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A Comprehensive Subclass of Analytic and Bi-Univalent Functions Associated with Subordination

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Abstract In the present paper, we define a new general subclass of bi-univalent functions involving a differential operator in the open unit disk $\mathbb{U}$ and determine estimates for the general Taylor-Maclaurin coefficients of the functions in this class. For this purpose, we use the Faber polynomial expansions. Several connections to some of the earlier known results are also pointed out.

1 Introduction

Let $\mathcal{A}$ denote the class of all analytic functions f defined in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ and normalized by the conditions $f(0) = 0$ and $f'(0) = 1$. Thus each $f \in \mathcal{A}$ has a Taylor-Maclaurin series expansion of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad (z \in \mathbb{U}). \tag{1.1}$$

Further, let $\mathcal{S}$ denote the class of all functions $f \in \mathcal{A}$ which are univalent in $\mathbb{U}$ (for details, see [10]; see also some of the recent investigations [3, 6, 7, 8, 23]). And let $\mathcal{C}$ be the class of functions $\Phi(z) = 1 + \sum_{n=1}^{\infty} \Phi_n z^n$ that are analytic in $\mathbb{U}$ and satisfy the condition $\mathrm{Re}(\Phi(z)) > 0$ in $\mathbb{U}$. By the Caratheodory's lemma (see [10]) we have $|\Phi(z)| \leq 2$.

Let the functions f, g be analytic in $\mathbb{U}$. If there exists a Schwarz function ϖ, which is analytic in $\mathbb{U}$ under the conditions

$$\varpi(0) = 0, \;\; |\varpi(z)| \leq 1,$$

such that

$$f(z) = g(\varpi(z)), \;\; z \in \mathbb{U},$$

then, the function f is subordinate to g in $\mathbb{U}$, and we write $f(z) \prec g(z)$.

By the Koebe one-quarter theorem (for details, (see [10]), we know that the image of $\mathbb{U}$ under every function $f \in \mathcal{A}$ contains a disk of radius $\frac{1}{4}$. According to this, every function $f \in \mathcal{A}$ has an inverse map f^{-1} that satisfies the following conditions:

$$f^{-1}(f(z)) = z \quad (z \in \mathbb{U}),$$

and

$$f\left(f^{-1}(w)\right) = w \quad \left(|w| < r_0(f); r_0(f) \geq \frac{1}{4}\right).$$

In fact, the inverse function is given by

$$g(w) = f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots . \tag{1.2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1.1). Examples of functions in the class Σ are

$$\frac{z}{1-z}, \ -\log(1-z), \ \frac{1}{2}\log\left(\frac{1+z}{1-z}\right), \cdots .$$

It is worth noting that the familiar Koebe function is not a member of Σ, since it maps the unit disk $\mathbb{U}$ univalently onto the entire complex plane except the part of the negative real axis from $-1/4$ to $-\infty$. Thus, clearly, the image of the domain does not contain the unit disk $\mathbb{U}$. For a brief history and some intriguing examples of functions and characterization of the class Σ, see Srivastava et al. [20], Yousef et al. [24, 25, 26], and Frasin and Aouf [12].

In 1967, Lewin [18] investigated the bi-univalent function class Σ and showed that $|a_2| < 1.51$. Subsequently, Brannan and Clunie [9] conjectured that $|a_2| \leq \sqrt{2}$. On the other hand, Netanyahu [19] showed that $\max_{f\in\Sigma} |a_2| = \frac{4}{3}$. The best known estimate for functions in Σ has been obtained in 1984 by Tan [21], that is, $|a_2| < 1.485$. The coefficient estimate problem for each of the following Taylor-Maclaurin coefficients $|a_n|$ $(n \in \mathbb{N}\backslash\{1,2\})$ for each $f \in \Sigma$ given by (1.1) is presumably still an open problem.

The Faber polynomials introduced by Faber [11] play an important role in various areas of mathematical sciences, especially in geometric function theory. The recent publications [13] and [14] applying the Faber polynomial expansions to meromorphic bi-univalent functions motivated us to apply this technique to classes of analytic bi-univalent functions. In the literature, there are only a few works determining the general coefficient bounds $|a_n|$ for the analytic bi-univalent functions given by (1.1) using Faber polynomial expansions (see for example, [15, 16, 17]). Hamidi and Jahangiri [15] considered the class of analytic bi-close-to-convex functions. Jahangiri and Hamidi [17] considered the class defined by Frasin and Aouf [12], and Jahangiri et al. [16] considered the class of analytic bi-univalent functions with positive real-part derivatives.

