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SOFT NEUTROSOPHIC CLASSICAL SETS AND THEIR APPLICATIONS IN DECISION-MAKING

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Abstract. In this paper, we introduce the concept of soft neutrosophic classical sets and its set theoretical operations such as; union, intersection, complement, AND-product and OR-product. In addition to these concept and operations, we define four basic types of sets of degenerate elements in a soft neutrosophic classical set. Then, we propose a group decision making method based on soft neutrosophic classical sets, and give algorithm of proposed method. We also make an application of proposed method on a problem including soft neutrosophic classical data.

1 Introduction

Molodtsov [16] introduced soft set theory as a mathematical tool that independent from membership function in fuzzy set theory which defined by Zadeh [25] for coping with problems involving uncertainty and vagueness. Applications of soft set theory have been expanded in many different area. After Molodtsov's work, soft set theory has been growing very rapidly in terms of set theoretical and application. In 2003, Maji *et al.* [17, 18] applied soft set theory to decision making problems. Then, decision making, algebraic structures and set theoretical operations and properties on soft sets were made a lot of work. For instance; Ali *et al.* [1] contributed to soft set theory in terms of set theoretical by defining some novel operations on soft sets such as extending intersection and extending union, restricted intersection and restricted union, Çağman and Enginoğlu [7] modified definitions of soft set operations, Sezgin and Atagün [21] proved that De Morgan's law is hold in soft set theory for different operations on soft sets. Feng *et al.* [11] gave a systematic study on several types of soft subsets and various soft equal relations induced by them, Xia and Zuhua [24] gave some new operations such as the product, the anti product of two soft sets, inverse the image and pre-image of a soft set. Zhu and Wen [26] redefined the intersection, complement, and difference of soft sets. They also investigated the algebraic properties of these operations along with a known union operation and found that the new operation system on soft sets inherits all basic properties of operations on classical sets, which justifies their definitions. Çağman [8] made contributions to the theory of soft sets. Atagün and Aygün proposed two novel operations on soft set and he proved that set of all soft sets over same initial universe is an abelian group under defined new operations.

Concept of neutrosophic sets was defined by Smarandache [5, 6] as a new mathematical tool for dealing with problems involving incomplete, indeterminacy, inconsistent knowledge. Maji [19] introduced concept of neutrosophic soft set and some operations of neutrosophic soft sets. Karaaslan [13] redefined concept and operations of neutrosophic soft sets. Recently, the properties and applications on the neutrosophic soft sets have been studied increasingly. For examples; Broumi [3] defined concept of generalized neutrosophic soft set, Broumi *et al.* [4] gave a decision making method on the neutrosophic parameterized soft sets, Şahin and Küçük [20] introduced a new kind of decision making method based on the generalized neutrosophic soft set and its integration, Deli [9] defined concept of interval-valued neutrosophic soft set and its operations, Deli and Broumi [10] introduced neutrosophic soft matrices and gave a decision making method based on neutrosophic soft matrices. Karaaslan [14] defined concept of possibility neutrosophic soft set and its operations and gave a decision making method based on possibility neutrosophic soft set.

In this paper, we define concept of soft neutrosophic classical set and its operations. Basic idea of soft neutrosophic classical set is that; according to evaluations which are made in different times, in different environments and by different person...etc., an element belong to truth set, indeterminacy set and falsity set in same time. To explain this idea, let us give an example. Let $U = \{u_1, u_1, ..., u_n\}$ be a set of students and $L = \{l_1 = mathematics, l_2 = English, l_3 = philosophy\}$ be common lessons taken by all student in set U. Assume that there are five exams along one year for each lesson and these exams have parallel concepts individually. They are also evaluated the results of exams by three criteria that are successful, indeterminate (between successful and unsuccessful) and unsuccessful. For simple, let us consider mathematics lesson, in the end of the parallel five exams, we may see that some students both among successful students and among unsuccessful students or both among successful students and among indeterminate students etc. such situations can't be modeled with soft sets and other set theories. At this point of view, we need soft neutrosophic classical sets to model such situations. Therefore, in this work, we first define concept of soft neutrosophic classical set and its operations and obtain their some properties. Then we suggest a group decision making method so as to apply to the problems containing uncertainty, and give an application of the suggested method.

2 Preliminaries

In this section, we will recall some definitions and properties related to neutrosophic sets [22], soft sets [8, 16] and neutrosophic classical set [12] required for our study. Throughout this paper U, E and $\mathcal{P}(U)$ denote set of initial universe, set of parameters and power set of U, respectively.

Definition 2.1. [22] A neutrosophic set μ on the non-empty set U is defined by

$$\mu = \{\langle x, \mu_t(x), \mu_i(x), \mu_f(x)\rangle : x \in U\}$$

where $\mu_t, \mu_i, \mu_f : U \to]^-0, 1^+[$ and $^-0 \leq \mu_t(x) + \mu_i(x) + \mu_f(x) \leq 3^+$. Values of $\mu_t(x), \mu_i(x)$ and $\mu_f(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$. Therefore it is difficult to use the neutrosophic values in real life applications in scientific and engineering. For this reason we take values of $\mu_t(x), \mu_i(x)$ and $\mu_f(x)$ from the subset of $[0, 1]$.

Definition 2.2. [12] Let U be a non-empty fixed set and $\kappa_t, \kappa_i, \kappa_f \subseteq U$. A neutrosophic classical set κ is defined as follows:

$$\kappa = \{\langle x, \kappa_t, \kappa_i, \kappa_f\rangle : x \in U\}.$$

Here the set κ_t, κ_i and κ_f is called the set of member, indeterminacy and non-members of κ, respectively.

Definition 2.3. [16] Let U be an initial universe, E be a parameter set and A be non-nonempty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$.

Çağman [8] restructured definition of soft sets and its set theoretical operations which are more suitable for pure mathematics. In this paper, we will use definition of soft set given by Çağman in [8].

Definition 2.4. [8] A soft set φ over U is a set valued function from E to $\mathcal{P}(U)$. It can be written a set of ordered pairs

$$\varphi = \{(e, \varphi(e)) : e \in E\}.$$

Note that if $\varphi(e) = \emptyset$, then the element $(e, \varphi(e))$ is not appeared in φ. Set of all soft sets over U is denoted by $\mathcal{S}_U^E$.

Definition 2.5. [15] Let E be a parameter set, $S \subset E$ and $f : S \to E$ be an injective function. Then $S \cup f(S)$ is called extended parameter set of S and denoted by $\mathcal{E}_S$.

If $S = E$, then extended parameter set of S will be denoted by $\mathcal{E}$.

Definition 2.6. [15] Let E be a parameter set, $S \subseteq E$ and $S \cup f(S) = \mathcal{E}_S$ such that $f : S \to \mathcal{E}_S$ be an injective function. If $\varphi : S \to \mathcal{P}(U)$ and $\psi : f(S) \to \mathcal{P}(U)$ are two mappings such that $\varphi(e) \cap \psi(f(e)) = \emptyset$, then triple (φ, ψ, E) is called bipolar soft set. We can represent a bipolar soft set (φ, ψ, E) as following defined by a mapping:

$$f_S : E \to P(U) \times \mathcal{P}(U) \text{ such that } \varphi(e) = \emptyset \text{ and } \psi(f(e)) = U \text{ if } e \in E \setminus S \text{ and } f(e) \in E \setminus \mathcal{E}_S.$$

Also we can write a bipolar soft set f_S as a set of triples following form

$$f_S = (\varphi, \psi, E) = \left\{ (e, \varphi(e), \psi(f(e))) : e \in E \text{ and } \varphi(e) \cap \psi(f(e)) = \emptyset \right\}.$$

3 Soft neutrosophic classical sets

In this section, we define the concept of soft neutrosophic classical sets and set theoretical operations between soft neutrosophic sets. Then, we examine desired propositions of soft neutrosophic classical sets.

Definition 3.1. Let E be a parameter set and U be an initial universe. Then, a soft neutrosophic classical set (*snc*-set) κ over U is defined as follows:

$$\kappa = \{(e, \langle \kappa_t(e), \kappa_i(e), \kappa_f(e) \rangle) : e \in E\}$$

where $\kappa_t, \kappa_i, \kappa_f : E \to \mathcal{P}(U)$ and $\emptyset \subseteq \kappa_t(e) \cup \kappa_i(e) \cup \kappa_f(e) \subseteq U$. Also set $\kappa_t(e), \kappa_i(e)$ and $\kappa_f(e)$ is called membership set, indeterminacy set and non-membership set of snc- set κ related to parameter $e \in E$, respectively.
From now on, set of all *snc*-sets over U will be denoted by $\mathcal{SNC}_U^E$.

