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GENERAL EXTREMAL DEGREE BASED INDICES OF A GRAPH

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Abstract. The foremost molecular descriptors are bond additive, i.e., the sum of edge contributions. These descriptors are named as adriatic indices. The peculiarly interesting subclass of these descriptors consists of 148 discrete adriatic indices. In this research work, concentrated on the min-max type degree indices for certain classes of derived-regular graph and the relations among them have been found. This kind of results may improves the significance of indices.

1 Introduction

The graphs considered here are finite, undirected, without loops and multiple edges. Let $G = (V, E)$ be a connected graph with $|V(G)| = n$ vertices and $|E(G)| = m$ edges. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. Recently, automated computer treatment of chemical structures and QSAR are greatly applied through graph theory. Usually chemical structures under consideration are molecules, we call this type of chemical graph a molecular graph. They are created by replacing atoms and bonds with vertices and edges respectively. A molecular graph is a graph in which the vertices correspond to the atoms and the edges to the bonds of a molecule. A single number that can be computed from the molecular graph, and used to characterize some property of the underlying molecule is said to be a topological index or molecular structure descriptor. Numerous such descriptors have been considered in theoretical chemistry, and have found some applications, especially in QSPR/QSAR research [3, 7, 8].

Definitions: Let $\psi_k(G)$ be the min-max type degree index where $k \in \left\{-\frac{1}{2}, \frac{1}{2}, 1, -1, 2, -2\right\}$ defined as

$$\psi_k(G) \quad = \sum_{uv\in E(G)} \left(\frac{max(d_u, d_v)}{min(d_u, d_v)}\right)^k$$

Now,

- Let $k = -\frac{1}{2}$ correspond to min-max rodeg index defined as

$$\psi_{-\frac{1}{2}}(G) \quad = \sum_{uv\in E(G)} \sqrt{\frac{min(d_u, d_v)}{max(d_u, d_v)}}.$$

- Let $k = \frac{1}{2}$ correspond to max-min rodeg index defined as

$$\psi_{\frac{1}{2}}(G) \quad = \sum_{uv\in E(G)} \sqrt{\frac{max(d_u, d_v)}{min(d_u, d_v)}}.$$

- Let $k = -1$ correspond to min-max deg index defined as

$$\psi_{-1}(G) = \sum_{uv \in E(G)} \frac{min(d_u, d_v)}{max(d_u, d_v)}.$$

- Let $k = 1$ correspond to max-min deg index defined as

$$\psi_{1}(G) = \sum_{uv \in E(G)} \frac{max(d_u, d_v)}{min(d_u, d_v)}.$$

- Let $k = -2$ correspond to min-max sdeg index defined as

$$\psi_{-2}(G) = \sum_{uv \in E(G)} \left(\frac{min(d_u, d_v)}{max(d_u, d_v)}\right)^2.$$

- Let $k = 2$ correspond to max-min sdeg index defined as

$$\psi_{2}(G) = \sum_{uv \in E(G)} \left(\frac{max(d_u, d_v)}{min(d_u, d_v)}\right)^2.$$

- Let $\xi(G) = \psi_1(G) + \psi_{-1}(G)$ be the symmetric division deg index defined as

$$\xi(G) = \sum_{uv \in E(G)} \left(\frac{min(d_u, d_v)}{max(d_u, d_v)} + \frac{max(d_u, d_v)}{min(d_u, d_v)}\right).$$

These indices are the best predictor of enthalpy of vaporization for octane isomers, log water activity coefficient for polychlorobiphenyies and total surface area for polychlorobiphenyles. For more details, we refer to [4, 12, 14, 15].
In forthcoming sections, we establishing min-max degree based adriatic indices of regular and complete bipartite graph using some operators such as line, subdivision, semi total (vertex and edge)graph, total graphs, jump graph, para-line graph.

2 Line graph

The line graph $L(G)$ is the graph with vertex set $V(L) = E(G)$ and whose vertices correspond to the edges of G with two vertices being adjacent if and only if the corresponding edges in G have a vertex in common. For more details, we refer to [1, 13].

