. Clearly (2) is obvious.

**Lemma 3.6.** *Let $F$ be a hemi-slant Riemannian map from an almost contact metric manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$ with hemi-slant angle $\theta$. Then, we obtain*

$$g_m(\phi X, \phi Y) = cos^2\theta g_m(X, Y), \tag{3.10}$$

$$g_m(\omega X, \omega Y) = sin^2\theta g_m(X, Y), \tag{3.11}$$

*for any $X, Y \in \Gamma(D_1)$.*

The proof of above Lemma is exactly the same with slant immersions (see [13], for Sasakian case). Therefore, we omit its proof.

**Lemma 3.7.** *Let $F$ be a hemi-slant Riemannian map from an almost contact metric manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$ with the hemi-slant angle $\theta$. if $\omega$ is parallel with respect to $\nabla$ on $kerF_*$, then we have*

$$\mathcal{T}_{\phi X}\phi X = -cos^2\theta(\mathcal{T}_X X - \eta(X)\mathcal{T}_X\xi) \tag{3.12}$$

*for $X \in \Gamma(kerF_*)$.*

*Proof.* If $\omega$ is parallel, from (3.3), we get

$$\mathcal{C}\mathcal{T}_X Y = \mathcal{T}_X\phi Y, \tag{3.13}$$

for $X, Y \in \Gamma(kerF_*)$. Interchange $X$ and $Y$ in equation (3.13), we obtain

$$\mathcal{T}_X\phi Y = \mathcal{T}_Y\phi X. \tag{3.14}$$

Replacing $Y$ by $\phi X$ in above equation and then using Theorem (3.4), we obtain our result.

**Theorem 3.8.** *Let $F$ be a hemi-slant Riemannian map from an almost contact metric manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$ with the hemi-slant angle $\theta$. Then the distribution $D_2\oplus < \xi >$ is integrable.*

Proof. Let for $X, Y \in \Gamma(D_2\oplus < \xi >)$ and $W \in \Gamma(D_1)$, we get

$$\begin{aligned} 3d\omega(X, Y, W) &= X\omega(Y, W) + Y\omega(W, X) + W\omega(X, Y) - \omega([Y, W], X) \\ &\quad - \omega([X, Y], W) - \omega([W, X], Y), \end{aligned}$$

where $\omega(X, Y) = g_m(X, JY)$ is the fundamental $2-$ form of $M$ which vanish for an almost contact metric manifold. Thus we obtain

$$g_m([X, Y], \phi W) = 0. \tag{3.15}$$

Thus proof is complete.

**Theorem 3.9.** *Let $F$ be a hemi-slant Riemannian map from an almost contact metric manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$ with the hemi-slant angle $\theta$. Then the distribution $D_1\oplus < \xi >$ is integrable if and only if*

$$g_m(\mathcal{T}_Y\omega\phi Z - \mathcal{T}_Z\omega\phi Y, X) = g_n((\nabla F_*)(Y, \omega Z) - (\nabla F_*)(Y, \omega Z), F_*(JX)),$$

*for $X \in D_2$ and $Y, Z \in \Gamma(D_1\oplus < \xi >)$.*

Proof. Let for $X \in D_2$ and $Y, Z \in \Gamma(D_1\oplus < \xi >)$. Then using (2.1), (3.3) and (3.4), we obtain

$$\begin{aligned} g_m([Y, Z], X) = g_m(J[Y, Z], JX) &= -g_m(\nabla_Y\omega\phi Z, X) + g_m(\nabla_Z\omega\phi Y, X) \\ &\quad + g_m(\nabla_Y\omega Z, JX) - g_m(\nabla_Z\omega Y, JX). \end{aligned}$$

Now using (2.9) and Theorem 3.4 in above equation, we have

$$sin^2 g_m([Y, Z], X) = g_m(\mathcal{T}_Y\omega\phi Z - \mathcal{T}_Z\omega\phi Y, X) + g_m(\mathcal{H}\nabla_Y\omega Z - \mathcal{H}\nabla_Z\omega Y, JX)$$

