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SOME GROWTH ANALYSIS OF ENTIRE FUNCTIONS OF SEVERAL VARIABLES ON THE BASIS OF THEIR (p, q)-TH RELATIVE GOL'DBERG ORDER AND (p, q)-TH RELATIVE GOL'DBERG TYPE

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Abstract In this paper we study some growth properties of entire functions of several complex variables on the basis of their (p, q)-th relative Gol'dberg order and (p, q)-th relative Gol'dberg type entire functions of several complex variables where p and q are any positive integers.

1 Introduction and Definitions

Let $\mathbb{C}^n$ and R^n respectively denote the *complex* and *real* n*-space*. Also let us indicate the point $(z_1, z_2, \cdots, z_n)$, $(m_1, m_2, \cdots, m_n)$ of $\mathbb{C}^n$ or I^n by their corresponding unsuffixed symbols z, m respectively where I denotes the set of non-negative integers. The *modulus* of z, denoted by $|z|$, is defined as $|z| = \left(|z_1|^2 + \cdots + |z_n|^2\right)^{\frac{1}{2}}$. If the coordinates of the vector m are non-negative integers, then z^m will denote $z_1^{m_1} \cdots z_n^{m_n}$ and $\|m\| = m_1 + \cdots + m_n$.

If $D \subseteq \mathbb{C}^n$ ($\mathbb{C}^n$ denote the n*-dimensional complex space*) be an arbitrary *bounded complex* n*-circular domain* with center at the origin of coordinates then for any entire function $f(z)$ of n complex variables and $R > 0$, $M_{f,D}(R)$ may be define as $M_{f,D}(R) = \sup z \in D_R |f(z)|$ where a point $z \in D_R$ if and only if $\frac{z}{R} \in D$. If $f(z)$ is non-constant, then $M_{f,D}(R)$ is strictly increasing and its inverse $M_{f,D}^{-1} : (|f(0)|, \infty) \to (0, \infty)$ exists such that $\lim\limits_{R\to\infty} M_{f,D}^{-1}(R) = \infty$.

Considering this, the *Gol'dberg order* (resp. *Gol'dberg lower order*) {cf. [4], [5]} of an entire function $f(z)$ with respect to any *bounded complete* n*-circular domain* D is given by

$$\rho_{f,D} = \overline{\lim_{R\to+\infty}} \frac{\log^{[2]} M_{f,D}(R)}{\log R} (\text{resp. } \lambda_f = \underline{\lim_{R\to+\infty}} \frac{\log^{[2]} M_{f,D}(R)}{\log R}) .$$

where $\log^{[k]} R = \log\left(\log^{[k-1]} R\right)$ for $k = 1, 2, 3, \cdots$; $\log^{[0]} R = R$ and $\exp^{[k]} R = \exp\left(\exp^{[k-1]} R\right)$ for $k = 1, 2, 3, \cdots$; $\exp^{[0]} R = R$.

It is well known that $\rho_{f,D}$ is independent of the choice of the domain D, and therefore we write ρ_f instead of $\rho_{f,D}$ (resp. λ_f instead of $\lambda_{f,D}$){cf. [4], [5]} .

To compare the relative growth of two entire functions of n-complex variables having same non zero finite *Gol'dberg order*, one may introduce the definition of *Gol'dberg type* and *Gol'dberg lower type* in the following manner:

Definition 1.1. {cf. [4], [5]} The *Gol'dberg type* and *Gol'dberg lower type* respectively denoted by $\Delta_{f,D}$ and $\overline{\Delta}_{f,D}$ of an entire function $f(z)$ of n-complex variables with respect to any *bounded complete* n*-circular domain* D are defined as follows:

$$\Delta_{f,D} = \overline{\lim_{R\to+\infty}} \frac{\log M_{f,D}(R)}{(R)^{\rho_f}} \quad \text{and} \quad \overline{\Delta}_{f,D} = \underline{\lim_{R\to+\infty}} \frac{\log M_{f,D}(R)}{(R)^{\rho_f}},\ 0 < \rho_f < +\infty .$$

Analogously to determine the relative growth of two entire functions of n-complex variables having same non zero finite *Gol'dberg lower order,* one may introduce the definition of *Gol'dberg weak type* in the following way:

Definition 1.2. The *Gol'dberg weak type* denoted by $\tau_{f,D}$ of an entire function $f(z)$ of n-complex variables with respect to any *bounded complete n-circular domain D* is defined as follows:

$$\tau_{f,D} = \lim_{R\to+\infty} \frac{\log M_{f,D}(R)}{(R)^{\lambda_f}} \ , \ 0 < \ \lambda_f \ < +\infty \ .$$

Also one may define the growth indicator $\overline{\tau}_{f,D}$ in the following manner :

$$\overline{\tau}_{f,D} = \overline{\lim_{R\to+\infty}} \frac{\log M_{f,D}(R)}{(R)^{\lambda_f}} \ , \ 0 < \ \lambda_f \ < +\infty$$

Gol'dberg has shown that [5] *Gol'dberg type* depends on the domain D. Hence all the growth indicators define in Definition 1.1 and Definition 1.2 are also depend on D.