Definition 3.2. Let κ be a soft neutrosophic classical set over U. If, for $u \in U$, $u \in (\kappa_t(e) \cap \kappa_i(e)) \cup (\kappa_i(e) \cap \kappa_f(e)) \cup (\kappa_t(e) \cap \kappa_f(e))$, then $u \in U$ is called degenerate element of U related to parameter $e \in E$ and set of degenerate elements related to parameter $e \in E$ is denoted by $\mathcal{D}_e$.
We define the following four basic types of sets of degenerate elements:
Let $e \in E$ and $u \in \mathcal{D}_e$. Then,

(i) If $u \in (\kappa_t(e) \cap \kappa_i(e))/(\kappa_t(e) \cap \kappa_i(e) \cap \kappa_f(e))$, then $u \in U$ is called positive weak degenerate element, and set of all positive weak degenerate elements is denoted by $\mathcal{D}_e^{w^+}$.

(ii) If $u \in (\kappa_i(e) \cap \kappa_f(e))/(\kappa_t(e) \cap \kappa_i(e) \cap \kappa_f(e))$, then $u \in U$ is called negative weak degenerate element, and set of all negative weak degenerate elements is denoted by $\mathcal{D}_e^{w^-}$.

(iii) If $u \in (\kappa_t(e) \cap \kappa_f(e))/(\kappa_t(e) \cap \kappa_i(e) \cap \kappa_f(e))$, then $u \in U$ is called strict degenerate element, and set of all strict degenerate elements is denoted by $\mathcal{D}_e^s$.

(iv) If $u \in (\kappa_t(e) \cap \kappa_i(e) \cap \kappa_f(e))$, then $u \in U$ is called full strict degenerate element, and set of all full strict degenerate elements is denoted by $\mathcal{D}_e^{\hat{s}}$.

Sets of degenerate elements related to parameter $e \in E$ can be shown as in Figure 1:

Example 3.3. Let $E = \{e_1, e_2, e_3, e_4\}$ be a parameter set and $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$.
Suppose that,

$$\begin{aligned}
\kappa(e_1) &= \langle \{u_1, u_2, u_7\}, \{u_2, u_4, u_5\}, \{u_3, u_5, u_6, u_8\} \rangle \\
\kappa(e_2) &= \langle \{u_1, u_2, u_4\}, \{u_3, u_5, u_6, u_7\}, \{u_1, u_7, u_8\} \rangle \\
\kappa(e_3) &= \langle \{u_2, u_3, u_8\}, \{u_1, u_7\}, \{u_4, u_5, u_6\} \rangle \\
\kappa(e_4) &= \langle \{u_6, u_7, u_8\}, \{u_3, u_5, u_6\}, \{u_1, u_2, u_3, u_4\} \rangle
\end{aligned}$$

then,

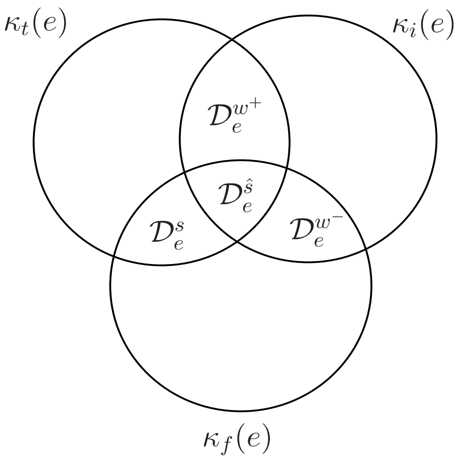

Figure 1. Sets of degenerate elements related to parameter $e \in E$.

$$\begin{aligned}\kappa = \ & \Big\{\big(e_1, \langle\{u_1,u_2,u_7\},\{u_2,u_4,u_5\},\{u_3,u_5,u_6,u_8\}\rangle\big),\\ & \big(e_2, \langle\{u_1,u_2,u_4\},\{u_3,u_5,u_6,u_7\},\{u_1,u_7,u_8\}\rangle\big),\\ & \big(e_3, \langle\{u_2,u_3,u_8\},\{u_1,u_7\},\{u_4,u_5,u_6\}\rangle\big),\\ & \big(e_4, \langle\{u_6,u_7,u_8\},\{u_3,u_5,u_6\},\{u_1,u_2,u_4\}\rangle\big)\Big\}\end{aligned}$$

here,

- $\mathcal{D}_{e_1} = \{u_2,u_5\}, \mathcal{D}_{e_2} = \{u_1,u_7\}, \mathcal{D}_{e_3} = \emptyset, \mathcal{D}_{e_4} = \{u_6\}$
- $\mathcal{D}_{e_1}^{w^+} = \{u_2\}, \mathcal{D}_{e_1}^{w^-} = \{u_5\}, \mathcal{D}_{e_1}^{s} = \emptyset$ and $\mathcal{D}_{e_1}^{\hat{s}} = \emptyset$,
- $\mathcal{D}_{e_2}^{w^+} = \emptyset, \mathcal{D}_{e_2}^{w^-} = \{u_7\}, \mathcal{D}_{e_2}^{s} = \{u_1\}$ and $\mathcal{D}_{e_2}^{\hat{s}} = \emptyset$
- $\mathcal{D}_{e_3}^{w^+} = \emptyset, \mathcal{D}_{e_3}^{w^-} = \emptyset$, $\mathcal{D}_{e_3}^{s} = \emptyset$ and $\mathcal{D}_{e_3}^{\hat{s}} = \emptyset$
- $\mathcal{D}_{e_4}^{w^+} = \{u_6\}, \mathcal{D}_{e_4}^{w^-} = \emptyset$, $\mathcal{D}_{e_4}^{s} = \emptyset$ and $\mathcal{D}_{e_4}^{\hat{s}} = \emptyset$

Definition 3.4. Let $\kappa \in \mathcal{SNC}_U^E$. If $\mathcal{D}_e = \emptyset$ for all $e \in E$, then snc-set κ is called consistent snc-set ($csnc-$set) and denoted by $\hat{\kappa}$.

Example 3.5. Let us consider parameter set E and initial universe set U in Example 3.3. If

$$\begin{aligned}\kappa(e_1) &= \langle\{u_1,u_7\},\{u_2,u_4,u_5\},\{u_3,u_6,u_8\}\rangle\\ \kappa(e_2) &= \langle\{u_2,u_4\},\{u_3,u_5,u_6\},\{u_1,u_7,u_8\}\rangle\\ \kappa(e_3) &= \langle\{u_2,u_3,u_8\},\{u_1,u_7\},\{u_4,u_5,u_6\}\rangle\\ \kappa(e_4) &= \langle\{u_6,u_7,u_8\},\{u_5\},\{u_1,u_2,u_3,u_4\}\rangle\end{aligned}$$

then,

$$\begin{aligned}\hat{\kappa} \ = \ & \Big\{\big(e_1, \langle\{u_1,u_7\},\{u_2,u_4,u_5\},\{u_3,u_6,u_8\}\rangle\big),\\ & \big(e_2, \langle\{u_2,u_4\},\{u_3,u_5,u_6\},\{u_1,u_7,u_8\}\rangle\big),\\ & \big(e_3, \langle\{u_2,u_3,u_8\},\{u_1,u_7\},\{u_4,u_5,u_6\}\rangle\big),\\ & \big(e_4, \langle\{u_6,u_7,u_8\},\{u_5\},\{u_1,u_2,u_3,u_4\}\rangle\big)\Big\}\end{aligned}$$

is a $csnc-$set.

Corollary 3.6. *Let $\kappa \in \mathcal{SNC}_U^E$. If $\mathcal{D}_e^{w^+} = \emptyset$, $\mathcal{D}_e^{w^-} = \emptyset$, $\mathcal{D}_e^{s} = \emptyset$ and $\mathcal{D}_e^{\hat{s}} = \emptyset$ for all $e \in E$, κ is a csnc−set. On the other hand, if κ is csnc-set, then $\kappa_t(e) \cap \kappa_i(e) = \emptyset$, $\kappa_i(e) \cap \kappa_f(e) = \emptyset$ and $\kappa_t(e) \cap \kappa_f(e) = \emptyset$.*

Now, in the following proposition, we shows relation between soft set and snc-set and between bipolar soft set and snc-set

Proposition 3.7. *Let E be a parameter set, U be an initial universe and κ be a csnc−set over U. Then,*

(i) If for all $e \in E$, $\kappa_i(e) = \emptyset$, f is a bipolar soft set

(ii) If for all $e \in E$, $\kappa_i(e) = \emptyset$ and $\kappa_\kappa(e) = \emptyset$, κ is a soft set.