Using equation (2.1), we have

$$g_m(\mathcal{T}_Y\omega\phi Z - \mathcal{T}_Z\omega\phi Y, X) = g_n((\nabla F_*)(Y, \omega Z) - (\nabla F_*)(Y, \omega Z), F_*(JX)),$$

for $X \in D_2$ and $Y, Z \in \Gamma(D_1)$.
Now, here we give results for leaf of the distribution $D_1\oplus < \xi >$ and for the leaf of the distribution $D_2\oplus < \xi >$.

**Theorem 3.10.** *Let $F$ be a hemi-slant Riemannian map from almost contact manifolds $(M, J, \xi, \eta, g_m)$ into Riemannian manifolds $(N, g_n)$ with the hemi-slant angle $\theta$. Then the distribution $D_1 \oplus <\xi>$ defines a totally geodesic foliation on $M$ if and only if*

$$g_m(\mathcal{T}_Z \omega\phi Y, X) = g_n((\nabla F_*)(Y, \omega Z), F_*(JX)),$$

*for $X \in D_2$ and $Y, Z \in \Gamma(D_1 \oplus <\xi>)$.*

In a similar pattern, we have the following result.

**Theorem 3.11.** *Let $F$ be a hemi-slant Riemannian map from an almost contact metric manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$ with the hemi-slant angle $\theta$. Then the distribution $D_2 \oplus <\xi>$ defines a totally geodesic foliation on $M$ if and only if*

$$g_m(\mathcal{T}_X \omega\phi Z, Y) = g_n((\nabla F_*)(X, \omega Z), F_*(JX)),$$

*for $X, Y \in D_2 \oplus <\xi>$ and $Z \in \Gamma(D_1)$.*

**Theorem 3.12.** *Let $F$ be a hemi-slant Riemannian map from an almost contact metric manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$ with the hemi-slant angle $\theta$. Then the distribution $kerF_*$ defines a totally geodesic foliation on $M$ if and only if*

$$g_m(T_X JY, BW) = g_n((\nabla F_*)(X, JY), CW),$$

*and*

$$\begin{aligned} g_n(\nabla_{\omega\phi Z} F_*(CW), F_*(\omega X)) + g_n(\omega_{\omega Z} F_*(\omega X), F_*(C^2 W)) = {} & g_m(BW, A_{\omega\phi Z}\omega X) \\ & - g_m(\mathcal{V}\nabla_{\omega\phi Z} BW + A_{\omega\phi Z} CW, \phi X) \\ & + g_m(\phi X, A_{\omega Z} C^2 W) + g_m(\omega Z, \mathcal{T}_X \omega BW) \\ & - g_m(\mathcal{V}\nabla_{\omega Z}\phi X + A_{\omega Z}\omega X, BCW) \end{aligned}$$

*for $X \in \Gamma(kerF_*)$, $Y \in D_2$, $Z \in D_1$ and $W \in \Gamma((kerF_*)^\perp)$*

Proof. Let us suppose $X \in \Gamma(kerF_*)$, $Y \in D_2$ and $W \in \Gamma((kerF_*)^\perp)$, using (2.3), (2.8) and (3.4), we get

$$\begin{aligned} g_m(\nabla_X Y, W) &= g_m(J\nabla_X Y, JW) \\ &= g_m(\mathcal{H}\nabla_X JY, CW) + g_m(\mathcal{T}_X JY, BW), \end{aligned}$$

from (2.1) and (2.3), we have

$$g_m(\nabla_X Y, W) = g_n((\nabla F_*)(X, JY), CW) + g_m(\mathcal{T}_X JY, BW).$$

