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4-Prime cordiality of m copies of some graphs

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Abstract. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a map. For each edge uv, assign the label gcd $(f(u), f(v))$. f is called k-prime cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, $i, j \in \{1, 2, \ldots, k\}$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labeled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labeled with 1 and not labeled with 1. A graph with a k-prime cordial labeling is called a k-prime cordial graph. In this paper we investigate 4-prime cordial labeling behavior of complete graph, book, flower, mC_n and some more graphs.

1 Introduction

All graphs in this paper are finite, simple and undirected. Let G be a (p, q) graph where p refers the number of vertices of G and q refers the number of edge of G. The number of vertices of a graph G is called order of G, and the number of edges is called size of G. In 1987, Cahit introduced the concept of cordial labeling of graphs [1]. Sundaram, Ponraj, Somasundaram [5] have introduced the notion of prime cordial labeling. A prime cordial labeling of a graph G with vertex set V is a bijection $f : V(G) \to \{1, 2, \ldots, |V|\}$ such that if each edge uv is assigned the label 1 if gcd $(f(u), f(v)) = 1$ and 0 if gcd $(f(u), f(v)) > 1$, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. Also they discussed the prime cordial labeling behavior of various graphs. Recently Ponraj et al. [7], introduced k-prime cordial labeling of graphs. In this paper we investigate 4-prime cordial labeling behavior of complete graph, book, flower, mC_n and some more graphs. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for smallest integer greater than or equal to x. Terms not defined here follow from Harary [3] and Gallian [2].

2 4-Prime cordial labeling

Remark 2.1. A 2-prime cordial labeling is a product cordial labeling. [6]

Definition 2.2. The Join of two graphs $G_1 + G_2$ is obtained from G_1 and G_2 and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$.

Definition 2.3. The graph $W_n = C_n + K_1$ is called a wheel. In a wheel, a vertex of degree 3 is called a rim vertex. A vertex which is adjacent to all the rim vertices is called the central vertex. The edges with one end incident with the rim and the other incident with the central vertex are called spokes.

Definition 2.4. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.

Definition 2.5. The graph $P_n \odot K_1$ is called a comb.

Definition 2.6. The graph $C_n \odot K_1$ is called a crown.

Definition 2.7. The Cartesian product graph $G_1 \square G_2$ is defined as follows: Consider any two points $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \square V_2$. Then u and v are adjacent in $G_1 \square G_2$

whenever $[u_1 = v_1$ and $u_2v_2 \in E(G_2)]$ or $[u_2 = v_2$ and $u_1v_1 \in E(G_1)]$. The graph $L_n = P_n \square P_2$ is called a ladder.

Theorem 2.8. m copies of the complete bipartite graph $K_{n,n}$ is 4-prime cordial for all even values of m.

Proof. It is easy to verify that K_1, K_2, K_3 are 4-prime cordial graphs. On the other hand suppose there exists a 4-prime cordial labeling f for $n \geq 4$ then we have the following cases.Let $V(K_{n,n}) = V_1 \cup V_2$. Consider the first copy. Assign the label 2 to the all the vertices of V_1 and 4 to the all the vertices of V_2. Next consider the second copy. In this copy assign the label 1 to the all the vertices of V_1 and 3 to the all the vertices of V_2. In the third copy assign the label to the vertices as in the first copy and the fourth copy as in the second copy. Proceeding like this assign the labels to the consecutive copies. That is assign the label to the vertices of the i^{th} copy as in $(i-2)^{th}$ copy $(3 \leq i \leq m)$. his vertex labels is obviously a 4-prime cordial labeling since $v_f(1) = v_f(2) = v_f(3) = v_f(4) = \frac{mn}{2}$ and $e_f(0) = e_f(1) = \frac{mn^2}{2}$. □

Theorem 2.9. (i) mL_n is 4-prime cordial for all even values of m and all values of n.
(ii) If m is odd, then mL_n is 4-prime for all values of n.

Proof. Clearly mL_n has and edges. The proof is divided into two cases depends on the parity of m.

Case 1. m is even.

In the first copy assign the label 2 to all the left side vertices and 4 to the all the right side vertices. We now move to the second copy. In this case assign the label as in previous second copy. Proceeding like this we assign the $\frac{m}{2}$ copies. Next consider the $\frac{m}{2} + 1^{th}$ copy. In this copy assign the labels 1,3 to the first two vertices of the left side. Then assing the next two vertices and so on. That is assign the label to the left sides as in the pattern 1,3,1,3,1,3...1,3. Similarly assign the right sides in the way 3,1,3,1,3,1,....,3,1. This procedure is continue the remaining $\frac{m}{2} - 1$ ladders. If f is this vertex labeling then $v_f(1) = v_f(2) = v_f(3) = v_f(4) =$ and $e_f(0) = e_f(1) =$.

Case 2. m is odd.