2 The class $\mathfrak{B}_\Sigma(\mu, \lambda, \Phi, \xi)$

Yousef et al. [25] have introduced and studied the following subclass of analytic bi-univalent functions:

Definition 2.1. For $\lambda \geq 1$, $\mu \geq 0$, $\delta \geq 0$ and $0 \leq \alpha < 1$, a function $f \in \Sigma$ given by (1.1) is said to be in the class $\mathfrak{B}_\Sigma^\mu(\alpha, \lambda, \delta)$ if the following conditions hold for all $z, w \in \mathbb{U}$:

$$\text{Re}\left((1-\lambda)\left(\frac{f(z)}{z}\right)^\mu + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} + \xi\delta z f''(z)\right) > \alpha \tag{2.1}$$

and

$$\text{Re}\left((1-\lambda)\left(\frac{g(w)}{w}\right)^\mu + \lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1} + \xi\delta w g''(w)\right) > \alpha, \tag{2.2}$$

where the function $g(w) = f^{-1}(w)$ is defined by (1.2) and $\xi = \frac{2\lambda+\mu}{2\lambda+1}$.

Using the Faber polynomial expansion of functions $f \in \mathcal{A}$ of the form (1.1), the coefficients of its inverse map $g = f^{-1}$ may be expressed as in [1]:

$$g(w) = f^{-1}(w) = w + \sum_{n=2}^{\infty} \frac{1}{n} K_{n-1}^{-n}(a_2, a_3, ...)\, w^n, \tag{2.3}$$

where

$$K_{n-1}^{-n} = \frac{(-n)!}{(-2n+1)!\,(n-1)!}a_2^{n-1} + \frac{(-n)!}{(2\,(-n+1))!\,(n-3)!}a_2^{n-3}a_3 + \frac{(-n)!}{(-2n+3)!\,(n-4)!}a_2^{n-4}a_4 \tag{2.4}$$

$$+ \frac{(-n)!}{(2\,(-n+2))!\,(n-5)!}a_2^{n-5}\left[a_5 + (-n+2)\,a_3^2\right] + \frac{(-n)!}{(-2n+5)!\,(n-6)!}a_2^{n-6}$$

$$[a_6 + (-2n+5)\,a_3a_4] + \sum_{j\geq 7} a_2^{n-j}V_j,$$

such that V_j with $7 \leq j \leq n$ is a homogeneous polynomial in the variables $a_2, a_3, ..., a_n$ [2].

In particular, the first three terms of K_{n-1}^{-n} are

$$K_1^{-2} = -2a_2,\ K_2^{-3} = 3\left(2a_2^2 - a_3\right),\ K_3^{-4} = -4\left(5a_2^3 - 5a_2a_3 + a_4\right). \tag{2.5}$$

In general, for any $p \in \mathbb{N} := \{1, 2, 3, ...\}$, an expansion of K_n^p is as in [1],

$$K_n^p = pa_n + \frac{p(p-1)}{2}D_n^2 + \frac{p!}{(p-3)!3!}D_n^3 + \cdots + \frac{p!}{(p-n)!n!}D_n^n, \tag{2.6}$$

where $D_n^p = D_n^p\,(a_2, a_3, ...)$, and by [22], $D_n^m\,(a_1, a_2, ..., a_n) = \sum_{n=1}^{\infty} \frac{m!}{i_1!...i_n!}a_1^{i_1}...a_n^{i_n}$ while $a_1 = 1$, and the sum is taken over all non-negative integers $i_1, ..., i_n$ satisfying $i_1 + i_2 + \cdots + i_n = m$, $i_1 + 2i_2 + \cdots + ni_n = n$, it is clear that $D_n^m\,(a_1, a_2, ..., a_n) = a_1^n$.

Now, we are ready to establish a new subclass of analytic and bi-univalent functions based on subordination.

Definition 2.2. For $\lambda \geq 1$, $\mu \geq 0$, and $\delta \geq 0$, A function $f \in \Sigma$ is said to be in the class $\mathfrak{B}_\Sigma(\mu, \lambda, \Phi, \xi)$, if the following subordinations are satisfied:

$$(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} + \xi\delta z f''(z) \prec \Phi(z) \tag{2.7}$$

and

$$(1-\lambda)\left(\frac{g(w)}{w}\right)^{\mu} + \lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1} + \xi\delta w g''(w) \prec \Phi(w) \tag{2.8}$$

where the function $g(w) = f^{-1}(w)$ is defined by (1.2) and $\xi = \frac{2\lambda+\mu}{2\lambda+1}$.