Theorem 2.1. *Let G be a r- regular graph with $n \geq 2$ vertices. Then*

$$\psi_k(L(G)) = \frac{nr}{2}(r-1)$$

and

$$\xi(L(G)) = nr(r-1)$$

Proof. Let G be a r- regular graph with $n \geq 2$ vertices. By algebraic method, we have $|V(L(G))| = \frac{nr}{2}$ and $|E(L(G))| = \frac{nr}{2}(r-1)$. Since line graph of a r - regular graph is $(2r-2)$ - regular.

$$Let, \psi_k(G) = \sum_{uv \in E(G)} \left(\frac{max(d_u, d_v)}{min(d_u, d_v)}\right)^k$$

$$\psi_{-\frac{1}{2}}(L(G)) = \sum_{uv \in E(G)} \left(\frac{2r-2}{2r-2}\right)^{-\frac{1}{2}} \left(\frac{nr}{2}(r-1)\right)$$

$$= \frac{nr}{2}(r-1).$$

Similarly, if $k = \frac{1}{2}, 1, -1, 2, -2$ then $\psi_k(L(G)) = \frac{nr}{2}(r-1)$.

$$\begin{aligned}
Let, \xi(G) &= \sum_{uv \in E(G)} \left(\frac{min(d_u, d_v)}{max(d_u, d_v)} + \frac{max(d_u, d_v)}{min(d_u, d_v)} \right) \\
\xi(L(G)) &= \sum_{uv \in E(G)} \left(\frac{2r-2}{2r-2} + \frac{2r-2}{2r-2} \right) \left(\frac{nr}{2}(r-1) \right) \\
&= nr(r-1).
\end{aligned}$$

Theorem 2.2. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. Then*

$$\begin{aligned}
\psi_k(L(K_{r,s})) &= \frac{rs}{2}(r+s-2) \\
&\textit{and} \\
\xi(L(K_{r,s})) &= rs(r+s-2).
\end{aligned}$$

Proof. Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. By algebraic method, we have $|V(L(K_{r,s}))| = rs$ and $|E(L(K_{r,s}))| = \frac{1}{2} rs(r+s-2)$. Since line graph of complete bipartite graph $K_{r,s}$ is a $(r+s-2)$-regular graph.

$$\begin{aligned}
Let, \psi_k(G) &= \sum_{uv \in E(G)} \left(\frac{max(d_u, d_v)}{min(d_u, d_v)} \right)^k \\
\psi_{-\frac{1}{2}}(L(K_{r,s})) &= \sum_{uv \in E(G)} \left(\frac{r+s-2}{r+s-2} \right)^{-\frac{1}{2}} \left(\frac{rs}{2}(r+s-2) \right) \\
&= \frac{rs}{2}(r+s-2).
\end{aligned}$$

Similarly, if $k = \frac{1}{2}, 1, -1, 2, -2$ then $\psi_k(L(K_{r,s})) = \frac{rs}{2}(r+s-2)$.

$$\begin{aligned}
Let, \xi(G) &= \sum_{uv \in E(G)} \left(\frac{min(d_u, d_v)}{max(d_u, d_v)} + \frac{max(d_u, d_v)}{min(d_u, d_v)} \right) \\
\xi(L(K_{r,s})) &= \sum_{uv \in E(G)} \left(\frac{r+s-2}{r+s-2} + \frac{r+s-2}{r+s-2} \right) \left(\frac{rs}{2}(r+s-2) \right) \\
&= rs(r+s-2).
\end{aligned}$$

□

By above result with $r = s$, we have complete regular bipartite graph $K_{r,r}$ with $r > 1$.

3 Subdivision graph

The subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edges by a path of length two, or equivalently, by inserting an additional vertex into each edge of G. For more details, we refer to [9].

Theorem 3.1. *Let G be a r- regular graph with $n \geq 2$ vertices.*

$$\psi_k(S(G)) = \begin{cases} nr(\frac{2}{r})^k, & \textit{if } r=1 \\ nr, & \textit{if } r=2 \\ nr(\frac{r}{2})^k, & \textit{if } r>2 \end{cases}$$

and

$$\xi(S(G)) = \begin{cases} 2nr, & \text{if } r{=}2 \\ \frac{(r^2+4)n}{2}, & \text{if } r{=}1 \text{ and } r{>}2 \end{cases}$$

Proof. Let G be a r- regular graph with $n \geq 2$ vertices. By algebraic method, we have $|V(S(G))| = n + \frac{nr}{2}$ and $|E(S(G))| = nr$.