Now, for $Z \in D_1$, using (2.1), (2.10), (3.3), (3.4) and Theorem 3.3, we obtain

$$\begin{aligned} sin^2\theta g_m(\nabla_X Z) = {} & -g_m(\mathcal{H}\nabla_X \omega\phi Z, W) + g_m(\mathcal{H}\nabla_X \omega Z, CW) \\ & + g_m(\mathcal{T}_X \omega Z, BW). \end{aligned}$$

Since $[W, X] \in \Gamma(kerF_*)$, for $W \in \Gamma((kerF_*)^\perp)$ and $X \in \Gamma(kerF_*)$, we get

$$g_m(\mathcal{H}\nabla_X \omega\phi Z, W) = -g_m(\nabla_{\omega\phi Z} W, X).$$

Now, using (2.1), (2.10), (2.11), (3.3) and (3.4), we have

$$\begin{aligned} g_m(\mathcal{H}\nabla_X \omega\phi Z, W) = {} & -g_m(A_{\omega\phi Z} BW, \omega X) - g_m(\mathcal{V}\nabla_{\omega\phi Z} BW, \phi X) \\ & - g_m(\mathcal{H}\nabla_{\omega\phi Z} CW, \omega X) - g_m(A_{\omega\phi Z} CW, \phi X). \end{aligned}$$

Then using (2.1), we get

$$\begin{aligned} g_m(\mathcal{H}\nabla_X \omega\phi Z, W) = {} & -g_m(A_{\omega\phi Z} BW, \omega X) - g_m(\mathcal{V}\nabla_{\omega\phi Z} BW + A_{\omega\phi Z} CW, \phi X) \\ & - g_n(\nabla_{\omega\phi Z} F_{ast}(CW), F_*(\omega X)). \end{aligned}$$

In similar pattern, we can easily get

$$g_m(\mathcal{H}\nabla_X\omega Z, CW) = -g_m(A_{\omega Z}\phi X, C^2W) + g_m(\mathcal{V}\nabla_{\omega Z}\phi X + A_{\omega Z}\omega X, BCW) \\ + g_n(\nabla_{\omega Z}F_*(\omega X), F_*(C^2W)).$$

Thus putting (3.18) and (3.19) in (3.17), we obtain

$$sin^2\theta g_m(\nabla_X Z, W) = -g_m(A_{\omega\phi Z}BW, \omega X) - g_m(\mathcal{V}\nabla_{\omega\phi Z}BW + A_{\omega\phi Z}CW, \phi X) \\ - g_n(\nabla_{\omega\phi Z}F_*(CW), F_*(\omega X)) - g_m(A_{\omega Z}\phi X, C^2W) \\ + g_m(\mathcal{V}\nabla_{\omega Z}\phi X + A_{\omega Z}\omega X, BCW) + g_n(\nabla_{\omega Z}F_*(\omega X), F_*(C^2W)) \\ + g_m(\mathcal{T}_X\omega Z, BW).$$

Then using (3.16) and (3.20), we get required result.

**Theorem 3.13.** *Let $F$ be a hemi-slant Riemannian map from an almost contact metric manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$ with the hemi-slant angle $\theta$. Then the distribution $(kerF_*)^\perp$ defines a totally geodesic foliation on $M$ if and only if*

$$g_m(BY, A_X JZ) = g_n(\nabla_X F_*(CY), F_*(JZ)),$$

*and*

$$g_m(A_X BY, \omega Z) = g_n(F_*(CY), \nabla_X F_*(\omega Z)) - g_n(F_*(Y), \nabla_X F_*(\omega\phi Z)),$$

*for $X, Y \in \Gamma((kerF_*)^\perp)$ and $Z \in \Gamma(D_1)$.*

Proof. For $X, Y \in \Gamma(kerF_*)^\perp$ and $W \in \Gamma(D_2)$, using (2.16), (2.10) and (2.11), we have

$$g_m(\nabla_X Y, W) = g_m(\mathcal{H}\nabla_X CY + A_X BY, JW).$$

Then using (2.1), we get

$$g_m(\nabla_X Y, W) = g_n(\nabla_X F_*(CY), F_*(JW)) + g_m(A_X BY, JW). \tag{3.16}$$