However, extending the notion of *Gol'dberg order*, Datta and Maji [1] defined the concept of (p,q)-*th Gol'dberg order* (resp. (p,q)-*th Gol'dberg lower order)* of an entire function $f(z)$ for any *bounded complete n-circular domain D* where $p \geq q \geq 1$ in the following way:

$$\rho_{f,D}(p,q) = \overline{\lim_{R\to+\infty}} \frac{\log^{[p]} M_{f,D}(R)}{\log^{[q]} R} = \overline{\lim_{R\to+\infty}} \frac{\log^{[p]} R}{\log^{[q]} M_{f,D}^{-1}(R)}$$

$$(\text{resp. } \lambda_{f,D}(p,q) = \underline{\lim_{R\to+\infty}} \frac{\log^{[p]} M_{f,D}(R)}{\log^{[q]} R} = \underline{\lim_{R\to+\infty}} \frac{\log^{[p]} R}{\log^{[q]} M_{f,D}^{-1}(R)}) \ .$$

These definitions extended the *generalized Gol'dberg order* $\rho_{f,D}^{[l]}$ (resp. *generalized Gol'dberg lower order* $\lambda_{f,D}^{[l]}$) of an entire function $f(z)$ for any *bounded complete n-circular domain D* for each integer $l \geq 2$ since these correspond to the particular case $\rho_{f,D}^{[l]} = \rho_{f,D}(l,1)$ (resp. $\lambda_{f,D}^{[l]} = \lambda_{f,D}(l,1)$). Clearly $\rho_{f,D}(2,1) = \rho_{f,D}$ (resp. $\lambda_{f,D}(2,1) = \lambda_{f,D}$). Further in the line of Gol'dberg {cf. [4], [5]}, one can easily verify that $\rho_{f,D}(p,q)$ (resp. $\lambda_{f,D}(p,q)$) is independent of the choice of the domain D, and therefore one can write $\rho_f(p,q)$ (resp. $\lambda_f(p,q)$) instead of $\rho_{f,D}(p,q)$ (resp. $\lambda_{f,D}(p,q)$).

In this connection let us recall that if $0 < \rho_f(p,q) < \infty$, then the following properties hold

$$\rho_f(p-n,q) = \infty \text{ for } n < p, \ \rho_f(p,q-n) = 0 \text{ for } n < q, \text{ and}$$

$$\rho_f(p+n,q+n) = 1 \text{ for } n = 1,2,...$$

Similarly for $0 < \lambda_f(p,q) < \infty$, one can easily verify that

$$\lambda_f(p-n,q) = \infty \text{ for } n < p, \ \lambda_f(p,q-n) = 0 \text{ for } n < q, \text{ and}$$

$$\lambda_f(p+n,q+n) = 1 \text{ for } n = 1,2,.... \ .$$

Recalling that for any pair of integer numbers m,n the Kroenecker function is defined by $\delta_{m,n} = 1$ for $m = n$ and $\delta_{m,n} = 0$ for $m \neq n$, the aforementioned properties provide the following definition.

Definition 1.3. For any bounded complete n-circular domain D, an entire function $f(z)$ of n-complex variables is said to have index-pair $(1,1)$ if $0 < \rho_f(1,1) < \infty$. Otherwise, $f(z)$ is said to have index-pair $(p,q) \neq (1,1)$, $p \geq q \geq 1$, if $\delta_{p-q,0} < \rho_f(p,q) < \infty$ and $\rho_f(p-1,q-1) \notin R^+$.

Definition 1.4. For any bounded complete n-circular domain D, an entire function $f(z)$ of n-complex variables is said to have lower index-pair $(1,1)$ if $0 < \lambda_f(1,1) < \infty$. Otherwise, $f(z)$ is said to have lower index-pair $(p,q) \neq (1,1)$, $p \geq q \geq 1$, if $\delta_{p-q,0} < \lambda_f(p,q) < \infty$ and $\lambda_f(p-1,q-1) \notin R^+$.

To compare the relative growth of two entire functions having same non zero finite (p,q)-*Gol'dberg order*, one may introduce the definition of (p,q)-*Gol'dberg type* and (p,q)-*Gol'dberg lower type* in the following manner:

Definition 1.5. The (p,q)-*th Gol'dberg type* and (p,q)-th *Gol'dberg lower type* respectively denoted by $\Delta_{f,D}(p,q)$ and $\overline{\Delta}_{f,D}(p,q)$ of an entire function $f(z)$ of n-complex variables with respect to any *bounded complete* n*-circular domain* D are defined as follows:

$$\Delta_{f,D}(p,q) = \overline{\lim_{R\to+\infty}} \frac{\log^{[p-1]} M_{f,D}(R)}{\left[\log^{[q-1]} R\right]^{\rho_f(p,q)}}$$

$$\text{and } \overline{\Delta}_{f,D}(p,q) = \underline{\lim}_{R\to+\infty} \frac{\log^{[p-1]} M_{f,D}(R)}{\left[\log^{[q-1]} R\right]^{\rho_f(p,q)}}, \; 0 < \rho_f(p,q) < +\infty,$$

where $p \geq q \geq 1$.

Analogously to determine the relative growth of two entire functions of n-complex variables having same non zero finite (p,q)-th *Gol'dberg lower order,* one may introduce the definition of (p,q)- th *Gol'dberg weak type* in the following way:

Definition 1.6. The (p,q)- th *Gol'dberg weak type* denoted by $\tau_{f,D}(p,q)$ of an entire function $f(z)$ of n-complex variables with respect to any *bounded complete* n*-circular domain* D is defined as follows:

$$\tau_{f,D}(p,q) = \underline{\lim}_{R\to+\infty} \frac{\log^{[p-1]} M_{f,D}(R)}{\left[\log^{[q-1]} R\right]^{\lambda_f(p,q)}}, \; 0 < \lambda_f(p,q) < +\infty .$$

Also one may define the growth indicator $\overline{\tau}_{f,D}(p,q)$ in the following manner :

$$\overline{\tau}_{f.D}(p,q) = \overline{\lim_{R\to+\infty}} \frac{\log^{[p-1]} M_f(R)}{\left[\log^{[q-1]} R\right]^{\lambda_f(p,q)}}, \; 0 < \lambda_f(p,q) < +\infty,$$

where $p \geq q \geq 1$.