We have two partitions of the vertex set $V(S(G)))$ as follows:

$V_1 = \{v \in V(S(G)) : d_{S(G)}(v) = 2\}$; $|V_1| = \frac{nr}{2}$ and

$V_2 = \{v \in V(S(G)) : d_{S(G)}(v) = r\}$; $|V_2| = n$.

Also we have the edge set $E_1 = \{uv \in E(S(G)) : d_{S(G)}(u) = 2, d_{S(G)}(v) = r\}$; $|E(S(G))| = |E_1| = nr$.

The proof technique is homologous to earlier theorem. □

Theorem 3.2. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. Then*

$$\psi_k(S(K_{r,s})) = \begin{cases} rs\left(\frac{2}{r}+\frac{2}{s}\right)^k, & \text{if } r{=}1{=}s \\ 2rs, & \text{if } r{=}2{=}s \\ rs\left(\frac{r}{2}+\frac{s}{2}\right)^k, & \text{if } r{>}2 \text{ and } s{>}2 \end{cases}$$

and

$$\xi(S(K_{r,s})) = \begin{cases} s\left(\frac{4+r^2}{2}\right)+r\left(\frac{4+s^2}{2}\right), & \text{if } r{=}1{=}s \\ 4rs, & \text{if } r{=}2{=}s \\ s\left(\frac{4+r^2}{2}\right)+r\left(\frac{4+s^2}{2}\right), & \text{if } r{>}2 \text{ and } s{>}2 \end{cases}$$

Proof. Let $K_{r,s}$ be complete bipartite graph with $(r+s)$ vertices and $|V_1^*| = r, |V_2^*| = s$, $V(K_{r,s}) = V_1^* \cup V_2^*$ for $1 \leq r \leq s$ vertices. Every vertex of V_1^* is incident with s edges and every vertex of V_2^* is incident with r edges. By algebraic method, we have $|V(S(K_{r,s}))| = r+s+rs$, and $|E(S(K_{r,s}))| = 2rs$.

We have three partitions of the vertex set $V(S(K_{r,s}))$ as follows:

$V_1 = \{v \in V(S(K_{r,s})) : d_{S(K_{r,s})}(v) = r\}$; $|V_1| = s$,

$V_2 = \{v \in V(S(K_{r,s})) : d_{S(K_{r,s})}(v) = s\}$; $|V_2| = r$ and

$V_3 = \{v \in V(S(K_{r,s})) : d_{S(K_{r,s})}(v) = 2\}$; $|V_3| = rs$.

Also we have two partitions of the edge set $E(S(K_{r,s}))$ as follows:

$E_1 = \{uv \in E(S(K_{r,s})) : d_{S(K_{r,s})}(u) = 2, d_{S(K_{r,s})}(v) = r\}; |E_1| = rs$, and

$E_2 = \{uv \in E(S(K_{r,s})) : d_{S(K_{r,s})}(u) = 2, d_{S(K_{r,s})}(v) = s\}; |E_2| = rs.$

Then $(S(K_{r,s})) = \sum_{ue}[d_{S(K_{r,s})}(u) * d_{S(K_{r,s})}(e)].$

The proof technique is homologous to earlier theorem. □

4 Vertex-semi total graph

The vertex-semi total graph $T_1(G)$ with vertex set $V(G) \cup E(G)$ and edge set $E(S) \cup E(G)$ is the graph obtained from G by adding a new vertex corresponding to each edge of G and by joining each new vertex to the end vertices of the edge corresponding to it.