Now, using theorem 3.4 for $Z \in \Gamma(D_1)$, we obtain

$$sin^2\theta g_m(\nabla_X Y, W) = g_m(Y, \mathcal{H}\nabla_X\omega\phi Z) - g_m(CY, \mathcal{H}\nabla_X\omega Z) - g_m(BY, A_X\omega Z).$$

From equations (2.1) and (2.3), we can easily get

$$sin^2\theta g_m(\nabla_X Y, W) = g_n(F_*(Y), \nabla_X F_*(\omega\phi Z)) - g_n(F_*(CY), \nabla_X F_*(\omega Z)) \\ - g_m(BY, A_X\omega Z). \tag{3.17}$$

Then from (3.16) and (3.17), we get our results.

## 4 Harmonicity of hemi-slant Riemannian maps from a Sasakian manifold into a Riemannian manifold

In this section, we find necessary and sufficient condition for hemi-slant Riemannian maps from a Sasakian manifold into a Riemannian manifold to be harmonic and totally geodesic. Also, we obtain a characterization theorem for the proper hemi-slant Riemannian maps from a Sasakian manifold into a Riemannian manifold and we give some examples of such maps.

**Theorem 4.1.** *Let $F$ be a hemi-slant Riemannian map from a Sasakian manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$ with the hemi-slant angle $\theta$. Then $F$ is harmonic if and only if*

$$trace|_{D_2}\{F_*(\mathcal{T}_{(.)})(.) - CA_{J(.)}(.) - \omega\mathcal{V}\nabla_{J(.)}(.) + \nabla_{J(.)}F_*J(.)\} + trace|_{D_1} \\ \{F_*(C\mathcal{T}_{(.)})\phi(.) + \omega\mathcal{V}\nabla_{(.)}\phi(.) + C\mathcal{H}\nabla_{(.)}\omega(.) + J\mathcal{T}_{(.)}(.) - sec^2\theta\mathcal{T}_{\phi(.)}\phi(.) \\ - csc^2\theta(CA_{\omega(.)}(.) + \omega\mathcal{V}\nabla_{\omega(.)}(.) + \theta A_{\omega(.)}\phi(.)) + csc^2\theta\nabla_{\omega(.)}F_*(\omega(.))\} = 0.$$

Proof. Let $kerF_*$ has an orthonormal frame $\{e_1, e_2, ....., e_{s_1}, \bar{e}_1, \bar{e}_1, ....., \bar{e}_{s_2}, sec\theta\phi\bar{e}_1, sec\theta\phi\bar{e}_2, ......, sec\theta\phi\bar{e}_{s_2}, \xi\}$ such that $\{e_1, e_2, ....., e_{s_1}\}$ is an orthonormal frame of $D_2$ and $\{\bar{e}_1, \bar{e}_1, ....., \bar{e}_{s_2}, sec\theta\phi\bar{e}_1, sec\theta\phi\bar{e}_2, ......, sec\theta\phi\bar{e}_{s_2}\}$ is an orthonormal frame of $D_1$ and $< \xi >$ is vertical vector field orthogonal to $D_1$ and $D_2$. Therefore it follows that $\{Je_1, Je_2, ....., Je_s, csc\theta\omega\bar{e_1}, csc\theta\omega\bar{e_2}, ....., csc\theta\omega\bar{e_{s_2}}\}$ is an orthonormal frame of $(kerF_*)^{\perp}$. Let for $X \in \Gamma(D_2)$ and $Y \in \Gamma(D_1)$, we define $\Omega(X,Y)$ as

$$\Omega(X,Y) = (\nabla F_*)(X,X) + (\nabla F_*)(JX,JX) + (\nabla F_*)(Y,Y) + sec^2\theta(\nabla F_*)(\phi Y,\phi Y) + csc^2\theta(\nabla F_*)(\omega Y,\omega Y).$$