Definition 1.5 and Definition 1.6 are extended the *generalized Gol'dberg type* $\Delta^{[l]}_{f,D}$ (resp.*generalized Gol'dberg lower type* $\overline{\Delta}^{[l]}_{f,D}$) and *generalized Gol'dberg weak order* $\tau^{[l]}_{f,D}$ of an entire function $f(z)$ of n-complex variables with respect to any *bounded complete* n*-circular domain* D for each integer $l \geq 2$ since these correspond to the particular case $\Delta^{[l]}_{f,D} = \Delta_{f,D}(l,1)$ (resp. $\overline{\Delta}^{[l]}_{f,D} = \overline{\Delta}_f(l,1)$) and $\tau^{[l]}_{f,D} = \tau_f(l,1)$ (resp. $\overline{\tau}^{[l]}_{f,D} = \overline{\tau}_f(l,1)$). Clearly $\Delta_{f,D}(2,1) = \Delta_{f,D}$ (resp. $\overline{\Delta}_{f,D}(2,1) = \overline{\Delta}_{f,D}$) and $\tau_{f,D}(2,1) = \tau_{f,D}$ (resp. $\overline{\tau}_{f,D}(2,1) = \overline{\tau}_{f,D}$).

Since Gol'dberg has shown that [5] *Gol'dberg type* depends on the domain D, therefore all the growth indicators define in Definition 1.5 and Definition 1.6 are also depend on D.

For any two entire functions $f(z)$ and $g(z)$ of n-complex variables and for any bounded complete n-circular domain D with center at all the origin $\mathbb{C}^n$, Mondal and Roy [6] introduced the concept *relative Gol'dberg order* which is as follows:

$$\begin{aligned}\rho_{g,D}(f) &= \inf\{\mu > 0 : M_{f,D}(R) < M_{g,D}(R^\mu) \text{ for all } R > R_0(\mu) > 0\} \\ &= \overline{\lim_{R\to+\infty}} \frac{\log M^{-1}_{g,D} M_{f,D}(R)}{\log R} .\end{aligned}$$

In [6], Mandal and Roy also proved that the *relative Gol'dberg order* of $f(z)$ with respect to $g(z)$ is independent of the choice of the domain D. So the *relative Gol'dberg order* of $f(z)$ with respect to $g(z)$ may be denoted as $\rho_g(f)$ instead of $\rho_{g,D}(f)$.

Likewise, one can define the *relative Gol'dberg lower order* $\lambda_{g,D}(f)$ in the following manner:

$$\lambda_{g,D}(f) = \varliminf_{R\to+\infty} \frac{\log M_{g,D}^{-1} M_{f,D}(R)}{\log R}.$$

In the line of Mandal and Roy {cf. [6]} , one can also verify that $\lambda_{g,D}(f)$ is independent of the choice of the domain D, and therefore one can write $\lambda_g(f)$ instead of $\lambda_{g,D}(f)$.

In the case of *relative Gol'dberg order,* it therefore seems reasonable to define suitably the (p,q)*-th relative Gol'dberg order* of entire function of n*-complex variables* and for any *bounded complete* n*-circular domain* D with center at the origin in $\mathbb{C}^n$. With this in view one may introduce definition of (p,q)*-th relative Gol'dberg order* $\rho_{g,D}^{(p,q)}(f)$ of an entire function $f(z)$ with respect to another entire function $g(z)$ where both $f(z)$ and $g(z)$ are of n-complex variables and D be any bounded complete n-circular domain with center at the origin in $\mathbb{C}^n$, in the light of index-pair. Our next definition avoids the restriction $p > q$ and gives the more natural particular case of *Generalized Gol'dberg order* i.e, $\rho_{g,D}^{[l,1]}(f) = \rho_{g,D}^{[l]}(f)$.

Definition 1.7. Let $f(z)$ and $g(z)$ be any two entire functions of n-complex variables with index-pair (m,q) and (m,p), respectively, where p,q,m are positive integers such that $m \geq q \geq 1$ and $m \geq p \geq 1$ and D be any bounded complete n-circular domain with center at the origin in $\mathbb{C}^n$. Then the (p,q)-th relative Gol'dberg order of $f(z)$ with respect to $g(z)$ is defined as

$$\begin{aligned}
\rho_{g,D}^{(p,q)}(f) &= \inf \left\{ \begin{array}{c} \mu > 0 : M_{f,D}(r) < M_{g,D}\left(\exp^{[p]}\left(\mu \log^{[q]} R\right)\right) \\ \text{for all } R > R_0(\mu) > 0 \end{array} \right\} \\
&= \varlimsup_{R\to+\infty} \frac{\log^{[p]} M_{g,D}^{-1} M_{f,D}(R)}{\log^{[q]} R} = \varlimsup_{R\to+\infty} \frac{\log^{[p]} M_{g,D}^{-1}(R)}{\log^{[q]} M_{f,D}^{-1}(R)}.
\end{aligned}$$