As in case 1 assign the labels to the all the vertices of the first, second, third, ..., $\frac{(m-1)}{2}^{th}$ copy. We now consider the last m^{th} copy. In the first column assign the label 2 to the first $\frac{n+1}{2}$ vertices and in the second column assign the label 4 to the first $\frac{n+1}{2}$. In the $\frac{n+3}{2}^{th}$ vertices of the first column is labelled by 1 and assign the label 3 to the $\frac{n+5}{2}^{th}$ vertices of the second copy. Next assign the label 1 to the $\frac{n+5}{2}^{th}$ vertices and 3 to the $\frac{n+3}{2}^{th}$ vertices of the first column. That is it is labelled as 1,3,1,3,1,3,...,1,3. In the second column the vertices $\frac{n+1}{2}^{th}$, $\frac{n+3}{2}^{th}$, ... vertices are labelled in the pattern 3,1,3,1,3,1,...,3,1. This verte labeling f is a 4-prime cordial labeling follows from Table 1, 2 □

Nature of m and n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$
m is even	$\frac{mn}{2}$	$\frac{mn}{2}$	$\frac{mn}{2}$	$\frac{mn}{2}$
m is odd and n is oddd	$\frac{mn-1}{2}$	$\frac{mn+1}{2}$	$\frac{mn-1}{2}$	$\frac{mn+1}{2}$

Table 1.

Nature of m and n	$e_f(0)$	$e_f(1)$
m is even	$\frac{m(3n-2)}{2}$	$\frac{m(3n-2)}{2}$
m is odd and n is oddd	$\frac{m(3n-2)+1}{2}$	$\frac{m(3n-2)-1}{2}$

Table 2.

Theorem 2.10. m copies of the comb $C_n \odot K_1$ is 4-prime cordial for all values of n.

Proof. **Case 1.** m is even.
Consider the first copy. Assign the label 2 to the all the pendent vertices and 4 to other vertices. Next we move the second copy. Assign the label 3 to all its pendent vertices and 1 to remaining vertices of this copy. In a similar way assign the label to the vertices of the third copy as in the first one and the fourth copy as in second copy. In general the vertices of the i^{th} copy are labelled as in $(i-2)^{th}$ copy $(3 \leq i \leq m)$.
Case 2. m is odd.
Let f be the 4-prime cordial labeling of m-1 copies of the comb $C_n \odot K_1$ as in theorem. Then the vertex labeling fvg is defined by

$$fvg(u) = \begin{cases} f(u) & \text{if} \quad \text{u in m-1 copies of } C_n \odot K_1 \\ g(u) & \text{if} \quad \text{u in } m^{th} \text{ copies of } C_n \odot K_1 \end{cases}$$

Obviously fvg is a 4-prime cordial labeling of $m(C_n \odot K_1)$. □

Theorem 2.11. $m(B_{n,n})$ is 4-prime cordial for all values m.

Proof. Let $V(mB_{n,n}) = \{u_i, v_i, u_i^j, v_i^j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(mB_{n,n}) = \{u_i u_i^j, u_i v_i, v_i v_i^j : 1 \leq i \leq m, 1 \leq j \leq n\}$. Clearly the order and size of $m(B_{n,n})$ are $2mn+2m$ and $2mn+2m-1$ respectively.
Case 1. m is even.
Consider the first copy. Assign the label 2,4 respectively to the vertices u_1 and v_1. Next asign the label 2 to the all the pendent vertices u_1^j $(1 \leq i \leq n)$ and assign the label 4 to the pendent vertices v_i^j $(1 \leq j \leq n)$. Next we consider the second copy. In this copy assign the label 1,3 to th vertices u_2 and v_2 respectively. Then assign the label 3 to the pendent vertices u_2^j $(1 \leq j \leq n)$ and 1 to the vertices v_2^j $(1 \leq j \leq n)$. Next assign the label to the vertices of the third copy as in first copy and fourth copy as in second copy and so on. That is assign the label to the vertices of the i^{th} copy as in $(i-2)^{th}$ copy. It is easy to verify that this vertex labeling is a 4-prime cordial labeling.
Case 2. m is odd.
Assign the label to the vertices of first, second, ... , $(m-1)^{th}$ copy as in case 1. As in theorem assign the m^{th} copy we get a 4-prime cordial labeling of $m(B_{n,n})$. □

Theorem 2.12. mB_n is 4-prime cordial labeling for all values of m.

Proof. Let $V(mB_n) = \{u_i, v_i, u_i^j, v_i^j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(mB_n) = \{u_i v_i, u_i u_i^j, v_i v_i^j : 1 \leq i \leq n, 1 \leq j \leq m\}$. Consider the first copy. Assign the label 2,4 respectively to the vertices u_1 and v_1. Next assign the label 2 to the vertices $u_1^1, u_2^1, u_3^1 ... u_n^1$ and 4 to the vertices $v_1^1, v_2^1, v_3^1 ... v_n^1$. We now move to the second copy. Assign the label 1,3 to the vertices vertices u_2 and v_2 respectively. Next assign the label 3 to the vertices $u_1^2, u_2^2, u_3^2 ... u_n^2$ and 1 to the vertices $v_1^2, v_2^2, v_3^2 ... v_n^2$. Next consider the i^{th} copy $(i \geq 3)$. Assign the label to the vertices of the i^{th} copy as in the $(i-2)^{th}$ copy $(3 \leq i \leq m)$. This vertex labeling f is a 4-prime cordial labeling since $v_f(1) = v_f(2) = v_f(3) = v_f(4) = \frac{m}{2}(n+1)$ and $e_f(0) = e_f(1) = \frac{m}{2}(2n+1)$.
Case 2. m is odd.
The 4-prime cordial labeling f of first $(m-1)$ copies together with the 4-prime cordial labeling of $B_{n,n}$ as in theorem we get a 4-prime cordial labeling in this case. □

Theorem 2.13. If n is odd, then $W_n \cup W_n$ is 4-prime cordial.