Proof. The proof is clear. □

Definition 3.8. Let $\kappa \in \mathcal{SNC}_U^E$. If, for all $e \in E$, $\kappa_t(e) = \emptyset$, $\kappa_i(e) = U$ and $\kappa_f(e) = U$, then κ is called null snc-set and denoted by Θ.

Proposition 3.9. *Let κ be a null snc-set. Then $u \in \mathcal{D}_e^{w^-}$ for all $u \in U$ and $e \in E$.*

Proof. Let κ be a null snc-set. Since $(\kappa_i(e) \cap \kappa_f(e))/(\kappa_t(e) \cap \kappa_i(e) \cap \kappa_f(e)) = U$ for all $e \in E$, $u \in \mathcal{D}_e^{w^-}$. □

Definition 3.10. Let $\kappa \in \mathcal{SNC}_U^E$. If, for all $e \in E$, $\kappa_t(e) = U$, $\kappa_i(e) = \emptyset$ and $\kappa_f(e) = \emptyset$, then κ is called absolute snc-set and denoted by Ξ.

Definition 3.11. Let $\kappa, \tau \in \mathcal{SNC}_U^E$. If for all $e \in E$, $\kappa_t(e) \subseteq \tau_t(e)$, $\kappa_i(e) \supseteq \tau_i(e)$ and $\kappa_f(e) \supseteq \tau_f(e)$, κ is called soft neutrosophic classical subset of τ and denoted by $\kappa \preceq \tau$.

Example 3.12. Let $E = \{e_1, e_2, e_3, e_4\}$ be a parameter set and $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$. Suppose that

$$\begin{aligned} \kappa = \Big\{ & \big(e_1, \langle \{u_1, u_2, u_7\}, \{u_2, u_4, u_5\}, \{u_3, u_5, u_6, u_8\} \rangle\big), \\ & \big(e_2, \langle \{u_1, u_2, u_4\}, \{u_3, u_5, u_6, u_7\}, \{u_1, u_7, u_8\} \rangle\big), \\ & \big(e_3, \langle \{u_2, u_3, u_8\}, \{u_1, u_7\}, \{u_4, u_5, u_6\} \rangle\big), \\ & \big(e_4, \langle \{u_6, u_7, u_8\}, \{u_3, u_5, u_6\}, \{u_1, u_2, u_3, u_4\} \rangle\big) \Big\} \end{aligned}$$

and

$$\begin{aligned} \tau = \Big\{ & \big(e_1, \langle \{u_1, u_2\}, \{u_2, u_4, u_5, u_6\}, \{u_3, u_5, u_6, u_7, u_8\} \rangle\big) \\ & \big(e_2, \langle \{u_1, u_4\}, \{u_2, u_3, u_5, u_6, u_7\}, \{u_1, u_7, u_6, u_8\} \rangle\big) \\ & \big(e_3, \langle \{u_3, u_8\}, \{u_1, u_6, u_7\}, \{u_2, u_4, u_5, u_6\} \rangle\big) \\ & \big(e_4, \langle \{u_7\}, \{u_3, u_5, u_6, u_7\}, \{u_1, u_2, u_3, u_4, u_6\} \rangle\big) \Big\}. \end{aligned}$$

Since $\tau_t(e) \subseteq \kappa_t(e)$, $\tau_i(e) \supseteq \kappa_i(e)$ and $\tau_f(e) \supseteq \kappa_f(e)$ for all $e \in E$, $\tau \preceq \kappa$.

Definition 3.13. Let $f, g \in \mathcal{SNC}_U^E$. Then, κ and τ are equal, denoted by $\kappa = \tau$, if $\kappa \preceq \tau$ and $\tau \preceq \kappa$.

Proposition 3.14. *Let $\kappa, \tau \in \mathcal{SNC}_U^E$. Then,*

(i) $\kappa \preceq \Xi$

(ii) $\Theta \preceq \kappa$

(iii) $\kappa \preceq \kappa$

(iv) If $\kappa \preceq \tau$ and $\tau \preceq \sigma$, then $\kappa \preceq \sigma$

Proof. It is clear from Definition 3.11. □

Definition 3.15. Let $\kappa, \tau \in \mathcal{SNC}_U^E$. Then, union operation between two *snc*-sets κ and τ, denoted by $\kappa \smile \tau$, is defined as follows:

$$\kappa \smile \tau = \Big\{ \big(e, \langle \kappa_t(e) \cup \tau_t(e), \kappa_i(e) \cap \tau_i(e), \kappa_f(e) \cap \tau_f(e) \rangle\big) : e \in E \Big\}.$$

Proposition 3.16. *Let $\kappa, \tau, \sigma \in \mathcal{SNC}_U^E$. Then,*

(i) $\kappa \smile \kappa = \kappa$

(ii) $\kappa \smile \Theta = \kappa$

(iii) $\kappa \smile \Xi = \Xi$

(iv) $\kappa \smile \tau = \tau \smile \kappa$

(v) $\kappa \smile \big(\tau \smile \sigma\big) = \big(\kappa \smile \tau\big) \smile \sigma$

Proof. It can be proved by Definition 3.15. □

Proposition 3.17. *Let $\kappa, \tau \in \mathcal{SNC}_U^E$ and $u \in U$ be a degenerate element in both κ and τ. Then $u \in U$ is a degenerate element in $\kappa \smile \tau$.*

Proof. In order to prove, we will investigate four cases:

- Case 1: Let $u \in \mathcal{D}_e^{w^+}$ for any $e \in E$. Since $u \in (\kappa_t(e) \cap \kappa_i(e))$ and $u \notin (\kappa_t(e) \cap \kappa_i(e) \cap \kappa_f(e))$, $u \in \kappa_t(e)$ and $u \in \kappa_i(e)$ and $u \notin \kappa_f(e)$. Similarly, since $u \in (\tau_t(e) \cap \tau_i(e))$ and $u \notin (\tau_t(e) \cap \tau_i(e) \cap \tau_f(e))$, $u \in \tau_t(e)$ and $u \in \tau_i(e)$ and $u \notin \tau_f(e)$. Thus, $u \in \kappa_t(e) \cup \tau_t(e)$, $u \in \kappa_i(e) \cap \tau_i(e)$ and $u \in (\kappa_t(e) \cup \tau_t(e)) \cap \kappa_i(e) \cap \tau_i(e)$. Inasmuch as $u \notin \kappa_f(e)$ and $u \notin \tau_f(e)$, $u \notin \kappa_f(e) \cap \tau_f(e)$. Therefore $u \notin (\kappa_t(e) \cup \tau_t(e)) \cap (\kappa_i(e) \cap \tau_i(e)) \cap (\kappa_f(e) \cap \tau_f(e))$. We have $u \in \Big([(\kappa_t(e) \cup \tau_t(e)) \cap (\kappa_i(e) \cap \tau_i(e))] \setminus [(\kappa_t(e) \cup \tau_t(e)) \cap (\kappa_i(e) \cap \tau_i(e)) \cap (\kappa_f(e) \cap \tau_f(e))]\Big)$ and so u is a positive weak degenerate element in $\kappa \smile \tau$.