Similarly, the (p,q)-th relative Gol'dberg lower order of $f(z)$ with respect to $g(z)$ is defined by:

$$\lambda_{g,D}^{(p,q)}(f) = \varliminf_{R\to+\infty} \frac{\log^{[p]} M_{g.D}^{-1} M_{f,D}(R)}{\log^{[q]} R} = \varliminf_{R\to+\infty} \frac{\log^{[p]} M_{g,D}^{-1}(R)}{\log^{[q]} M_{f,D}^{-1}(R)}.$$

In the line of Mandal and Roy {cf. [6]} one may prove that $\rho_{g,D}^{(p,q)}(f)$ (resp. $\lambda_{g,D}^{(p,q)}(f)$) is independent of the choice of the domain D, and therefore one can write $\rho_g^{(p,q)}(f)$ (resp. $\lambda_g^{(p,q)}(f)$) instead of $\rho_{g,D}^{(p,q)}(f)$ (resp. $\lambda_{g,D}^{(p,q)}(f)$).

Next we introduce the definition of (p,q)*-relative Gol'dberg type* and (p,q)*-relative Gol'dberg lower type* in order to compare the relative growth of two entire functions of n-complex variables having same non zero finite (p,q)*-relative Gol'dberg order* with respect to another entire function of n-complex variables.

Definition 1.8. Let $f(z)$ and $g(z)$ be any two entire functions of n-complex variables with index-pair (m,q) and (m,p), respectively, where p,q,m are positive integers such that $m \geq q \geq 1$ and $m \geq p \geq 1$ and D be any bounded complete n-circular domain with center at the origin in $\mathbb{C}^n$. Then the (p,q)*- relative Gol'dberg type* and (p,q)*- relative Gol'dberg lower type* of $f(z)$ with respect to $g(z)$ are defined as

$$\begin{aligned}
\Delta_{g,D}^{(p,q)}(f) &= \varlimsup_{R\to+\infty} \frac{\log^{[p-1]} M_{g,D}^{-1} M_{f,D}(R)}{\left[\log^{[q-1]} R\right]^{\rho_g^{(p,q)}(f)}} \text{ and} \\
\overline{\Delta}_{g,D}^{(p,q)}(f) &= \varliminf_{R\to+\infty} \frac{\log^{[p-1]} M_{g,D}^{-1} M_{f,D}(R)}{\left[\log^{[q-1]} R\right]^{\rho_g^{(p,q)}(f)}}, \ 0 < \rho_g^{(p,q)}(f) < +\infty.
\end{aligned}$$

Analogously to determine the relative growth of two entire functions of n-complex variables having same non zero finite $p,q)$*-th relative Gol'dberg lower order* with respect to another entire function of n-complex variables, one may introduce the definition of (p,q)- *th relative Gol'dberg weak type* in the following way:

Definition 1.9. Let $f(z)$ and $g(z)$ be any two entire functions of n-complex variables with index-pair (m,q) and (m,p) , respectively, where p,q,m are positive integers such that $m \geq q \geq 1$ and $m \geq p \geq 1$ and D be any bounded complete n-circular domain with center at the origin in $\mathbb{C}^n$. Then (p,q)- *th relative Gol'dberg weak type* denoted by $\tau_{g,D}^{(p,q)}(f)$ of an entire function $f(z)$ with respect to another entire function $g(z)$ is defined as follows:

$$\tau_{g,D}^{(p,q)}(f) = \underline{\lim}_{R\to+\infty} \frac{\log^{[p-1]} M_{g,D}^{-1} M_{f,D}(R)}{\left[\log^{[q-1]} R\right]^{\lambda_g^{(p,q)}(f)}}, \ 0 < \ \lambda_g^{(p,q)}(f) \ < +\infty .$$

Similarly the growth indicator $\overline{\tau}_{g,D}^{(p,q)}(f)$ of an entire function $f(z)$ with respect to another entire function $g(z)$ both of n-complex variables in the following manner :

$$\overline{\tau}_{g,D}^{(p,q)}(f) = \overline{\lim}_{R\to+\infty} \frac{\log^{[p-1]} M_{g,D}^{-1} M_{f,D}(R)}{\left[\log^{[q-1]} R\right]^{\lambda_g^{(p,q)}(f)}}, \ 0 < \ \lambda_g^{(p,q)}(f) \ < +\infty .$$

Therefore, for any two entire functions $f(z)$ and $g(z)$ both of n-complex variables, we note that

$$\rho_g^{(p,q)}(f) \neq \lambda_g^{(p,q)}(f), \Delta_{g,D}^{(p,q)}(f) > 0 \Rightarrow \overline{\tau}_{g,D}^{(p,q)}(f) = +\infty \text{ and}$$
$$\rho_g^{(p,q)}(f) \neq \lambda_g^{(p,q)}(f), \overline{\Delta}_{g,D}^{(p,q)}(f) > 0 \Rightarrow \tau_{g,D}^{(p,q)}(f) = +\infty .$$

Since Gol'dberg has shown that [5] *Gol'dberg type* depends on the domain D. Hence all the growth indicators define in Definition 1.8 and Definition 1.9 are also depend on D.