Theorem 4.1. *Let G be a r- regular graph with $n \geq 2$ vertices. Then*

$$\psi_k(T_1(G)) = \begin{cases} nr + \frac{nr}{2}, & \text{if } r=1 \\ nr(r)^k + \frac{nr}{2}, & \text{if } r>1 \end{cases}$$

and

$$\xi(T_1(G)) = \begin{cases} 2nr + nr, & \text{if } r=1 \\ n(r^2 + r), & \text{if } r>1 \end{cases}$$

Proof. Let G be a r- regular graph with $n \geq 2$ vertices. By algebraic method, we have $|V(T_1(G))| = \frac{nr}{2} + n$ and $|E(T_1(G))| = \frac{3nr}{2}$. We have two partitions of the vertex set $V(T_1(G)))$ as follows:

$V_1 = \{v \in V(T_1(G)) : d_{T_1(G)}(v) = 2\}$; $|V_1| = \frac{nr}{2}$ and

$V_2 = \{v \in V(T_1(G)) : d_{T_1(G)}(v) = 2r\}$; $|V_2| = n.$

Also we have two partitions of the edge set $E(T_1(G))$ as follows:

$E_1 = \{uv \in E(T_1(G)) : d_{T_1(G)}(u) = 2, d_{T_1(G)}(v) = 2r\}; |E_1| = nr$, and

$E_2 = \{uv \in E(T_1(G)) : d_{T_1(G)}(u) = d_{T_1(G)}(v) = 2r\}; |E_2| = \frac{nr}{2}.$

The proof manner is analogous to previous theorem. □

Theorem 4.2. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. Then*

$$\psi_k(T_1(K_{r,s})) = \begin{cases} 3rs, & \text{if } r=1=s \\ rs\left(1 + (r)^k + (\frac{s}{r})^k\right), & \text{if } r>2 \text{ and } s=1 \\ rs\left(1 + (s)^k + (\frac{s}{r})^k\right), & \text{if } s>2 \text{ and } r=1 \end{cases}$$

and

$$\xi(T_1(K_{r,s}))= \begin{cases} 6rs, & \textit{if } r=1=s \\ s(1+r^2+(r+s)^2), & \textit{if } r>2 \textit{ and } s=1 \\ (s^2+r^2)+(1+s^2)r+2rs, & \textit{if } s>2 \textit{ and } r=1 \end{cases}$$

Proof. If $K_{r,s}$ is a complete bipartite graph with $(r+s)$ - vertices and rs - edges, where $|V_1^*| = r, |V_2^*| = s$, $V(K_{r,s}) = V_1^* \cup V_2^*$ for $1 \leq r \leq s$ vertices. Every vertex of V_1^* is incident with s edges and every vertex of V_2^* is incident with r edges. By algebraic method, we have $|V(T_1(K_{r,s}))| = r+s+rs$, and $|E(T_1(K_{r,s}))| = 3rs$.
We have three partitions of the vertex set $V(T_1(K_{r,s}))$ as follows:

$V_1 = \{v \in V(T_1(K_{r,s})) : d_{T_1(K_{r,s})}(v) = 2r\};\ |V_1| = s,$

$V_2 = \{v \in V(T_1(K_{r,s})) : d_{T_1(K_{r,s})}(v) = 2s\};\ |V_2| = r$, and

$V_3 = \{v \in V(T_1(K_{r,s})) : d_{T_1(K_{r,s})}(v) = 2\};\ |V_2| = rs.$

Also we have three partitions of the edge set $E(T_1(K_{r,s}))$ as follows:

$E_1 = \{uv \in E(T_1(K_{r,s})) : d_{T_1(K_{r,s})}(u) = 2r, d_{T_1(K_{r,s})}(v) = 2\}; |E_1| = rs,$

$E_2 = \{uv \in E(T_1(K_{r,s})) : d_{T_1(K_{r,s})}(u) = 2s, d_{T_1(K_{r,s})}(v) = 2\}; |E_2| = rs$, and

$E_3 = \{uv \in E(T_1(K_{r,s})) : d_{T_1(K_{r,s})}(u) = 2r, d_{T_1(K_{r,s})}(v) = 2s\}; |E_3| = rs.$

Hence applying the topological indices definitions we get required results. □

By above result with $r = s$, we have complete regular bipartite graph $K_{r,r}$ with $r > 2$

5 Edge-semi total graph

An edge-semi total graph $T_2(G)$ with vertex set $V(G) \cup E(G)$ and edge set $E(S) \cup E(L)$ is the graph obtained from G by inserting a new vertex into each edge of G and by joining with edges those pairs of these new vertices which lie on adjacent edges of G.