Therefore using (2.1) and (2.16), we obtain

$$\Omega(X,Y) = -F_*(\nabla_X X) + \nabla_{JX} F_* JX - F_*(J\nabla_{JX} X) + F_*(J\nabla_Y JY) - sec^2\theta F_*(\nabla_{\phi Y}\phi Y) + csc^2\theta\nabla^F_{\omega Y} F_*(\omega Y) - csc^2\theta F_*(\nabla_{\omega Y}\omega Y).$$

Now, using (3.3), (3.4) and (2.8)-(2.10), we obtain

$$\begin{aligned}\Omega(X,Y) = &-F_*(\mathcal{T}_X X) + \nabla^F_{JX} F_* JX - F_*(CA_{JX} X) + F_*(\omega\mathcal{V}\nabla_{JX} X) + F_*(\mathcal{T}_Y\phi Y)\\ &+ F_*(\omega\mathcal{V}\nabla_Y\phi Y) + F_*(C\mathcal{H}\nabla_Y\omega Y) + F_*(J\mathcal{T}_Y\omega Y) - sec^2\theta F_*(\mathcal{T}_{\phi Y}\phi Y)\\ &+ csc^2\theta\nabla^F_{\omega Y} F_*(\omega Y) - csc^2\theta F_*(CA_{\omega Y} Y) - csc^2\theta F_*(\omega\mathcal{V}\nabla_{\omega Y} Y) + csc^2\theta F_*(A_{\omega Y}\phi Y).\end{aligned}$$

which proves our assertion.

**Theorem 4.2.** *Let $F$ be a hemi-slant Riemannian map from a Sasakian manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$ with the hemi-slant angle $\theta$. Then $F$ is totally geodesic on $M$ if and only if*

$$\omega\mathcal{T}_U JV + C\mathcal{H}\nabla_U JV = 0,$$

$$\mathcal{H}\nabla_W\omega\phi Z + C\mathcal{H}\nabla_W\omega Z + \omega\mathcal{T}_W\omega Z = 0,$$

$$\mathcal{H}\nabla_X\omega\phi Z + C\mathcal{H}\nabla_X\omega Z + \omega\mathcal{A}_X\omega Z = 0$$

*and*

$$F_*(\mathcal{A}_X\phi BY + \mathcal{H}\nabla_X\omega BY) + C\mathcal{H}\nabla_X CY + \omega\mathcal{A}_X CY = \nabla^f_X f_*(Y),$$

*for $W \in \Gamma(kerF_*), U, V \in \Gamma(D_1), Z \in \Gamma(D_2)$ and $X, Y \in \Gamma(kerF_*)^{\perp}$.*

Proof. Using (2.1) and (2.16) for $U, V \in \Gamma(D_2)$, we get

$$F_*(J\nabla_U JV) = (\nabla F_*)(U,V.$$

Now, using (2.9), (3.3) and (3.4), we have

$$F_*(\omega\mathcal{T}_U JV + C\mathcal{H}\nabla_U JV) = (\nabla F_*)(U,V). \tag{4.1}$$

Since $\xi \in \Gamma(kerF_*)$ and $Z \in \Gamma(D_1)$ and using (2.1), (2.16) and (3.3), which gives

$$F_*(\nabla_U\phi^2 Z + \nabla_U\omega\phi Z + J\nabla_U\omega Z) = (\nabla F_*)(U,Z).$$

Then using (2.9), (3.4) and Theorem 3.4, we can obtain easily

$$F_*(\mathcal{H}\nabla_U\omega\phi Z + C\mathcal{H}\nabla_U\omega Z + \omega\mathcal{T}_X\omega Z) = sin^2\theta(\nabla F_*)(U,Z). \tag{4.2}$$