Proof. Clealrly $W_n \cup W_n$ has 2n+2 vertices and 4n edges. Consider the first copy. Assin the label 2 to the central vertex and 4 to the first $\frac{n+1}{2}$ consecutive rim vertices. Next assign 2 to the reaining $\frac{n-1}{2}$ rim vertices of this copy. We now move to the second copy. In this copy assign the label 1 to the central vertex. Assign the label 3,1 to the next two rim vertices . Then assign the label 3,1 to the next two rim vertices and 3,1 to the next two contiunous in this process until we reach the last vertex. That is the rim vertices are labelled in the pattern 3,1,3,1,3,1... 3. It is easy to verify that the last vertex reveiced the label 3. For this vertex labeling all the edges in the first copy received the label 0 and second copy received the label 1. Hence $W_n \cup W_n$ is 4-prime cordial for the odd values n. □

Theorem 2.14. Let P_n be the path $u_1u_2...u_n$. Then the graph Q_n with the set $V(Q_n) = V(P_n) \cup \{v_i, w_i : 1 \leq i \leq n-1\}$ and $E(Q_n) = E(P_n) \cup \{v_iw_i : 1 \leq i \leq n\} \cup \{u_iv_i : 1 \leq i \leq n-1\} \cup \{u_{i+1}w_i : 1 \leq i \leq n-1\}$ is 4-prime cordial.

Proof. Clearly Q_n has 3n-2 vertices 4n-4 edges.
Case 1. n is odd.
First we consider path. Assign the labels 2,4 respectively to the vertices u_1 and u_2. Next assign the labels 2,4 to the vertices u_3, u_4 and so on until we reach the vertex $u_{\frac{n+1}{2}}$. That is the vertices $u_1, u_2, ..., u_n$ are labeled as 2,4,2,4,...,2,4,2. Note that $u_{\frac{n+1}{2}}$ received the label 2. Assign the label 3,1 to the next two vertices $u_{\frac{n+3}{2}}$ and $u_{\frac{n+5}{2}}$. Then assign the label 3,1 to the next two vertices $u_{\frac{n+7}{2}}$, $u_{\frac{n+9}{2}}$. Proceeding like this until we reach the vertex u_n. We now move to the vertices v_i and w_i. Assign the label 2 to the vertices $v_1, v_2, ..., v_{\frac{n-1}{2}}$ and 4 to the vertices $w_1, w_2, ..., w_{\frac{n-1}{2}}$. Next assign the labels 3,1 to the vertices $v_{\frac{n+1}{2}}$, $v_{\frac{n+3}{2}}$ and assign 3,1 to the next two vertices $v_{\frac{n+5}{2}}$, $v_{\frac{n+7}{2}}$ and so on. Proceeding like this until we reach the vertex u_n. In this process the vertices $u_{\frac{n+1}{2}}, u_{\frac{n+3}{2}}, ...u_n$ are received the labels 3,1,3,1.... Similarly assign the labels to the vertices $w_{\frac{n+1}{2}}, w_{\frac{n+3}{2}}$ ... in the pattern 1,3,1,3...
Case 2. n is even.
As in case 1 assign the labels to the vertices u_i $(1 \leq i \leq n-1)$, v_i and w_i $(1 \leq i \leq n-2)$. Next assign the labels 3,4 and 1 respectively to the vertices v_{n-1}, w_{n-1} and u_n respectiely. If $n \equiv 0 \pmod 4$ then interchange the labels of v_{n-1} and u_{n-1}. If $n \equiv 2 \pmod 4$ then interchange the labels of w_{n-2} and v_{n-1}. The table 3 establish that this vertex labeling is a 4-prime cordial labeling of Q_n. □

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
$n \equiv 1 \pmod 4$	$\frac{3n-3}{4}$	$\frac{3n+1}{4}$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$	$2n-2$	$2n-2$
$n \equiv 3 \pmod 4$	$\frac{3n-5}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$2n-2$	$2n-2$
$n \equiv 0 \pmod 4$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n+3}{4}$	$\frac{3n+3}{4}$	$2n-2$	$2n-2$
$n \equiv 2 \pmod 4$	$\frac{3n+1}{4}$	$\frac{3n+1}{4}$	$\frac{3n+1}{4}$	$\frac{3n+1}{4}$	$2n-2$	$2n-2$

Table 3.

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