Theorem 5.1. *Let G be a r- regular graph with $n \geq 2$ vertices. Then*

$$\psi_k(T_2(G)) = \frac{nr}{2}(r-1) + nr(2)^k$$

and

$$\xi(T_2(G)) = \frac{5nr}{2} + nr(r-1)$$

Proof. Let G be a r- regular graph with $n \geq 2$ vertices. By algebraic method, we have $|V(T_2(G))| = \frac{nr}{2} + n$ and $|E(T_2(G))| = \frac{nr}{2}(r+1)$. We have two partitions of the vertex set $V(T_2(G)))$ as follows:

$V_1 = \{v \in V(T_2(G)) : d_{T_2(G)}(v) = r\};\ |V_1| = n$ and

$V_2 = \{v \in V(T_2(G)) : d_{T_2(G)}(v) = 2r\};\ |V_2| = \frac{rn}{2}.$

Also we have two partitions of the edge set $E(T_2(G))$ as follows:

$E_1 = \{uv \in E(T_2(G)) : d_{T_2(G)}(u) = r, d_{T_2(G)}(v) = 2r\}; |E_1| = rn$, and

$E_2 = \{uv \in E(T_2(G)) : d_{T_2(G)}(u) = d_{T_2(G)}(v) = 2r\}; |E_2| = \dfrac{rn}{2}(r-1).$

Hence applying the topological indices definitions we get required results. □

Theorem 5.2. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. Then*

$$\psi_k(T_2(K_{r,s})) = rs\left(\frac{r+s}{r} + \frac{r+s}{s}\right)^k + \frac{1}{2}(r+s-2)$$

and

$$\xi(T_2(K_{r,s})) = \left(\frac{r^2+(r+s)^2}{r(r+s)} + \frac{s^2+(r+s)^2}{s(r+s)}\right)rs + rs(r+s-2)$$

Proof. Let $K_{r,s}$ be complete bipartite graph with $(r+s)$ vertices and $|V_1^*| = r, |V_2^*| = s$, $V(K_{r,s}) = V_1^* \cup V_2^*$ for $1 \leq r \leq s$ vertices. Every vertex of V_1^* is incident with s edges and every vertex of V_2^* is incident with r edges. By algebraic method, we have $|V(T_2(K_{r,s}))| = r+s+rs$, and $|E(T_2(K_{r,s}))| = sr[1 + \frac{1}{2}(r+s)]$.

We have three partitions of the vertex set $V(T_2(K_{r,s}))$ as follows:

$V_1 = \{v \in V(T_2(K_{r,s})) : d_{T_2(K_{r,s})}(v) = r\}$,

$V_2 = \{v \in V(T_2(K_{r,s})) : d_{T_2(K_{r,s})}(v) = s\}$, and

$V_3 = \{v \in V(T_2(K_{r,s})) : d_{T_2(K_{r,s})}(v) = r+s\}$.

Also we have three partitions of the edge set $E(T_2(K_{r,s}))$ as follows:

$E_1 = \{uv \in E(T_2(K_{r,s})) : d_{T_2(K_{r,s})}(u) = r, d_{T_2(K_{r,s})}(v) = r+s\}; |E_1| = rs$,

$E_2 = \{uv \in E(T_2(K_{r,s})) : d_{T_2(K_{r,s})}(u) = s, d_{T_2(K_{r,s})}(v) = r+s\}; |E_2| = rs$, and

$E_3 = \{uv \in E(T_2(K_{r,s})) : d_{T_2(K_{r,s})}(u) = d_{T_2(K_{r,s})}(v) = r+s\}; |E_3| = \frac{1}{2}rs\,[r+s-2]$.

Applying these cardinalities in topological indices definitions we get required results. □

By above result with $r = s$, we have complete regular bipartite graph $K_{r,r}$ with $r > 1$

6 Total graph

The total graph of a graph G is denoted by $T(G)$ with vertex set $V(G) \cup E(G)$ and any two vertices of $T(G)$ are adjacent if and only if they are either incident or adjacent in G. For more details, we refer to [1].