3 Coefficient bounds for the function class $\mathfrak{B}_\Sigma(\mu, \lambda, \Phi, \xi)$

Theorem 3.1. *For $\lambda \geq 1$, $\mu \geq 0$,and $\delta \geq 0$, let the function $f \in \mathfrak{B}_\Sigma(\mu, \lambda, \Phi, \xi)$ be given by (1.1). Then*

$$|a_2| \leq \min\left\{\frac{2}{\mu+\lambda+2\xi\delta}, \sqrt{\frac{8}{(\mu+2\lambda)\left(\mu+1+\frac{12\delta}{2\lambda+1}\right)}}\right\}$$

and

$$|a_3| \leq \min\left\{\frac{4}{(\mu+\lambda+2\xi\delta)^2}, \frac{8}{(\mu+2\lambda)\left(\mu+1+\frac{12\delta}{2\lambda+1}\right)}\right\} + \frac{2}{(\mu+2\lambda)\left(1+\frac{6\delta}{2\lambda+1}\right)}.$$

Proof. Let $f \in \mathfrak{B}_\Sigma(\mu, \lambda, \Phi, \xi)$.The inequalities (2.7) and (2.8) imply the existence of two positive real part functions

$$\varpi(z) = 1 + \sum_{n=1}^{\infty} t_n z^n$$

and

$$\varphi(w) = 1 + \sum_{n=1}^{\infty} s_n z^n$$

where Re $(\varpi(z)) > 0$ and Re $(\varphi(w)) > 0$ in $\mathcal{C}$ so that

$$(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} + \xi\delta z f''(z) = \Phi(\varpi(z)), \tag{3.1}$$

$$(1-\lambda)\left(\frac{g(w)}{w}\right)^{\mu} + \lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1} + \xi\delta w g''(w) = \Phi(\varphi(w)). \tag{3.2}$$

It follows from (3.1) and (3.2) that

$$(\mu + \lambda + 2\xi\delta)\, a_2 = \Phi_1 t_1 \tag{3.3}$$

$$(\mu + 2\lambda)\left[\frac{\mu-1}{2}a_2^2 + \left(1 + \frac{6\delta}{2\lambda+1}\right)a_3\right] = \Phi_1 t_2 + \Phi_2 t_1^2, \tag{3.4}$$

and

$$-(\mu + \lambda + 2\xi\delta)\, a_2 = \Phi_1 s_1, \tag{3.5}$$

$$(\mu + 2\lambda)\left[\left(\frac{\mu+3}{2} + \frac{12\delta}{2\lambda+1}\right)a_2^2 - \left(1 + \frac{6\delta}{2\lambda+1}\right)a_3\right] = \Phi_1 s_2 + \Phi_2 s_1^2. \tag{3.6}$$

From (3.3) and (3.5), we find

$$|a_2| \leq \frac{|\Phi_1 t_1|}{\mu + \lambda + 2\xi\delta} = \frac{|\Phi_1 s_1|}{\mu + \lambda + 2\xi\delta} \leq \frac{2}{\mu + \lambda + 2\xi\delta}. \tag{3.7}$$

From (3.4) and (3.6), we get

$$(\mu + 2\lambda)\left(\mu + 1 + \frac{12\delta}{2\lambda+1}\right)a_2^2 = \Phi_1(t_2 + s_2) + \Phi_2\left(t_1^2 + s_1^2\right)$$

or, equivalently

$$|a_2| \leq \sqrt{\frac{8}{(\mu + 2\lambda)\left(\mu + 1 + \frac{12\delta}{2\lambda+1}\right)}}. \tag{3.8}$$

Next, in order to find the bound on the coefficient $|a_3|$, we subtract (3.6) from (3.4). We thus get

$$2(\mu + 2\lambda)\left(1 + \frac{6\delta}{2\lambda+1}\right)\left(a_3 - a_2^2\right) = \Phi_1(t_2 - s_2) + \Phi_2\left(t_1^2 - s_1^2\right) \tag{3.9}$$

or

$$\begin{aligned} |a_3| &\leq |a_2|^2 + \frac{|\Phi_1(t_2 - s_2)|}{2(\mu + 2\lambda)\left(1 + \frac{6\delta}{2\lambda+1}\right)} \\ &= |a_2|^2 + \frac{2}{(\mu + 2\lambda)\left(1 + \frac{6\delta}{2\lambda+1}\right)}. \end{aligned} \tag{3.10}$$

Upon substituting the value of a_2^2 from (3.7) and (3.8) into (3.10), it follows that

$$|a_3| \leq \frac{4}{(\mu + \lambda + 2\xi\delta)^2} + \frac{2}{(\mu + 2\lambda)\left(1 + \frac{6\delta}{2\lambda+1}\right)}$$

and

$$|a_3| \leq \frac{8}{(\mu + 2\lambda)\left(\mu + 1 + \frac{12\delta}{2\lambda+1}\right)} + \frac{2}{(\mu + 2\lambda)\left(1 + \frac{6\delta}{2\lambda+1}\right)}$$