If $f(z)$ and $g(z)$ both of n-complex variables have got index-pair $(m,1)$ and (m,l) , respectively, then the above two definitions reduces to the definition of *generalized relative Gol'dberg type* $\Delta_{g,D}^{[l]}(f)$ (resp *generalized relative Gol'dberg lower type* $\overline{\Delta}_{g,D}^{[l]}(f)$) and *generalized relative Gol'dberg weak type* $\tau_{g,D}^{[l]}(f)$. If the entire functions $f(z)$ and $g(z)$ (both of n-complex variables) have the same index-pair $(p,1)$ where p is any positive integer, we get the definitions of *relative Gol'dberg type* as introduced by Roy [7] (resp.*relative Gol'dberg lower type*) and *relative Gol'dberg weak type*.

During the past decades, several authors {cf. [1],[2],[3],[6],[7], [8]} made closed investigations on the properties of entire functions of several complex variables using different growth indicator such as *Gol'dberg order,* (p,q)*-th Gol'dberg order* etc. In this paper we wish to establish some growth properties of entire functions of several complex variables (all the entire functions under consideration will be transcendental unless otherwise stated) on the basis of their (p,q)-th relative Gol'dberg order and (p,q)-th relative Gol'dberg type where p and q are any positive integers.

2 Main Results

In this section we present the main results of the paper.

Theorem 2.1. *Let $f(z)$, $g(z)$ and $h(z)$ be any three entire functions of n- complex variables and D be a bounded complete n-circular domain with center at origin in $\mathbb{C}^n$. Also let $0 < \overline{\sigma}_{h,D}^{(p,m)}(f) \leq \sigma_{h,D}^{(p,m)}(f) < \infty$, $0 < \overline{\sigma}_{k,D}^{(q,m)}(g) \leq \sigma_{k,D}^{(q,m)}(g) < \infty$ and $\rho_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$ where p,q,m are all positive integers. Then*

$$\frac{\overline{\sigma}_{h,D}^{(p,m)}(f)}{\sigma_{k,D}^{(q,m)}(g)} \leq \liminf_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\overline{\sigma}_{h,D}^{(p,m)}(f)}{\overline{\sigma}_{k,D}^{(q,m)}(g)}$$
$$\leq \limsup_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\sigma_{h,D}^{(p,m)}(f)}{\overline{\sigma}_{k,D}^{(q,m)}(g)} .$$

Proof. From the definition of $\sigma_{k,D}^{(q,m)}(g)$ and $\overline{\sigma}_{h,D}^{(p,m)}(f)$, we have for arbitrary positive ε and for all large values of R that

$$\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R) \geqslant \left(\overline{\sigma}_{h,D}^{(p,m)}(f) - \varepsilon\right)\left(\log^{[m-1]} R\right)^{\rho_h^{(q,m)}(f)} \tag{2.1}$$

and

$$\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R) \leq \left(\sigma_{k,D}^{(q,m)}(g) + \varepsilon\right)\left(\log^{[m-1]} R\right)^{\rho_k^{(q,m)}(g)} . \tag{2.2}$$

Now from (2.1), (2.2) and the condition $\rho_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$, it follows for all large values of R that

$$\frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \geqslant \frac{\left(\overline{\sigma}_{h,D}^{(p,m)}(f) - \varepsilon\right)}{\left(\sigma_{k,D}^{(q,m)}(g) + \varepsilon\right)} .$$

As $\varepsilon\,(>0)$ is arbitrary , we obtain that

$$\liminf_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \geqslant \frac{\overline{\sigma}_{h,D}^{(p,m)}(f)}{\sigma_{k,D}^{(q,m)}(g)} . \tag{2.3}$$

Again for a sequence of values of R tending to infinity we get that

$$\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R) \leq \left(\overline{\sigma}_{h,D}^{(p,m)}(f) + \varepsilon\right)\left(\log^{[m-1]} R\right)^{\rho_h^{(q,m)}(f)} \tag{2.4}$$

and for all sufficiently large values of R ,

$$\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R) \geqslant \left(\overline{\sigma}_{k,D}^{(q,m)}(g) - \varepsilon\right)\left(\log^{[m-1]} R\right)^{\rho_k^{(q,m)}(g)} . \tag{2.5}$$

Combining the condition $\rho_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$, (2.4) and (2.5) we get for a sequence of values of R tending to infinity that

$$\frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\left(\overline{\sigma}_{h,D}^{(p,m)}(f) + \varepsilon\right)}{\left(\overline{\sigma}_{k,D}^{(q,m)}(g) - \varepsilon\right)} .$$

Since $\varepsilon\,(>0)$ is arbitrary, it follows that

$$\liminf_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\overline{\sigma}_{h,D}^{(p,m)}(f)}{\overline{\sigma}_{k,D}^{(q,m)}(g)} . \tag{2.6}$$

Also for a sequence of values of R tending to infinity that

$$\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R) \leq \left(\overline{\sigma}_{k,D}^{(q,m)}(g) + \varepsilon\right)\left(\log^{[m-1]} R\right)^{\rho_k^{(q,m)}(g)} . \tag{2.7}$$

Now from (2.1), (2.7) and the condition $\rho_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$, we obtain for a sequence of values of R tending to infinity that

$$\frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \geq \frac{\left(\overline{\sigma}_{h,D}^{(p,m)}(f) - \varepsilon\right)}{\left(\overline{\sigma}_{k,D}^{(q,m)}(g) + \varepsilon\right)} .$$

As $\varepsilon\,(>0)$ is arbitrary, we get from above that

$$\limsup_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \geq \frac{\overline{\sigma}_{h,D}^{(p,m)}(f)}{\overline{\sigma}_{k,D}^{(q,m)}(g)} . \tag{2.8}$$