Theorem 6.1. *Let G be a r- regular graph with $n \geq 2$ vertices. Then*

$$\psi_k(T(G)) = \frac{nr^2}{2} + nr$$

and

$$\xi(T(G)) = nr^2 + 2nr$$

Proof. Let G be a r- regular graph with $n \geq 2$ vertices. By algebraic method, we have the vertex set $V_1 = \{v \in V(T(G)) : d_{T(G)}(v) = 2r\}$; $|V(T(G))| = |V_1| = \frac{nr}{2} + nr$, and the edge set $E_1 = \{uv \in E(T(G)) : d_{T(G)}(u) = d_{T(G)}(v) = 2r\}; |E(T(G))| = |E_1| = \frac{nr^2}{2} + nr$.

$$Let, \psi_k(G) = \sum_{uv \in E(G)} \left(\frac{max(d_u, d_v)}{min(d_u, d_v)}\right)^k$$

$$\psi_{-\frac{1}{2}}T(G) = \sum_{uv \in E(G)} \left(\frac{2r}{2r}\right)^{-\frac{1}{2}} \left(\frac{nr^2}{2} + nr\right)$$

$$= \frac{nr^2}{2} + nr.$$

Similarly, if $k = \frac{1}{2}, 1, -1, 2, -2$ then $\psi_k(G) = \frac{nr^2}{2} + nr$.

$$Let, \xi(G) = \sum_{uv \in E(G)} \left(\frac{min(d_u, d_v)}{max(d_u, d_v)} + \frac{max(d_u, d_v)}{min(d_u, d_v)} \right)$$

$$\xi(T(G)) = \sum_{uv \in E(G)} \left(\frac{2r}{2r} + \frac{2r}{2r} \right) \left(\frac{nr^2}{2} + nr \right)$$
$$= nr^2 + 2nr.$$

□

Theorem 6.2. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. Then*

$$\psi_k(T(K_{r,s})) = \begin{cases} 3rs + \frac{rs}{2}(r+s-2), & \text{if } r=1=s \\ rs\left(\frac{s}{r} + \frac{2r}{r+s} + \frac{2s}{r+s} \right)^k + \frac{rs}{2}(r+s-2), & \text{if } r=1 \text{ and } s>1 \\ rs\left(\frac{r}{s} + \frac{r+s}{2r} + \frac{r+s}{r+s} \right)^k + \frac{rs}{2}(r+s-2), & \text{if } r>1 \text{ and } s=1 \end{cases}$$

and

$$\xi(T(K_{r,s})) = \begin{cases} 6rs + rs(r+s-2), & \text{if } r=1=s \\ \frac{1}{2}\left(6(r^2+s^2) + (r+s)^2 + 2rs(r+s-2) \right), & \text{if } r=1 \text{ and } s>1 \\ \frac{1}{2}\left(6(r^2+s^2) + (r+s)^2 + 2rs(r+s-2) \right), & \text{if } r>1 \text{ and } s=1 \end{cases}$$

Proof. Let $K_{r,s}$ be complete bipartite graph with $(r+s)$ vertices and $|V_1^*| = r, |V_2^*| = s$, $V(K_{r,s}) = V_1^* \cup V_2^*$ for $1 \leq r \leq s$ vertices. Every vertex of V_1^* is incident with s edges and every vertex of V_2^* is incident with r edges. By algebraic method, we have $|V(T(K_{r,s}))| = r + s + rs$, and $|E(T(K_{r,s}))| = \frac{1}{2}rs(r+s-2) + 3rs$.
We have three partitions of the vertex set $V(T(K_{r,s}))$ as follows:

$V_1 = \{v \in V(T(K_{r,s})) : d_{T(K_{r,s})}(v) = r\}; |V_1| = r$,

$V_2 = \{v \in V(T(K_{r,s})) : d_{T(K_{r,s})}(v) = s\}; |V_2| = s$, and

$V_3 = \{v \in V(T(K_{r,s})) : d_{T(K_{r,s})}(v) = r + s\}; |V_3| = rs$.