- Case 2: Let $u \in \mathcal{D}_e^{w^-}$ for any $e \in E$. Since $u \in (\kappa_i(e) \cap \kappa_f(e))$ and $u \notin (\kappa_t(e) \cap \kappa_i(e) \cap \kappa_f(e))$, $u \in \kappa_i(e)$ and $u \in \kappa_f(e)$ and $u \notin \kappa_t(e)$. Similarly, since $u \in (\tau_i(e) \cap \tau_f(e))$ and $u \notin (\tau_t(e) \cap \tau_i(e) \cap \tau_f(e))$, $u \in \tau_i(e)$ and $u \in \tau_f(e)$ and $u \notin \tau_t(e)$. Thus, $u \in \kappa_i(e) \cap \tau_i(e)$, $u \in \kappa_f(e) \cap \tau_f(e)$ and $u \in (\kappa_i(e) \cap \tau_f(e)) \cap \kappa_i(e) \cap \tau_i(e)$. Inasmuch as $u \notin \kappa_f(e)$ and $u \notin \tau_f(e)$, $u \notin \kappa_f(e) \cap \tau_f(e)$. Therefore $u \notin (\kappa_t(e) \cup \tau_t(e)) \cap (\kappa_i(e) \cap \tau_i(e)) \cap (\kappa_f(e) \cap \tau_f(e))$. We have $u \in \Big([(\kappa_i(e) \cap \tau_i(e)) \cap (\kappa_f(e) \cap \tau_f(e))] \setminus [(\kappa_t(e) \cup \tau_t(e)) \cap (\kappa_i(e) \cap \tau_i(e)) \cap (\kappa_f(e) \cap \tau_f(e))]\Big)$ and so u is a negative weak degenerate element in $\kappa \smile \tau$.

- Case 3: Let $u \in \mathcal{D}_e^{s}$ for any $e \in E$. Since $u \in (\kappa_t(e) \cap \kappa_f(e))$ and $u \notin (\kappa_t(e) \cap \kappa_i(e) \cap \kappa_f(e))$, $u \in \kappa_t(e)$ and $u \in \kappa_f(e)$ and $u \notin \kappa_i(e)$. Similarly, since $u \in (\tau_t(e) \cap \tau_f(e))$ and $u \notin (\tau_t(e) \cap \tau_i(e) \cap \tau_f(e))$, $u \in \tau_t(e)$ and $u \in \tau_f(e)$ and $u \notin \tau_i(e)$. Thus, $u \in \kappa_t(e) \cup \tau_t(e)$, $u \in \kappa_f(e) \cap \tau_f(e)$ and $u \in (\kappa_t(e) \cup \tau_t(e)) \cap \kappa_f(e) \cap \tau_f(e)$. Since $u \notin \kappa_i(e)$ and $u \notin \tau_i(e)$, $u \notin \kappa_i(e) \cap \tau_i(e)$. Therefore $u \notin (\kappa_t(e) \cup \tau_t(e)) \cap (\kappa_i(e) \cap \tau_i(e)) \cap (\kappa_f(e) \cap \tau_f(e))$. We have $u \in \Big([(\kappa_t(e) \cup \tau_t(e)) \cap (\kappa_f(e) \cap \tau_f(e))] \setminus [(\kappa_t(e) \cup \tau_t(e)) \cap (\kappa_i(e) \cap \tau_i(e)) \cap (\kappa_f(e) \cap \tau_f(e))]\Big)$ and so u is a strict degenerate element in $\kappa \smile \tau$.

- Case 4: Let $u \in \mathcal{D}_e^{\hat{s}}$ for any $e \in E$. Since $u \in (\kappa_t(e) \cap \kappa_i(e) \cap \kappa_f(e))$, $u \in \kappa_t(e)$, $u \in \kappa_i(e)$ and $u \in \kappa_f(e)$. Similarly, since $u \in (\tau_t(e) \cap \tau_i(e) \cap \tau_f(e))$, $u \in \tau_t(e)$ and $u \in \tau_i(e)$ and $u \in \tau_f(e)$. Thus, $u \in \kappa_t(e) \cup \tau_t(e)$, $u \in \kappa_i(e) \cap \tau_i(e)$ and $u \in (\kappa_f(e) \cap \tau_f(e))$. Therefore $u \in [(\kappa_t(e) \cup \tau_t(e)) \cap (\kappa_i(e) \cap \tau_i(e)) \cap (\kappa_f(e) \cap \tau_f(e))]$. Hence u is a full strict degenerate element in $\kappa \smile \tau$.

□

Definition 3.18. Let $\kappa, \tau \in \mathcal{SNC}_U^E$. Then, intersection operation between two snc-sets κ and τ, denoted by $\kappa \frown \tau$, is defined as follows:

$$\kappa \frown \tau = \Big\{\big(e, \langle \kappa_t(e) \cap \tau_t(e), \kappa_i(e) \cup \tau_i(e), \kappa_f(e) \cup \tau_f(e)\rangle\big) : e \in E\Big\}.$$

Proposition 3.19. *Let $\kappa, \tau \in \mathcal{SNC}_U^E$ and $u \in U$ be a degenerate element in both κ and τ. Then $u \in U$ is a degenerate element in $\kappa \frown \tau$.*

Proof. The proof can be made in similar way to proof of Proposition 3.17. □

Proposition 3.20. *Let $\hat{\kappa}$ and $\hat{\tau}$ be two csnc-sets over U. Then, $\hat{\kappa} \smile \hat{\tau}$ is a csnc-set.*

Proof. Let $\hat{\kappa}$ and $\hat{\tau}$ be two csnc-sets over U. By Definition 3.4 and Corollary 3.6, $\hat{\kappa}_t(e) \cap \hat{\kappa}_t(e) = \emptyset$, $\hat{\kappa}_i(e) \cap \hat{\kappa}_i(e) = \emptyset$, $\hat{\kappa}_f(e) \cap \hat{\kappa}_f(e) = \emptyset$ and $\hat{\tau}_t(e) \cap \hat{\tau}_t(e) = \emptyset$, $\hat{\tau}_i(e) \cap \hat{\tau}_i(e) = \emptyset$, $\hat{\tau}_f(e) \cap \hat{\tau}_f(e) = \emptyset$ for all $e \in E$. Therefore, it is easy shown that

$$\begin{aligned}
(\hat{\kappa}_t(e) \cup \hat{\tau}_t(e)) \cap (\hat{\kappa}_i(e) \cap \hat{\tau}_i(e)) &= \emptyset \\
(\hat{\kappa}_t(e) \cup \hat{\tau}_t(e)) \cap (\hat{\kappa}_f(e) \cap \hat{\tau}_f(e)) &= \emptyset \\
(\hat{\kappa}_i(e) \cap \hat{\tau}_i(e)) \cap (\hat{\kappa}_f(e) \cap \hat{\tau}_f(e)) &= \emptyset.
\end{aligned}$$

Thus $\hat{\kappa} \smile \hat{\tau} = \Big\{(\hat{\kappa}_t(e) \cup \hat{\tau}_t(e)), (\hat{\kappa}_i(e) \cap \hat{\tau}_i(e)), (\hat{\kappa}_f(e) \cap \hat{\tau}_f(e))\Big\}$ is a csnc-set. □

Note that $\hat{\kappa} \frown \hat{\tau}$ may not be cscn-set.

Example 3.21. Let $E = \{e_1, e_2\}$ be a parameter set and $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ be an initial universe. Assume that

$$\hat{\kappa} = \Big\{\big(e_1, \{u_1, u_2\}, \{u_3, u_4, u_5\}, \{u_6, u_7, u_8\}\big),$$
$$\big(e_2, \{u_3, u_5\}, \{u_2, u_4\}, \{u_1, u_6, u_7, u_8\}\big)\Big\}$$

and

$$\hat{\tau} = \Big\{\big(e_1, \{u_2, u_3\}, \{u_5, u_6, u_7\}, \{u_1, u_4, u_8\}\big),$$
$$\big(e_2, \{u_1, u_5\}, \{u_3, u_7\}, \{u_2, u_4, u_6, u_8\}\big)\Big\}.$$

Then

$$\hat{\kappa} \frown \hat{\tau} = \Big\{\big(e_1, \{u_2\}, \{u_3, u_4, u_5, u_6, u_7\}, \{u_1, u_4, u_6, u_7, u_8\}\big),$$
$$\big(e_2, \{u_5\}, \{u_2, u_3, u_4, u_7\}, \{u_1, u_2, u_4, u_6, u_7, u_8\}\big)\Big\}.$$

Here, it is seen that $\hat{\kappa} \frown \hat{\tau}$ is not a csnc-set.