Proceeding in the same way, for $X \in \Gamma((kerF_*)^{\perp})$ and $Z \in \Gamma(D_1)$, we have

$$F_*(\mathcal{H}\nabla_X\omega\phi Z + C\mathcal{H}\nabla_X\omega Z + \omega\mathcal{A}_X\omega Z) = sin^2\theta(\nabla F_*)(X,Z). \tag{4.3}$$

Using (2.1), (2.16) and (3.3) for $X, Y \in \Gamma((kerF_*)^{\perp})$, we get

$$\nabla^F_X F_*(Y) + F_*(\nabla_X JBY) + F_*(J\nabla_X CY) = (\nabla F_*)(X,Y)$$

Now, using (2.10), (2.11), (3.3) and (3.4), we obtain

$$\nabla_X^F F_*(Y) - F_*(\mathcal{A}_X \phi BY - \mathcal{H}\nabla_X BY) - C\mathcal{H}\nabla_X CY - \omega(\mathcal{A}_X CY = (\nabla F_*)(X,Y). \quad (4.4)$$

We get results from equations (4.1)-(4.4).
Now, we investigate the geometry of hemi-slant submersions with totally umbilical fibers.
So, first we recall that a fiber of a Riemannian map $F$ is called totally umbilical if

$$T_U V = g_m(U,V)H \quad (4.5)$$

for any $U, V \in (KerF_*)$, where $H$ is the mean curvature vector field of the fiber $M$. This fiber is said to be minimal, if $H = 0$, identically.

**Theorem 4.3.** *Let $F$ be a proper hemi-slant Riemannian map with totally umbilical fibers from a Sasakian manifold $(M, J, \xi, \eta, g_m)$ into a Riemannian manifold $(N, g_n)$. Then either the anti-invariant distribution $D_2$ is one dimensional or the mean curvature vector field $H$ of any fiber $F^{-1}(p), p \in N$ is perpendicular to $JD^2$. Moreover, if $\phi$ is parallel, then $H \in \mu$, Furthermore, if $\omega$ is parallel then $T \equiv 0$.*

Proof. Since $F$ is a proper hemi-slant Riemannian map, then either $dim(D_2) = 1$ or $dim(D_2) > 1$. If $dim(D_2) = 1$, it is obvious if $dim(D_2) > 1$, then we can choose $X, Y \in D_2$ such that $\{X, Y\}$ is orthonormal.
Using (2.7), (3.3), (3.4) and (2.14)

$$\begin{aligned} T_X JY + \mathcal{H}\nabla_X JY &= \nabla_X JY \\ &= (\nabla_X J)Y + J\nabla_X Y \\ &= \phi\hat{\nabla}_X Y + \omega\hat{\nabla}_X Y + BT_X Y + CT_X Y + g(X,Y)\xi. \end{aligned}$$

Taking innerproduct with $X$, we obtain

$$g_m(T_X JY, X) = g_m(\phi\hat{\nabla}_X Y, X) + g_m(BT_X Y, X).$$

Using (2.14), we get

$$g_m(T_X JY, X) = -g_m(T_X Y, JX). \quad (4.6)$$

Then using (4.5) and (4.6), we obtain

$$\begin{aligned} g_m(H, JY) = g_m(T_X X, JY) &= -g_m(T_X JY, X) = g_m(T_X Y, JX) \\ &= g_m(X,Y) g_m(H, JX) = 0. \end{aligned}$$

So, we observe that

$$H \in JD_2. \quad (4.7)$$

Now, if $\phi$ is parallel then using (2.14) and Lemma 3.3 for $Z \in D_1$, we get

$$\begin{aligned} g_m(H, \omega Z) &= g_m(T_X X, \omega Z) = -g_m(T_X \omega Z, X) \\ &= -g_m(BT_X \omega Z, X) = -g_m(JT_X \omega Z, X) = g_m(T_X \omega Z, JX) = 0. \end{aligned}$$

So

$$H \in \omega D_1. \quad (4.8)$$

Using (4.7) and (4.8), we observe that $H \in \mu$. Further if $\omega$ is parallel then using (2.14) and Lemma 3.3 for unit vector field $X \in D_2$ and $W \in \mu$, we get

$$\begin{aligned} g_m(H, W) = g_m(T_X X, W) = g_m(JT_X X, JW) &= g_m(BT_X X + CT_X X, JW) \\ &= g_m(T_X \phi X, JW) \end{aligned}$$

since $\phi X = 0$. Thus, we get $H = 0$, that is, the fibers are minimal. Since the fibers are also totally umbilical, we obtain $T \equiv 0$ from (4.5).