Also we have four partitions of the edge set $E(T(K_{r,s}))$ as follows:

$E_1 = \{uv \in E(T(K_{r,s})) : d_{T(K_{r,s})}(u) = 2s, d_G(v) = 2r\}; |E_1| = rs$,

$E_2 = \{uv \in E(T(K_{r,s})) : d_{T(K_{r,s})}(u) = 2s, d_{T(K_{r,s})}(v) = r + s\}; |E_2| = rs$,

$E_3 = \{uv \in E(T(K_{r,s})) : d_{T(K_{r,s})}(u) = 2r, d_{T(K_{r,s})}(v) = r + s\}; |E_3| = rs$, and

$E_4 = \{uv \in E(T(K_{r,s})) : d_{T(K_{r,s})}(u) = d_{T(K_{r,s})}(v) = r + s\}; |E_4| = \frac{1}{2}rs\,(r+s-2)$.

Using these values in topological indices definitions we get required results. □

7 Jump graph

The jump graph $J(G)$ of a graph G is the graph defined on $E(G)$ and in which two vertices are adjacent if and only if they are not adjacent in G.

Theorem 7.1. *Let G be a r- regular graph with $n \geq 2$ vertices. Then*

$$\psi_k(J(G)) = \frac{nr(n-2r+1)}{4}$$
and
$$\xi(J(G)) = \frac{nr(n-2r+1)}{2}$$

Proof. Let G be a r- regular graph with $n \geq 2$ vertices. By algebraic method, we have $|V(J(G))| = \frac{nr}{2}$ and $|E(J(G))| = \frac{nr(n-2r+1)}{4}$. Since jump graph of a r - regular graph is $n-2r+1$ - regular.

Theorem 7.2. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. Then*

$$\psi_k(J(K_{r,s})) = \frac{rs}{2}(rs-r-s+1)$$
and
$$\xi(J(K_{r,s})) = rs(rs-r-s+1)$$

Proof. Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. By algebraic method, we have $|V(J(K_{r,s}))| = rs$ and $|E(J(K_{r,s}))| = \frac{1}{2}rs(rs-r-s+1)$. Since jump graph of complete bipartite graph $K_{r,s}$ is a $(rs+r+s+1)$-regular graph.

$$Let, \psi_k(G) = \sum_{uv\in E(G)} \left(\frac{max(d_u,d_v)}{min(d_u,d_v)}\right)^k$$

$$\begin{aligned}\psi_{-\frac{1}{2}}J(G) &= \sum_{uv\in E(G)} \left(\frac{rs+r+s+1}{rs+r+s+1}\right)^{-\frac{1}{2}} \left(\frac{rs(rs-r-s+1)}{2}\right)\\ &= \frac{rs(rs-r-s+1)}{2}.\end{aligned}$$

Similarly, if $k = \frac{1}{2}, 1, -1, 2, -2$ then $\psi_k(G) = \frac{rs(rs-r-s+1)}{2}$.

$$Let, \xi(G) = \sum_{uv\in E(G)} \left(\frac{min(d_u,d_v)}{max(d_u,d_v)} + \frac{max(d_u,d_v)}{min(d_u,d_v)}\right)$$

$$\begin{aligned}\xi(J(G)) &= \sum_{uv\in E(G)} \left(\frac{rs+r+s+1}{rs+r+s+1} + \frac{rs+r+s+1}{rs+r+s+1}\right)\left(\frac{rs(rs-r-s+1)}{2}\right)\\ &= rs(rs-r-s+1).\end{aligned}$$

□

8 Para-line graph

Let us define the para-line graph of a graph G. Given a graph G, insert two vertices to each edge xy of G. Those two vertices will be denoted by $(x,y),(y,x)$ where (x,y) (resp. (y,x) is the one incident to x (resp. y). We define the vertex set and the edge set as follows:

$$V(P(G)) = (x,y) \in V(G) \times V(G); xy \in E(G)$$
$$E(P(G)) = (((x,w),(x,z)); (x,w),(x,z) \in V(P(G)), w \neq z) \cup ((x,y),(y,x); xy \in E(G)).$$

The resultant graph is called a para-line graph and denoted by $P(G)$.