Definition 3.22. Let $\kappa \in \mathcal{SNC}_U^E$. Then, complement of κ, denoted by $\kappa^{\tilde{c}}$, is defined as follows

$$\kappa^{\tilde{c}} = \Big\{\big(e, \langle \kappa_f(e), (\kappa_i(e))^c, \kappa_t(e)\rangle\big) : e \in E\Big\}.$$

Example 3.23. Let $E = \{e_1, e_2, e_3, e_4\}$ be a parameter set and $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$. Suppose that

$$\begin{aligned}
\kappa = \Big\{&\big(e_1, \langle\{u_1, u_2, u_7\}, \{u_2, u_4, u_5\}, \{u_3, u_5, u_6, u_8\}\rangle\big), \\
&\big(e_2, \langle\{u_1, u_2, u_4\}, \{u_3, u_5, u_6, u_7\}, \{u_1, u_7, u_8\}\rangle\big), \\
&\big(e_3, \langle\{u_2, u_3, u_8\}, \{u_1, u_7\}, \{u_4, u_5, u_6\}\rangle\big), \\
&\big(e_4, \langle\{u_6, u_7, u_8\}, \{u_3, u_5, u_6\}, \{u_1, u_2, u_3, u_4\}\rangle\big)\Big\}
\end{aligned}$$

and

$$\tau = \Big\{(e_1, \langle\{u_2, u_3, u_7\}, \{u_2, u_4, u_6\}, \{u_3, u_8\}\rangle,) \\ (e_2, \langle\{u_1, u_4\}, \{u_5, u_7, u_8\}, \{u_1, u_6, u_8\}\rangle,) \\ (e_3, \langle\{u_1, u_3, u_5, u_8\}, \{u_1, u_6, u_7\}, \{u_2, u_5\}\rangle,) \\ (e_4, \langle\{u_3, u_7, u_8\}, \{u_3, u_6, u_7\}, \{u_1, u_2, u_4\}\rangle)\Big\}$$

be two snc-sets. Then,

$$\kappa \smile \tau = \Big\{(e_1, \langle\{u_1, u_2, u_3, u_7\}, \{u_2, u_4\}, \{u_3, u_8\}\rangle) \\ (e_2, \langle\{u_1, u_2, u_4\}, \{u_5, u_7\}, \{u_1, u_8\}\rangle) \\ (e_3, \langle\{u_1, u_2, u_3, u_5, u_8\}, \{u_1, u_7\}, \{u_5\}\rangle) \\ (e_4, \langle\{u_3, u_6, u_7, u_8\}, \{u_3, u_6\}, \{u_1, u_2, u_4\}\rangle)\Big\}$$

$$\kappa \frown \tau = \Big\{(e_1, \langle\{u_2, u_7\}, \{u_2, u_4, u_5, u_6\}, \{u_3, u_5, u_6, u_8\}\rangle), \\ (e_2, \langle\{u_1, u_4\}, \{u_3, u_5, u_6, u_7, u_8\}, \{u_1, u_6, u_7, u_8\}\rangle), \\ (e_3, \langle\{u_3, u_8\}, \{u_1, u_6, u_7\}, \{u_2, u_4, u_5, u_6\}\rangle), \\ (e_4, \langle\{u_7, u_8\}, \{u_3, u_5, u_6, u_7\}, \{u_1, u_2, u_3, u_4\}\rangle)\Big\}$$

and

$$\kappa^{\tilde{c}} = \Big\{(e_1, \langle\{u_3, u_5, u_6, u_8\}, \{u_1, u_3, u_6, u_7, u_8\}, \{u_1, u_2, u_7\}\rangle), \\ (e_2, \langle\{u_1, u_7, u_8\}, \{u_1, u_2, u_4, u_8\}, \{u_1, u_2, u_4\}\rangle), \\ (e_3, \langle\{u_4, u_5, u_6\}, \{u_2, u_3, u_4, u_5, u_8\}, \{u_2, u_3, u_8\}\rangle), \\ (e_4, \langle\{u_1, u_2, u_3, u_4\}, \{u_1, u_2, u_4, u_7, u_8\}, \{u_6, u_7, u_8\}\rangle)\Big\}$$

Proposition 3.24. *Let* $\kappa, \tau, \sigma \in \mathcal{SNC}_U^E$. *Then,*

(i) $\kappa \frown \kappa = \kappa$

(ii) $\kappa \frown \Theta = \Theta$

(iii) $\kappa \frown \Xi = \kappa$

(iv) $\kappa \frown \tau = \tau \frown \kappa$

(v) $\kappa \frown \big(\tau \frown \sigma\big) = \big(\kappa \frown \tau\big) \frown \sigma.$

Proof. It can be proved by Definition 3.18. □

Although union of a soft set and its complement is universal soft set, and intersection of them is empty soft set, this situation is not available in a snc-set. Namely sometimes $\kappa \smile \kappa^{\tilde{c}} \neq \Xi$ and $\kappa \frown \kappa^{\tilde{c}} \neq \Theta$.

Proposition 3.25. *Let* $\kappa, \tau \in \mathcal{SNC}_U^E$. *Then,*

(i) $(\kappa \smile \tau)^{\tilde{c}} = \kappa^{\tilde{c}} \frown \tau^{\tilde{c}},$

(ii) $(\kappa \frown \tau)^{\tilde{c}} = \kappa^{\tilde{c}} \smile \tau^{\tilde{c}}.$

Proof. (i)

$$\begin{aligned}(\kappa \smile \tau)^{\tilde{c}} &= \Big\{\big(e, \langle \kappa_t(e) \cup \tau_t(e), \kappa_i(e) \cap \tau_i(e), \kappa_f(e) \cap \tau_f(e)\rangle\big) : e \in E\Big\}^{\tilde{c}} \\ &= \Big\{\big(e, \langle \kappa_f(e) \cap \tau_f(e), (\kappa_i(e) \cap \tau_i(e))^c, \kappa_t(e) \cup \tau_t(e)\rangle\big) : e \in E\Big\} \\ &= \Big\{\big(e, \langle \kappa_f(e) \cap \tau_f(e), \kappa_i(e)^c \cup \tau_i(e)^c, \kappa_t(e) \cup \tau_t(e)\rangle\big) : e \in E\Big\} \\ &= \kappa^{\tilde{c}} \frown \tau^{\tilde{c}}.\end{aligned}$$

(ii)

$$\begin{aligned}(\kappa \frown \tau)^{\tilde{c}} &= \Big\{\big(e, \langle \kappa_t(e) \cap \tau_t(e), \kappa_i(e) \cup \tau_i(e), \kappa_f(e) \cup \tau_f(e)\rangle\big) : e \in E\Big\}^{\tilde{c}} \\ &= \Big\{\big(e, \langle \kappa_f(e) \cup \tau_f(e), (\kappa_i(e) \cup \tau_i(e))^c, \kappa_t(e) \cap \tau_t(e)\rangle\big) : e \in E\Big\} \\ &= \Big\{\big(e, \langle \kappa_f(e) \cup \tau_f(e), \kappa_i(e)^c \cap \tau_i(e)^c, \kappa_t(e) \cap \tau_t(e)\rangle\big) : e \in E\Big\} \\ &= \kappa^{\tilde{c}} \smile \tau^{\tilde{c}}.\end{aligned}$$

□

Definition 3.26. [7] Let (φ, E) be a soft set over U. Then a subset of $U \times E$ is uniquely defined by

$$R_A = \{(u, e) | e \in E, u \in F(e)\}$$

which is called a relation form of (φ, E)

Here we introduce a similar notation for snc-sets as follows.

Definition 3.27. Let $\kappa \in \mathcal{SNC}_U^E$. Then a subset of $U \times E$ is uniquely defined by

$$R_\kappa = \Big\{(R_\kappa^t(e), R_\kappa^i(e), R_\kappa^f(e)) : e \in E\Big\}.$$

which is called a relation form of κ. Here $R_\kappa^t(e) = \{(u, e) : u \in \kappa_t(e)\}$, $R_\kappa^i(e) = \{(u, e) : u \in \kappa_i(e)\}$ and $R_\kappa^f(e) = \{(u, e) : u \in \kappa_f(e)\}$.

Definition 3.28. [2] Let (φ, E) and (ψ, E) be two soft sets and their corresponding relation forms are R_φ and R_ψ, respectively. A new relation form $R_{\varphi\psi}$ is defined as:

$$(u, e) \in R_{\varphi\psi} \Leftrightarrow (u, e) \in R_\varphi \triangle R_\psi$$

where $R_\varphi \triangle R_\psi$ is a symmetric difference of R_φ and R_ψ. Then the soft set (ω, E), whose relation form is $R_{\varphi\psi}$ is called the inverse product of soft sets (φ, E) and (ψ, E) and denoted by

$$(\varphi, E) \bullet_i (\psi, E) = (\omega, E).$$

Based on the Definition 3.28, we introduce similar notation for snc-sets.