Which completes the proof of Theorem 3.1. □

Theorem 3.2. *Let* $f \in \mathfrak{B}_\Sigma(\mu, \lambda, \Phi, \xi)$. *If* $a_m = 0$ *with* $2 \leq m \leq n-1$, *then*

$$|a_n| \leq \frac{2}{\mu + (n-1)\lambda + n(n-1)\xi\delta} \ (n \geq 4). \tag{3.11}$$

Proof. By using the Faber polynomial expansion of functions $f \in \mathcal{A}$ of the form (1.1) and its inverse map $g = f^{-1}$, we can write

$$(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1} + \xi\delta z f''(z) = 1 + \sum_{n=2}^{\infty} F_{n-1}(a_2, a_3, ..., a_n) z^{n-1} \tag{3.12}$$

and

$$(1-\lambda)\left(\frac{g(w)}{w}\right)^{\mu} + \lambda g'(w)\left(\frac{g(w)}{w}\right)^{\mu-1} + \xi\delta w g''(w) = 1 + \sum_{n=2}^{\infty} F_{n-1}(A_2, A_3, ..., A_n) w^{n-1} \tag{3.13}$$

where

$$F_1 = (\mu + \lambda + 2\xi\delta)a_2, \ F_2 = (\mu + 2\lambda)\left[\frac{\mu-1}{2}a_2^2 + \left(1 + \frac{6\delta}{2\lambda+1}\right)a_3\right] \tag{3.14}$$

and, in general (see [5])

$$F_{n-1}(a_2, a_3, ..., a_n) = [\mu + (n-1)\lambda + n(n-1)\xi\delta] \times [(\mu-1)!]$$
$$\times \sum_{i_1 + 2i_2 + \cdots + (n-1)i_{n-1} = n-1} \left(\frac{\mu + n\lambda}{\mu + n\lambda + n(n+1)\xi\delta}\right)^{1-i_{n-1}} \cdot$$
$$\frac{a_2^{i_1} a_3^{i_2} ... a_n^{i_{n-1}}}{i_1! i_2! \cdots i_{n-1}! [\mu - (i_1 + i_2 + \cdots + i_{n-1})]!}.$$

Next, by using the Faber polynomial expansion of functions $\varpi, \varphi \in \mathcal{C}$, we also obtain

$$\Phi(\varpi(z)) = 1 + \sum_{n=1}^{\infty}\sum_{k=1}^{\infty} \Phi_k F_n^k(t_1, t_2, ..., t_n) z^n, \tag{3.15}$$

and

$$\Phi(\varphi(z)) = 1 + \sum_{n=1}^{\infty}\sum_{k=1}^{\infty} \Phi_k F_n^k(s_1, s_2, ..., s_n) w^n. \tag{3.16}$$

Comparing the corresponding coefficients yields

$$[\mu + (n-1)\lambda + n(n-1)\xi\delta] a_n = \sum_{k=1}^{n-1} \Phi_k F_{n-1}^k(t_1, t_2, ..., t_{n-1}) \ (n \geq 2)$$

and

$$[\mu + (n-1)\lambda + n(n-1)\xi\delta] A_n = \sum_{k=1}^{n-1} \Phi_k F_{n-1}^k(s_1, s_2, ..., s_{n-1}) \ (n \geq 2). \tag{3.17}$$

Note that for $a_m = 0$, $2 \leq m \leq n-1$, we have $A_n = -a_n$ and so

$$[\mu + (n-1)\lambda + n(n-1)\xi\delta] a_n = \Phi_1 t_{n-1},$$

$$-[\mu + (n-1)\lambda + n(n-1)\xi\delta] a_n = \Phi_1 s_{n-1}, \tag{3.18}$$

Now taking the absolute values of either of the above two equations and using the facts that $|\Phi_1| \leq 2$, $|t_{n-1}| \leq 1$, and $|s_{n-1}| \leq 1$, we obtain

$$|a_n| \leq \frac{|\Phi_1 t_{n-1}|}{\mu + (n-1)\lambda + n(n-1)\xi\delta} = \frac{|\Phi_1 s_{n-1}|}{\mu + (n-1)\lambda + n(n-1)\xi\delta} \tag{3.19}$$

$$\leq \frac{2}{\mu + (n-1)\lambda + n(n-1)\xi\delta} \tag{3.20}$$

This evidently completes the proof of Theorem 3.2. □

Remark 3.3. As a final remark, for $\delta = 0$ in

(i) Theorem 3.1 we obtain Theorem 1 in [4].

(ii) Theorem 3.2 we obtain Theorem 2 in [4].

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