Also for all sufficiently large values of R ,

$$\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(r) \leq \left(\sigma_{h,D}^{(p,m)}(f) + \varepsilon\right) \left(\log^{[m-1]} R\right)^{\rho_h^{(q,m)}(f)} . \tag{2.9}$$

As the condition $\rho_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$, it follows from (2.5) and (2.9) for all large values of R that

$$\frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(r)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(r)} \leq \frac{\left(\sigma_{h,D}^{(p,m)}(f) + \varepsilon\right)}{\left(\overline{\sigma}_{k,D}^{(q,m)}(g) - \varepsilon\right)} .$$

Since $\varepsilon\,(> 0)$ is arbitrary, we obtain that

$$\limsup_{R \to +\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(r)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(r)} \leq \frac{\sigma_{h,D}^{(p,m)}(f)}{\overline{\sigma}_{k,D}^{(q,m)}(g)} . \tag{2.10}$$

Thus the theorem follows from (2.3) , (2.6) , (2.8) and (2.10) .

Theorem 2.2. *Let $f(z)$, $g(z)$ and $h(z)$ be any three entire functions of n- complex variables and D be a bounded complete n-circular domain with center at origin in $\mathbb{C}^n$. Also let $0 < \sigma_{h,D}^{(p,m)}(f) < \infty$, $0 < \sigma_{k,D}^{(q,m)}(g) < \infty$ and $\rho_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$ where p, q, m are all positive integers. Then*

$$\liminf_{R \to +\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\sigma_{h,D}^{(p,m)}(f)}{\sigma_{k,D}^{(q,m)}(g)} \leq \limsup_{R \to +\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} .$$

Proof. From the definition of $\sigma_{k,D}^{(q,m)}(g)$, we get for a sequence of values of R tending to infinity that

$$\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(r) \geqslant \left(\sigma_{k,D}^{(q,m)}(g) - \varepsilon\right) \left(\log^{[m-1]} R\right)^{\rho_k^{(q,m)}(g)} . \tag{2.11}$$

Now from (2.9), (2.11) and the condition $\rho_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$, it follows for a sequence of values of R tending to infinity that

$$\frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\left(\sigma_h^{(p,m)}(f) + \varepsilon\right)}{\left(\sigma_k^{(q,m)}(g) - \varepsilon\right)} .$$

As $\varepsilon\,(> 0)$ is arbitrary, we obtain that

$$\liminf_{R \to +\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\sigma_h^{(p,m)}(f)}{\sigma_k^{(q,m)}(g)} . \tag{2.12}$$

Again for a sequence of values of R tending to infinity that

$$\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(r) \geqslant \left(\sigma_{h,D}^{(p,m)}(f) - \varepsilon\right) \left(\log^{[m-1]} R\right)^{\rho_h^{(q,m)}(f)} . \tag{2.13}$$

So combining the condition $\rho_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$, (2.2) and (2.13) we get for a sequence of values of R tending to infinity that

$$\frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(r)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(r)} \geqslant \frac{\left(\sigma_{h,D}^{(p,m)}(f) - \varepsilon\right)}{\left(\sigma_{k,D}^{(q,m)}(g) + \varepsilon\right)} .$$

Since $\varepsilon\,(> 0)$ is arbitrary, it follows that

$$\limsup_{R \to +\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(r)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(r)} \geqslant \frac{\sigma_{h,D}^{(p,m)}(f)}{\sigma_{k,D}^{(q,m)}(g)} . \tag{2.14}$$

Thus the theorem follows from (2.12) and (2.14) .

The following theorem is a natural consequence of Theorem 2.1 and Theorem 2.2.

Theorem 2.3. *Let $f(z)$, $g(z)$ and $h(z)$ be any three entire functions of n- complex variables and D be a bounded complete n-circular domain with center at origin in $\mathbb{C}^n$. Also let $0 < \overline{\sigma}_{h,D}^{(p,m)}(f) \leq \sigma_{h,D}^{(p,m)}(f) < \infty$, $0 < \overline{\sigma}_{k,D}^{(q,m)}(g) \leq \sigma_{k,D}^{(q,m)}(g) < \infty$ and $\rho_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$ where p, q, m are all positive integers. Then*

$$\liminf_{R\to+\infty}\frac{\log^{[p-1]} M_{h,D}^{-1}M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1}M_{g,D}(R)} \leq \min\left\{\frac{\overline{\sigma}_{h,D}^{(p,m)}(f)}{\overline{\sigma}_{k,D}^{(q,m)}(g)}, \frac{\sigma_{h,D}^{(p,m)}(f)}{\sigma_{k,D}^{(q,m)}(g)}\right\}$$
$$\leq \max\left\{\frac{\overline{\sigma}_{h,D}^{(p,m)}(f)}{\overline{\sigma}_{k,D}^{(q,m)}(g)}, \frac{\sigma_{h,D}^{(p,m)}(f)}{\sigma_{k,D}^{(q,m)}(g)}\right\} \leq \limsup_{R\to+\infty}\frac{\log^{[p-1]} M_{h,D}^{-1}M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1}M_{g,D}(R)}.$$

The proof is omitted.

Now in the line of Theorem 2.1, Theorem 2.2, Theorem 2.3 respectively one can easily prove the following three theorems using the notion of (p,q)-*th relative Gol'dberg weak type* and therefore their proofs are omitted.