Definition 3.29. Let $\kappa, \tau \in \mathcal{SNC}_U^E$ and their corresponding relation forms are R_κ and R_τ, respectively. A new relation form $R_{\kappa\tau}$ is defined as:

$$R_{\kappa\tau} = \{(R_\kappa^t(e) \triangle R_\tau^t(e), R_\kappa^i(e) \triangle R_\tau^i(e), R_\kappa^f(e) \triangle R_\tau^f(e)) : e \in E\}$$

where $\triangle$ denotes symmetric difference operation in classical sets. Let σ be a snc-set, whose relation form $R_{\kappa\tau}$. Then, σ is inverse relation of snc-sets κ and τ, and denoted by

$$\kappa *_i \tau = \sigma.$$

Here, it is easily seen that $\sigma \in \mathcal{SNC}_U^E$.

Example 3.30. Let $E = \{e_1, e_2, e_3\}$ be a parameter set and $U = \{u_1, u_2, u_3, u_4, u_5\}$. Suppose that

$$\begin{aligned}\kappa &= \Big\{(e_1,\langle\{u_1,u_2\},\{u_2,u_4\},\{u_3,u_5\}\rangle),(e_2,\langle\{u_1,u_4\},\{u_3\},\{u_1,u_2,u_5\}\rangle),\\ &(e_3,\langle\{u_2,u_3\},\{u_1,u_5\},\{u_3,u_4\}\rangle)\Big\}\end{aligned}$$

and

$$\begin{aligned}\tau &= \Big\{(e_1,\langle\{u_2,u_3\},\{u_2,u_5\},\{u_1,u_4\}\rangle),(e_2,\langle\{u_4\},\{u_1,u_2,u_3\},\{u_5\}\rangle,)\\ &(e_3,\langle\{u_1,u_4\},\{u_2,u_4,u_5\},\{u_1,u_3\}\rangle)\Big\}\end{aligned}$$

be two snc-sets. Then,

$$\begin{aligned}R_\kappa &= \Big\{(\{(u_1,e_1),(u_2,e_1)\},\{(u_2,e_1),(u_4,e_1)\},\{(u_3,e_1),(u_5,e_1)\}),\\ &(\{(u_1,e_2),(u_4,e_2)\},\{(u_3,e_2)\},\{(u_1,e_2),(u_2,e_2),(u_5,e_2)\}),\\ &(\{(u_2,e_3),(u_3,e_3)\},\{(u_1,e_3),(u_5,e_3)\},\{(u_1,e_3),(u_4,e_3)\})\Big\},\end{aligned}$$

$$\begin{aligned}R_\tau &= \Big\{(\{(u_2,e_1),(u_3,e_1)\},\{(u_2,e_1),(u_5,e_1)\},\{(u_1,e_1),(u_4,e_1)\}),\\ &(\{(u_4,e_2)\},\{(u_1,e_2),(u_2,e_2),(u_3,e_2)\},\{(u_5,e_2)\}),\\ &(\{(u_1,e_3),(u_4,e_3)\},\{(u_2,e_3),(u_4,e_3),(u_5,e_3)\},\{(u_1,e_3),(u_3,e_3)\})\Big\}\end{aligned}$$

and

$$\begin{aligned}R_{\kappa\tau} &= \Big\{(\{(u_1,e_1),(u_3,e_1)\},\{(u_4,e_1),(u_5,e_1)\},\{(u_3,e_1),(u_5,e_1),(u_1,e_1),(u_4,e_1)\}),\\ &(\{(u_1,e_2)\},\{(u_1,e_2),(u_2,e_2)\},\{(u_1,e_2),(u_2,e_2)\}),\\ &(\{(u_1,e_3),(u_2,e_3),(u_3,e_3),(u_4,e_3)\},\{(u_1,e_3),(u_2,e_3),(u_4,e_3)\},\\ &\{(u_3,e_3),(u_4,e_3)\})\Big\}.\end{aligned}$$

Corresponding snc-set of $R_{\kappa\tau}$ is

$$\begin{aligned}\sigma &= \Big\{(e_1,\langle\{u_1,u_3,u_4,u_5\},\{u_4,u_5\},\{u_1,u_3,u_4,u_5\}\rangle),(e_2,\langle\{u_1\},\{u_1,u_2\},\{u_1,u_2\}\rangle,)\\ &(e_3,\langle\{u_1,u_2,u_3,u_4\},\{u_1,u_2,u_4\},\{u_3,u_4\}\rangle)\Big\}.\end{aligned}$$

Therefore $\kappa *_i \tau = \sigma$.

Proposition 3.31. *Let $\kappa,\tau \in \mathcal{SNC}_U^E$. Then,*

$$\kappa *_i \tau = \tau *_i \kappa.$$

Proof. The proof is obvious from definition of inverse product of snc-sets. □

Proposition 3.32. *Let κ,τ and $\sigma \in \mathcal{SNC}_U^E$. Then,*

$$(\kappa *_i \tau) *_i \sigma = \kappa *_i (\tau *_i \sigma).$$

Proof. Let κ,τ and $\sigma \in \mathcal{SNC}_U^E$. Then

$$\begin{aligned}(\kappa *_i \tau) *_i \sigma &= \Big\{(R^t_\kappa(e)\triangle R^t_\tau(e))\triangle R^t_\sigma(e), R^i_\kappa(e)\triangle R^i_\tau(e)\triangle R^i_\sigma(e),\\ &R^f_\kappa(e)\triangle R^f_\tau(e))\triangle R^f_\sigma(e) : e\in E\Big\}.\end{aligned}$$

Firstly, we will shown that $(R^t_\kappa(e)\triangle R^t_\tau(e))\triangle R^t_\sigma(e) = R^t_\kappa(e)\triangle(R^t_\tau(e)\triangle R^t_\sigma(e))$, $(R^i_\kappa(e)\triangle R^i_\tau(e))\triangle R^i_\sigma(e) = R^i_\kappa(e)\triangle(R^i_\tau(e)\triangle R^i_\sigma(e))$ and $(R^f_\kappa(e)\triangle R^f_\tau(e))\triangle R^f_\sigma(e) = R^f_\kappa(e)\triangle(R^f_\tau(e)\triangle R^f_\sigma(e))$. Let $\kappa *_i \tau = \theta$, where θ is a snc-set over U and its relation forms are $R^t_{\kappa\tau}(e)$, $R^i_{\kappa\tau}(e)$ and $R^f_{\kappa\tau}(e)$, then

$$(u,e) \in R^t_{\kappa\tau}(e) \Leftrightarrow (u,e) \in R^t_\kappa(e)\triangle R^t_\tau(e)$$

where $R^t_\kappa(e)$ and $R^t_\tau(e)$ are relation forms of membership set of κ and τ, respectively. Let $\theta *_i \sigma = \epsilon$ and its relation form of membership set is $R^t_{(\kappa\tau)\sigma}(e)$, where

$$(u,e) \in R^t_{(\kappa\tau)\sigma}(e) \Leftrightarrow (u,e) \in R^t_{\kappa\tau}(e)\triangle R^t_\sigma(e) \Leftrightarrow (u,e) \in (R^t_\kappa(e)\triangle R^t_\tau(e))\triangle R^t_\sigma(e).$$

Let us consider the snc-set $\kappa *_i (\tau *_i \sigma)$. Assume that $\tau *_i \sigma = \gamma$. Here $\gamma \in \mathcal{SNC}^E_U$ and its relation form of membership set is $R^t_{\tau\sigma}(e)$. From definition of $R^t_{\tau\sigma}(e)$,

$$(u,e) \in R^t_{\tau\sigma}(e) \Leftrightarrow (u,e) \in R^t_\tau(e)\triangle R^t_\sigma(e)$$

where $R^t_\tau(e)$ and $R^t_\sigma(e)$ are relation forms of membership set of τ and σ. Let $\kappa * \gamma = \rho$. Then, its relation form of membership set is $R^t_{\kappa(\tau\sigma)}(e)$, where

$$(u,e) \in R^t_{\kappa(\tau\sigma)}(e) \Leftrightarrow R^t_\kappa(e)\triangle R^t_\gamma(e) \Leftrightarrow R^t_\kappa(e)\triangle(R^t_\tau(e)\triangle R^t_\sigma(e)).$$