Theorem 8.1. *Let G be a r- regular graph with $n \geq 2$ vertices. Then*

$$\psi_k(P(G)) = mr$$

and

$$\xi(P(G)) = 2mr$$

Proof. Let G be a r- regular graph with $n \leq 2$ vertices. By algebraic method, we have $|V(P(G))| = 2m$ and $|E(P(G))| = mr$. Since para-line graph of a r - regular graph is r - regular.

$$Let, \psi_k(G) = \sum_{uv \in E(G)} \left(\frac{max(d_u, d_v)}{min(d_u, d_v)} \right)^k$$

$$\psi_{-\frac{1}{2}} P(G) = \sum_{uv \in E(G)} \left(\frac{r}{r} \right)^{-\frac{1}{2}} \left(mr \right)$$

$$= mr.$$

Similarly, if $k = \frac{1}{2}, 1, -1, 2, -2$ then $\psi_k(G) = mr$.

$$Let, \xi(G) = \sum_{uv \in E(G)} \left(\frac{min(d_u, d_v)}{max(d_u, d_v)} + \frac{max(d_u, d_v)}{min(d_u, d_v)} \right)$$

$$\xi(P(G)) = \sum_{uv \in E(G)} \left(\frac{r}{r} + \frac{r}{r} \right) \left(mr) \right)$$

$$= 2mr.$$

□

Theorem 8.2. *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. Then*

$$\psi_k(P(K_{r,s})) = \begin{cases} rs + \frac{rs(r-1)}{2} + \frac{rs(s-1)}{2}, & \text{if } r=s \\ \left(\frac{s}{r}\right)^k rs + \frac{rs(r-1)}{2} + \frac{rs(s-1)}{2}, & \text{if } s>r \\ \left(\frac{r}{s}\right)^k rs + \frac{rs(r-1)}{2} + \frac{rs(s-1)}{2}, & \text{if } r>s \end{cases}$$

and

$$\xi(P(K_{r,s})) = \begin{cases} 2rs + rs(r-1) + rs(s-1), & \text{if } r=s \\ \left(\frac{s}{r} + \frac{r}{s}\right) rs + rs(r-1) + rs(s-1), & \text{if } s>r \\ \left(\frac{r}{s} + \frac{s}{r}\right) rs + rs(r-1) + rs(s-1), & \text{if } r>s \end{cases}$$

Proof. Let $K_{r,s}$ be complete bipartite graph with $(r+s)$ vertices and $|V_1^*| = r, |V_2^*| = s$, $V(K_{r,s}) = V_1^* \cup V_2^*$ for $1 \leq r \leq s$ vertices. Every vertex of V_1^* is incident with s edges and every vertex of V_2^* is incident with r edges. By algebraic method, we have $|V(P(K_{r,s}))| = 2rs$, and $|E(P(K_{r,s}))| = \frac{rs(r+s)}{2}$.
We have three partitions of the vertex set $V(P(K_{r,s}))$ as follows:

$V_1 = \{v \in V(P(K_{r,s})) : d_{P(K_{r,s})}(v) = r\}$,

$V_2 = \{v \in V(P(K_{r,s})) : d_{P(K_{r,s})}(v) = r\}$, and

$V_3 = \{v \in V(P(K_{r,s})) : d_{P(K_{r,s})}(v) = s\}$.

Also we have three partitions of the edge set $E(P(K_{r,s}))$ as follows:

$E_1 = \{uv \in E(P(K_{r,s})) : d_{P(K_{r,s})}(u) = r, d_{P(K_{r,s})}(v) = s\}; |E_1| = rs,$

$E_2 = \{uv \in E(P(K_{r,s})) : d_{P(K_{r,s})}(u) = s, d_{P(K_{r,s})}(v) = r\}; |E_2| = rs(\frac{r-1}{2})$, and

$E_3 = \{uv \in E(P(K_{r,s})) : d_{P(K_{r,s})}(u) = d_{P(K_{r,s})}(v) = s\}; |E_3| = rs(\frac{s-1}{2})$.

Applying these cardinalities in topological indices definitions we get required results. □

Remark In theorem 2.1, 2.2, 6.1,7.1, 7,2, 8.1 we notice that that, min-max type degree index of graphs (line, total, jump, para-line) values are expressed as twice of the corresponding SDD index.

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