Theorem 2.4. *Let $f(z)$, $g(z)$ and $h(z)$ be any three entire functions of n- complex variables and D be a bounded complete n-circular domain with center at origin in $\mathbb{C}^n$. Also let $0 < \tau_{h,D}^{(p,m)}(f) \leq \overline{\tau}_{h,D}^{(p,m)}(f) < \infty$, $0 < \tau_{k,D}^{(q,m)}(g) \leq \overline{\tau}_{k,D}^{(q,m)}(g) < \infty$ and $\lambda_h^{(p,m)}(f) = \lambda_k^{(q,m)}(g)$ where p, q, m are all positive integers. Then*

$$\frac{\tau_{h,D}^{(p,m)}(f)}{\overline{\tau}_{k,D}^{(q,m)}(g)} \leq \liminf_{R\to+\infty}\frac{\log^{[p-1]} M_{h,D}^{-1}M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1}M_{g,D}(R)} \leq \frac{\tau_{h,D}^{(p,m)}(f)}{\tau_{k,D}^{(q,m)}(g)}$$
$$\leq \limsup_{R\to+\infty}\frac{\log^{[p-1]} M_{h,D}^{-1}M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1}M_{g,D}(R)} \leq \frac{\overline{\tau}_{h,D}^{(p,m)}(f)}{\tau_{k,D}^{(q,m)}(g)}.$$

Theorem 2.5. *Let $f(z)$, $g(z)$ and $h(z)$ be any three entire functions of n- complex variables and D be a bounded complete n-circular domain with center at origin in $\mathbb{C}^n$. Also let $0 < \overline{\tau}_{h,D}^{(p,m)}(f) < \infty$, $0 < \overline{\tau}_{k,D}^{(q,m)}(g) < \infty$ and $\lambda_h^{(p,m)}(f) = \lambda_k^{(q,m)}(g)$ where p, q, m are all positive integers. Then*

$$\liminf_{R\to+\infty}\frac{\log^{[p-1]} M_{h,D}^{-1}M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1}M_{g,D}(R)} \leq \frac{\overline{\tau}_{h,D}^{(p,m)}(f)}{\overline{\tau}_{k,D}^{(q,m)}(g)} \leq \limsup_{R\to+\infty}\frac{\log^{[p-1]} M_{h,D}^{-1}M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1}M_{g,D}(R)}.$$

Theorem 2.6. *Let $f(z)$, $g(z)$ and $h(z)$ be any three entire functions of n- complex variables and D be a bounded complete n-circular domain with center at origin in $\mathbb{C}^n$. Also let $0 < \tau_{h,D}^{(p,m)}(f) \leq \overline{\tau}_{h,D}^{(p,m)}(f) < \infty$, $0 < \tau_{k,D}^{(q,m)}(g) \leq \overline{\tau}_{k,D}^{(q,m)}(g) < \infty$ and $\lambda_h^{(p,m)}(f) = \lambda_k^{(q,m)}(g)$ where p, q, m are all positive integers. Then*

$$\liminf_{R\to+\infty}\frac{\log^{[p-1]} M_{h,D}^{-1}M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1}M_{g,D}(R)} \leq \min\left\{\frac{\tau_{h,D}^{(p,m)}(f)}{\tau_{k,D}^{(q,m)}(g)}, \frac{\overline{\tau}_{h,D}^{(p,m)}(f)}{\overline{\tau}_{k,D}^{(q,m)}(g)}\right\}$$
$$\leq \max\left\{\frac{\tau_{h,D}^{(p,m)}(f)}{\tau_{k,D}^{(q,m)}(g)}, \frac{\overline{\tau}_{h,D}^{(p,m)}(f)}{\overline{\tau}_{k,D}^{(q,m)}(g)}\right\} \leq \limsup_{R\to+\infty}\frac{\log^{[p-1]} M_{h,D}^{-1}M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1}M_{g,D}(R)}.$$

We may now state the following theorems without their proofs based on (p,q)-*th relative Gol'dberg type* and (p,q)-*th relative Gol'dberg weak type*:

Theorem 2.7. *Let $f(z)$, $g(z)$ and $h(z)$ be any three entire functions of n- complex variables and D be a bounded complete n-circular domain with center at origin in $\mathbb{C}^n$. Also let $0 <$*

$\overline{\sigma}_{h,D}^{(p,m)}(f) \leq \sigma_{h,D}^{(p,m)}(f) < \infty$, $0 < \tau_{k,D}^{(q,m)}(g) \leq \overline{\tau}_{k,D}^{(q,m)}(g) < \infty$ *and* $\rho_h^{(p,m)}(f) = \lambda_k^{(q,m)}(g)$ *where* p, q, m *are all positive integers. Then*

$$\frac{\overline{\sigma}_{h,D}^{(p,m)}(f)}{\overline{\tau}_{k,D}^{(q,m)}(g)} \leq \liminf_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\overline{\sigma}_{h,D}^{(p,m)}(f)}{\tau_{k,D}^{(q,m)}(g)}$$
$$\leq \limsup_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\sigma_{h,D}^{(p,m)}(f)}{\tau_{k,D}^{(q,m)}(g)}.$$