Similarly, we can prove that $(R^i_\kappa(e)\triangle R^i_\tau(e))\triangle R^i_\sigma(e) = R^i_\kappa(e)\triangle(R^i_\tau(e)\triangle R^i_\sigma(e))$ and $(R^f_\kappa(e)\triangle R^f_\tau(e))\triangle R^f_\sigma(e) = R^f_\kappa(e)\triangle(R^f_\tau(e)\triangle R^f_\sigma(e))$. Thus, we have $(\kappa *_i \tau) *_i \sigma = \kappa *_i (\tau *_i \sigma)$. □

Proposition 3.33. *Let* $\kappa, \tau \in \mathcal{SNC}^E_U$. *Then,*

$$(\kappa *_i \tau)^{\tilde{c}} = \kappa^{\tilde{c}} *_i \tau^{\tilde{c}}$$

Proof. The proof is clear from Definition 3.22 and 3.28. □

Definition 3.34. Let $\kappa, \tau \in \mathcal{SNC}^E_U$. Then, natural union of $snc-$sets κ and τ, denoted by $\kappa \smile_n \tau$, and is defined as follows:

$$\kappa \smile_n \tau = \Big\{\big(e, \langle \kappa_t(e) \cup \tau_t(e), \kappa_i(e) \cup \tau_i(e), \kappa_f(e) \cup \tau_f(e)\rangle\big) : e \in E\Big\}.$$

Definition 3.35. Let $\kappa, \tau \in \mathcal{SNC}^E_U$. Then, natural intersection of $snc-$sets κ and τ, denoted by $\kappa \frown_n \tau$, and is defined as follows:

$$\kappa \frown_n \tau = \Big\{\big(e, \langle \kappa_t(e) \cap \tau_t(e), \kappa_i(e) \cap \tau_i(e), \kappa_f(e) \cap \tau_f(e)\rangle\big) : e \in E\Big\}.$$

Proposition 3.36. *Let* $\kappa, \tau \in \mathcal{SNC}^E_U$. *Then,*

(i) $\kappa \smile_n \tau \preceq \kappa \smile \tau$

(ii) $\kappa \frown_n \tau \succeq \kappa \frown \tau$

(iii) $\kappa \frown_n \tau \preceq \kappa \smile \tau$

(iv) $(\kappa \smile_n \tau)^{\tilde{c}} = \kappa^{\tilde{c}} \frown_n \tau^{\tilde{c}}$

(v) $(\kappa \frown_n \tau)^{\tilde{c}} = \kappa^{\tilde{c}} \smile_n \tau^{\tilde{c}}$.

Proof. The proof can be obtained from Definition 3.11,3.34 and 3.35. □

Definition 3.37. Let $\kappa, \tau \in \mathcal{SNC}^E_U$. Then, OR-product of κ and τ, denoted by $\kappa \oplus \tau$, is defined by

$$\kappa \oplus \tau = \Big\{\big((e_1,e_2), \langle \kappa_t(e_1) \cup \tau_t(e_2), \kappa_i(e_1) \cap \tau_i(e_2), \kappa_\kappa(e_1) \cap \tau_\kappa(e_2)\rangle\big) : e_1, e_2 \in E\Big\}.$$

Definition 3.38. Let $\kappa, \tau \in \mathcal{SNC}^E_U$. Then, AND-product of κ and τ, denoted by $\kappa \otimes \tau$, is defined by

$$\kappa \otimes \tau = \Big\{\big((e_1,e_2), \langle \kappa_t(e_1) \cap \tau_t(e_2), \kappa_i(e_1) \cup \tau_i(e_2), \kappa_\kappa(e_1) \cup \tau_\kappa(e_2)\rangle\big) : e_1, e_2 \in E\Big\}.$$

Proposition 3.39. *Let $\kappa, \tau, \sigma \in \mathcal{SNC}_U^E$. Then,*

(i) $\kappa \oplus \tau = \tau \oplus \kappa$

(ii) $\kappa \oplus \big(\tau \oplus \sigma\big) = \big(\kappa \oplus \tau\big) \oplus \sigma$

(iii) $\kappa \otimes \tau = \tau \otimes \kappa$

(iv) $\kappa \otimes \big(\tau \otimes \sigma\big) = \big(\kappa \otimes \tau\big) \otimes \sigma$.

Proof. It can be proved by using Definition 3.37 and 3.38. □

Proposition 3.40. *Let $\kappa, \tau \in \mathcal{SNC}_U^E$. Then,*

(i) $(\kappa \oplus \tau)^{\tilde{c}} = \kappa^{\tilde{c}} \otimes \tau^{\tilde{c}}$

(ii) $(\kappa \otimes \tau)^{\tilde{c}} = \kappa^{\tilde{c}} \oplus \tau^{\tilde{c}}$.

Proof. (i)

$$\begin{aligned}(\kappa \oplus \tau)^{\tilde{c}} &= \Big\{\big((e_1, e_2), \langle \kappa_t(e_1) \cup \tau_t(e_2), \kappa_i(e_1) \cap \tau_i(e_2), \kappa_\kappa(e_1) \cap \tau_\kappa(e_2)\rangle\big) : \\ &\quad e_1, e_2 \in E\Big\}^{\tilde{c}} \\ &= \Big\{\big((e_1, e_2), \langle \kappa_\kappa(e_1) \cap \tau_\kappa(e_2), (\kappa_i(e_1) \cup \tau_i(e_2))^c, \kappa_t(e_1) \cup \tau_t(e_2)\rangle\big) : \\ &\quad e_1, e_2 \in E\Big\} \\ &= \kappa^{\tilde{c}} \otimes \tau^{\tilde{c}}.\end{aligned}$$

(ii)

$$\begin{aligned}(\kappa \otimes \tau)^{\tilde{c}} &= \Big\{\big((e_1, e_2), \langle \kappa_t(e_1) \cap \tau_t(e_2), \kappa_i(e_1) \cup \tau_i(e_2), \kappa_\kappa(e_1) \cup \tau_\kappa(e_2)\rangle\big) : \\ &\quad e_1, e_2 \in E\Big\}^{\tilde{c}} \\ &= \Big\{\big((e_1, e_2), \langle \kappa_\kappa(e_1) \cup \tau_\kappa(e_2), (\kappa_i(e_1) \cap \tau_i(e_2))^c, \kappa_t(e_1) \cap \tau_t(e_2)\rangle\big) : \\ &\quad e_1, e_2 \in E\Big\} \\ &= \kappa^{\tilde{c}} \oplus \tau^{\tilde{c}}.\end{aligned}$$

□

Definition 3.41. Let $\kappa, \tau \in \mathcal{SNC}_U^E$. Then, natural OR-product of κ and τ, denoted by $\kappa \oplus_n \tau$, is defined by

$$\kappa \oplus_n \tau = \Big\{\big((e_1, e_2), \langle \kappa_t(e_1) \cup \tau_t(e_2), \kappa_i(e_1) \cup \tau_i(e_2), \kappa_\kappa(e_1) \cup \tau_\kappa(e_2)\rangle\big) : e_1, e_2 \in E\Big\}.$$

Definition 3.42. Let $\kappa, \tau \in \mathcal{SNC}_U^E$. Then, natural AND-product of κ and τ, denoted by $\kappa \otimes_n \tau$, is defined by

$$\kappa \otimes_n \tau = \Big\{\big((e_1, e_2), \langle \kappa_t(e_1) \cap \tau_t(e_2), \kappa_i(e_1) \cap \tau_i(e_2), \kappa_\kappa(e_1) \cap \tau_\kappa(e_2)\rangle\big) : e_1, e_2 \in E\Big\}.$$

Proposition 3.43. *Let $\kappa, \tau, \sigma \in \mathcal{SNC}_U^E$. Then,*

(i) $\kappa \oplus_n \tau = \tau \oplus_n \kappa$

(ii) $\kappa \oplus_n \big(\tau \oplus_n \sigma\big) = \big(\kappa \oplus \tau\big) \oplus \sigma$

(iii) $\kappa \otimes_n \tau = \tau \otimes_n \kappa$

(iv) $\kappa \otimes_n \big(\tau \otimes_n \sigma\big) = \big(\kappa \otimes_n \tau\big) \otimes_n \sigma$

(v) $(\kappa \oplus_n \tau)^{\tilde{c}} = \kappa^{\tilde{c}} \otimes_n \tau^{\tilde{c}}$

(vi) $(\kappa \otimes_n \tau)^{\tilde{c}} = \kappa^{\tilde{c}} \oplus_n \tau^{\tilde{c}}$

Proof. It can be proved by using Definition 3.41 and 3.42. □

4 Group decision making method

In this section, we propose a decision making method based on the snc-sets. Firstly, we define some notions that necessary to construct algorithm of decision making method. Finally, we will present an application of soft neutrosophic classical set theory in a decision making problem.