## 5 Example

**Example 5.1.** For an Euclidean space $R^{2n+1}$ with standard coordinates $(x_1, x_2....x_{2n}, x_{2n+1})$, we can choose an almost contact structure $J$ on $R^{2n+1}$ as follows:

$$J(a_1\frac{\partial}{\partial x_1}+....+a_{2n+1}\frac{\partial}{\partial x_{2n+1}})=(-a_2\frac{\partial}{\partial x_1}+a_1\frac{\partial}{\partial x_2}+....-a_{2n}\frac{\partial}{\partial x_{2n}}+a_{2n-1}\frac{\partial}{\partial x_{2n-1}})$$

where $a_1, a_2, ...., a_{2n}, a_{2n+1}$ are $C^\infty$ real valued function defined on $R^{2n+1}$. Let $\xi = \frac{\partial}{\partial x_{2n+1}}$, $\eta = dx_{2n+1}$ and $g$ is usual inner product on $R^{2n+1}$. Then $(R^{2n+1}, J, \xi, \eta, g)$ is an almost contact metric structure on $R^{2n+1}$. Throughout this section we will use this notion.

**Example 5.2.** Define a map $F : R^9 \to R^8$ by
$F(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = (\frac{x_1-x_3}{\sqrt{2}}, 0, x_4, x_5, x_6, \frac{x_7+x_8}{\sqrt{2}}, 0, 0)$
Then the map $F$ is hemi-slant Riemannian map such that
$D_1 = span\{\frac{1}{\sqrt{2}}(\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_3}), \frac{\partial}{\partial x_2}\}$, $D_2 = span\{\frac{1}{\sqrt{2}}(\frac{\partial}{\partial x_7}-\frac{\partial}{\partial x_8})\}$ and $\xi = \frac{\partial}{\partial x_9}$ with hemi-slant angle $\theta = \frac{\pi}{4}$.

**Example 5.3.** Define a map $F : R^9 \to R^8$ by
$F(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = (\frac{x_1+x_3}{\sqrt{2}}, 0, x_4, x_5, x_6, \frac{x_7-x_8}{\sqrt{2}}, 0, 0)$
Then the map $F$ is hemi-slant Riemannian map such that
$D_1 = span\{\frac{1}{\sqrt{2}}(\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_3}), \frac{\partial}{\partial x_2}\}$, $D_2 = span\{\frac{1}{\sqrt{2}}(\frac{\partial}{\partial x_7}+\frac{\partial}{\partial x_8})\}$ and $\xi = \frac{\partial}{\partial x_9}$ with hemi-slant angle $\theta = \frac{\pi}{4}$.

**Example 5.4.** Every semi-invariant Riemannian map from an almost contact metric manifolds into a Riemannian manifold is a hemi-slant Riemannian map with $\theta = \frac{\pi}{2}$.

**Example 5.5.** Every slant Riemannian map from an almost contact metric manifold into a Riemannian manifold is a hemi-slant Riemannian map with $D_2 = 0$.

**Example 5.6.** Every slant submersion from an almost contact metric manifold into a Riemannian manifold is a hemi-slant Riemannian map with $(rangeF_*)^\perp = 0$ and $D_2 = 0$.

**Example 5.7.** Every hemi-slant submersion from an almost contact metric manifold into a Riemannian manifold is a hemi-slant Riemannian map with $(rangeF_*)^\perp = 0$.

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