Theorem 2.8. *Let* $f(z)$, $g(z)$ *and* $h(z)$ *be any three entire functions of* n*- complex variables and* D *be a bounded complete* n*-circular domain with center at origin in* $\mathbb{C}^n$*. Also let* $0 < \sigma_{h,D}^{(p,m)}(f) < \infty$, $0 < \overline{\tau}_{k,D}^{(q,m)}(g) < \infty$ *and* $\rho_h^{(p,m)}(f) = \lambda_k^{(q,m)}(g)$ *where* p, q, m *are all positive integers. Then*

$$\liminf_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\sigma_{h,D}^{(p,m)}(f)}{\overline{\tau}_{k,D}^{(q,m)}(g)} \leq \limsup_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)}.$$

Theorem 2.9. *Let* $f(z)$, $g(z)$ *and* $h(z)$ *be any three entire functions of* n*- complex variables and* D *be a bounded complete* n*-circular domain with center at origin in* $\mathbb{C}^n$*. Also let* $0 < \overline{\sigma}_{h,D}^{(p,m)}(f) \leq \sigma_{h,D}^{(p,m)}(f) < \infty$, $0 < \tau_{k,D}^{(q,m)}(g) \leq \overline{\tau}_{k,D}^{(q,m)}(g) < \infty$ *and* $\rho_h^{(p,m)}(f) = \lambda_k^{(q,m)}(g)$ *where* p, q, m *are all positive integers. Then*

$$\liminf_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \min\left\{\frac{\overline{\sigma}_{h,D}^{(p,m)}(f)}{\tau_{k,D}^{(q,m)}(g)}, \frac{\sigma_{h,D}^{(p,m)}(f)}{\overline{\tau}_{k,D}^{(q,m)}(g)}\right\}$$
$$\leq \max\left\{\frac{\overline{\sigma}_{h,D}^{(p,m)}(f)}{\tau_{k,D}^{(q,m)}(g)}, \frac{\sigma_{h,D}^{(p,m)}(f)}{\overline{\tau}_{k,D}^{(q,m)}(g)}\right\} \leq \limsup_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)}.$$

Theorem 2.10. *Let* $f(z)$, $g(z)$ *and* $h(z)$ *be any three entire functions of* n*- complex variables and* D *be a bounded complete* n*-circular domain with center at origin in* $\mathbb{C}^n$*. Also let* $0 < \tau_{h,D}^{(p,m)}(f) \leq \overline{\tau}_{h,D}^{(p,m)}(f) < \infty$, $0 < \overline{\sigma}_{k,D}^{(q,m)}(g) \leq \sigma_{k,D}^{(q,m)}(g) < \infty$ *and* $\lambda_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$ *where* p, q, m *are all positive integers. Then*

$$\frac{\tau_{h,D}^{(p,m)}(f)}{\sigma_{k,D}^{(q,m)}(g)} \leq \liminf_{Rr\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\tau_{h,D}^{(p,m)}(f)}{\overline{\sigma}_{k,D}^{(q,m)}(g)}$$
$$\leq \limsup_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\overline{\tau}_{h,D}^{(p,m)}(f)}{\overline{\sigma}_{k,D}^{(q,m)}(g)}.$$

Theorem 2.11. *Let* $f(z)$, $g(z)$ *and* $h(z)$ *be any three entire functions of* n*- complex variables and* D *be a bounded complete* n*-circular domain with center at origin in* $\mathbb{C}^n$*. Also let* $0 < \overline{\tau}_{h,D}^{(p,m)}(f) < \infty$, $0 < \sigma_{k,D}^{(q,m)}(g) < \infty$ *and* $\lambda_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$ *where* p, q, m *are all positive integers. Then*

$$\liminf_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)} \leq \frac{\overline{\tau}_{h,D}^{(p,m)}(f)}{\sigma_{k,D}^{(q,m)}(g)} \leq \limsup_{R\to+\infty} \frac{\log^{[p-1]} M_{h,D}^{-1} M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1} M_{g,D}(R)}.$$

Theorem 2.12. *Let* $f(z)$, $g(z)$ *and* $h(z)$ *be any three entire functions of* n*- complex variables and* D *be a bounded complete* n*-circular domain with center at origin in* $\mathbb{C}^n$*. Also let* $0 < \tau_{h,D}^{(p,m)}(f) \leq \overline{\tau}_{h,D}^{(p,m)}(f) < \infty$, $0 < \overline{\sigma}_{k,D}^{(q,m)}(g) \leq \sigma_{k,D}^{(q,m)}(g) < \infty$ *and* $\lambda_h^{(p,m)}(f) = \rho_k^{(q,m)}(g)$ *where*

p, q, m *are all positive integers. Then*

$$\liminf_{R\to+\infty}\frac{\log^{[p-1]} M_{h,D}^{-1}M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1}M_{g,D}(R)} \leq \min\left\{\frac{\tau_{h,D}^{(p,m)}(f)}{\overline{\sigma}_{k,D}^{(q,m)}(g)}, \frac{\overline{\tau}_{h,D}^{(p,m)}(f)}{\sigma_{k,D}^{(q,m)}(g)}\right\}$$

$$\leq \max\left\{\frac{\tau_{h,D}^{(p,m)}(f)}{\overline{\sigma}_{k,D}^{(q,m)}(g)}, \frac{\overline{\tau}_{h,D}^{(p,m)}(f)}{\sigma_{k,D}^{(q,m)}(g)}\right\} \leq \limsup_{R\to+\infty}\frac{\log^{[p-1]} M_{h,D}^{-1}M_{f,D}(R)}{\log^{[q-1]} M_{k,D}^{-1}M_{g,D}(R)}.$$

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