Definition 4.1. Let $U = \{u_1, u_2, ..., u_n\}$ be an initial universe and $E = \{e_1, e_2, ..., e_m\}$ be a parameter set. κ be snc-set over U related to parameter set E. Then non-degenerate value of $u_i \in U$ related to parameter $e_j \in E$, denoted by ν_{ij}, and is defined as follows:

$$\nu_{ij} = \begin{cases} 1, & u_i \in \kappa_t(e_j) \setminus (\kappa_i(e_j) \cup \kappa_f(e_j)) \\ 0, & u_i \in (\kappa_i(e_j) \setminus (\kappa_t(e_j) \cup \kappa_f(e_j))) \cup D^s_{e_j} \cup (U \setminus (\kappa_t(e_j) \cup \kappa_i(e_j) \cup \kappa_f(e_j))) \\ -1, & u_i \in \kappa_f(e_j) \setminus (\kappa_t(e_j) \cup \kappa_i(e_j)) \\ 1/2, & u_i \in D^{w^+}_{e_j} \\ -1/2, & u_i \in D^{w^-}_{e_j} \\ 1/6, & u_i \in D^{\tilde{s}}_{e_j} \end{cases}$$

Definition 4.2. [19] The score of an object $u_i \in U$ is S_i and is calculated as $S_i = \sum_{j=1}^{m} \nu_{ij}$.

Definition 4.3. Let $U = \{u_1, u_2, ..., u_n\}$ be an initial universe and S_i be the scores of $u_i \in U$, for $i = 1, 2, ..., n$. Then $max\{S_i : i = 1, 2, ..., n\}$ and $min\{S_i : i = 1, 2, ...n\}$ are called positive consistent element and negative consistent element, respectively.

From now on, positive consistent element and negative consistent element of a snc-set will be denoted by $\overline{p}$ and $\overline{n}$, respectively.

Example 4.4. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a initial universe and $E = \{e_1, e_2, e_3\}$ be a set of parameters. Let us consider given snc-set κ as follows:

$$\kappa = \Big\{\big(e_1, \langle\{u_1, u_2, u_3\}, \{u_3, u_4, u_5\}, \{u_4, u_6\}\rangle\big), \big(e_2, \langle\{u_3, u_5\}, \{u_4, u_5\}, \{u_1, u_2, u_3, u_5\}\rangle\big), \big(e_3, \langle\{u_4, u_6\}, \{u_1, u_2, u_6\}, \{u_3, u_5, u_1\}\rangle\big)\Big\}.$$

Then,

$$\begin{aligned} S_1 = \sum_{j=1}^{3} \nu_{1j} &= \nu_{11} + \nu_{12} + \nu_{13} \\ &= 1 - 1 - 0.5 = -0.5 \\ S_2 &= 1 - 1 + 0.5 = 0.5 \\ S_3 &= 0.5 + 0 - 1 = -0.5 \\ S_4 &= -0.5 + 0.5 + 1 = 1 \\ S_5 &= 0.5 + 0.17 - 1 = -0.33 \\ S_6 &= -1 + 1 + 0.5 = 0.5. \end{aligned}$$

In here, alternative u_4 is positive consistent element and u_3 and u_1 are negative consistent elements.

Now, we present an algorithm to select the optimum element as;

Algorithm

Step 1: Input the snc-set κ and τ,
Step 2: Find OR-product of snc-set κ and τ,
Step 3: Compute the score S_i of u_i $\forall u_i \in U$
Step 4: Find positive consistent element $\overline{p}$ as an optimum element.

Let us use the algorithm to solve the following problem.

Example 4.5. Assume that, there is a company which has three decision-makers who are working as experts to choose one or more desirable alternatives from set of alternatives for their own companies in humanity research department. In humanity research department experts determine required parameters for selection of element to the firm. Each of experts constructs own snc-set by using the determined parameters. Then they apply the naturel union operations on snc-sets constructed by own. Obtained the natural union set is a snc-set and it can not be consistent snc-set, in this case, some elements can be degenerate elements in natural union set. Now, $C = \{c_1, c_2\}$, $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ and $E = \{e_1 = presentable, e_2 = good\ diction, e_3 = reference\}$ be set of experts, set of alternatives and set of parameters, respectively. Then

Step 1: Each of experts input snc-set κ and τ as follow:

$$\begin{aligned}\kappa = \ & \Big\{\big(e_1, \langle\{u_1, u_2, u_7, u_9\}, \{u_4, u_5\}, \{u_6, u_8, u_{10}\}\rangle\big),\\ & \big(e_2, \langle\{u_1, u_2, u_4, u_6\}, \{u_8, u_9, u_{10}\}, \{u_3, u_5, u_7\}\rangle\big),\\ & \big(e_3, \langle\{u_5, u_8, u_{10}\}, \{u_1, u_7, u_6\}, \{u_2, u_3, u_4, u_9\}\rangle\big)\Big\}\end{aligned}$$

and

$$\begin{aligned}\tau = \ & \Big\{\big(e_1, \langle\{u_1, u_3, u_7, u_{10}\}, \{u_2, u_4, u_8, u_9\}, \{u_3, u_5, u_6\}\rangle\big),\\ & \big(e_2, \langle\{u_2, u_4, u_6\}, \{u_1, u_9, u_{10}\}, \{u_3, u_5, u_7, u_8\}\rangle\big),\\ & \big(e_3, \langle\{u_5, u_8\}, \{u_1, u_4, u_7, u_6, u_{10}\}, \{u_2, u_3, u_9\}\rangle\big)\Big\}.\end{aligned}$$

Note that, here κ, τ be two csnc-sets, that is they aren't any degenerate element.

Step 2: Let us find OR-product of the snc-sets κ and τ as; follows:

$$\begin{aligned}\kappa \oplus \tau = \ & \{((e_1, e_1), \langle\{u_1, u_2, u_3, u_7, u_9, u_{10}\}, \{u_4\}, \{u_6\}\rangle),\\ & ((e_1, e_2), \langle\{u_1, u_2, u_4, u_6, u_7, u_9\}, \emptyset, \{u_8\}\rangle),\\ & ((e_1, e_3), \langle\{u_1, u_2, u_5, u_7, u_8, u_9\}, \emptyset, \emptyset\rangle),\\ & ((e_2, e_1), \langle\{u_1, u_2, u_3, u_4, u_6, u_7, u_{10}\}, \{u_9\}, \{u_3, u_5\}\rangle),\\ & ((e_2, e_2), \langle\{u_1, u_2, u_4, u_6\}, \{u_9, u_{10}\}, \{u_3, u_5, u_7\}\rangle),\\ & ((e_2, e_3), \langle\{u_1, u_2, u_4, u_5, u_6, u_8\}, \{u_{10}\}, \{u_3\}\rangle),\\ & ((e_3, e_1), \langle\{u_1, u_3, u_5, u_7, u_8, u_{10}\}, \emptyset, \{u_3\}\rangle),\\ & ((e_3, e_2), \langle\{u_2, u_4, u_5, u_6, u_8, u_{10}\}, \{u_1\}, \{u_3\}\rangle),\\ & ((e_3, e_3), \langle\{u_5, u_8, u_{10}\}, \{u_1, u_6, u_7\}, \{u_2, u_3, u_9\}\rangle),\end{aligned}$$

Step 3: Score of each elements of U is computed as follows:

$$S_1 = 7,\ S_2 = 6, S_3 = -2,\ S_4 = 5,\ S_5 = 3,$$

$$S_6 = 4,\ S_7 = 4,\ S_8 = 4,\ S_9 = 2,\ S_{10} = 5,$$

Step 4: Since $max\{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}\} = S_1$, positive consistent element $\overline{p} = S_1$, then u_1 is optimum element.

5 Conclusion

In this paper, we present the concept of snc-sets and basic operations such as; union, intersection, AND product, OR-product. We also propose an efficient approach for group decision making problems based on snc-sets. It can be applied to problems of many fields that contain uncertainty such as; multi-criteria decision making, game theory, algebraic structure and so on.

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