ISOMETRIC IMMERSION OF 3-DIMENSIONAL NORMAL ALMOST CONTACT METRIC MANIFOLDS

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Abstract. In this paper we study a three dimensional normal almost contact metric manifold which is isometrically immersed in a four-dimensional Riemannian manifold of constant curvature 1.

1 Introduction

Let $M$ be an almost contact metric manifold and $(\phi, \xi, \eta)$ its almost contact structure. This means, $M$ is an odd-dimensional differentiable manifold and $\phi$, $\xi$, $\eta$ are tensor fields on $M$ of types $(1,1)$, $(1,0)$ and $(0,1)$ respectively, such that

$$\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta \circ \phi = 0. \quad (1.1)$$

Let $\mathbb{R}$ be the real line and $t$ a coordinate on $\mathbb{R}$. Define an almost complex structure $J$ on $M \times \mathbb{R}$ by

$$J(X, \lambda \frac{d}{dt}) = (\phi X - \lambda \xi, \eta(X) \frac{d}{dt}), \quad (1.2)$$

where the pair $(X, \lambda \frac{d}{dt})$ denotes a tangent vector on $M \times \mathbb{R}$, $X$ and $\lambda \frac{d}{dt}$ being tangent to $M$ and $\mathbb{R}$ respectively.

$M$ and $(\phi, \xi, \eta)$ are said to be normal if the structure $J$ is integrable ([1],[2]). The necessary and sufficient condition for $(\phi, \xi, \eta)$ to be normal is

$$\mathcal{N}(\phi, \phi) + 2d\eta \otimes \xi = 0, \quad (1.3)$$

where the pair $[\phi, \phi]$ is the Nijenhuis tensor of $\phi$ defined by

$$[\phi, \phi](X, Y) = [\phi X, \phi Y] + \phi^2[X, Y] - \phi[\phi X, Y] - \phi[X, \phi Y] \quad (1.4)$$

for any $X, Y \in \chi(M)$; $\chi(M)$ being the Lie algebra of vector fields on $M$.

We say that the form $\eta$ has rank $r = 2s$ if $(d\eta)^s \neq 0$ and $\eta \wedge (d\eta)^s = 0$ and has rank $r = 2s+1$ if $\eta \wedge (d\eta)^s \neq 0$ and $(d\eta)^{s+1} = 0$. We also say $r$ is rank of the structure $(\phi, \xi, \eta)$.

A Riemannian metric $g$ on $M$ satisfying the condition

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \quad (1.5)$$

for any $X, Y \in \chi(M)$, is said to be compatible with the structure $(\phi, \xi, \eta)$. If $g$ is such a metric, then the quadruple $(\phi, \xi, \eta, g)$ is called an almost contact metric structure on $M$ and $M$ is an almost contact metric manifold. On such a manifold we also have

$$\eta(X) = g(X, \xi) \quad (1.6)$$
for any $X \in \chi(M)$ and we can always define the 2-form $\Phi$ by

$$\Phi(Y, Z) = g(Y, \phi Z),$$  \hspace{1cm} (1.7)

where $X, Y \in \chi(M)$.

It is no hard to see that if $\dim M = 3$, then two Riemannian metric $g$ and $g'$ are compatible with the same almost contact structure $(\phi, \xi, \eta)$ on $M$ if and only if

$$g' = \sigma g + (1 - \sigma) \eta \otimes \eta$$

for a certain positive function $\sigma$ on $M$.

A normal almost contact metric structure $(\phi, \xi, \eta, g)$ satisfying additionally the condition $d\eta = \phi$ is called Sasakian. Of course, any such structure on $M$ has rank 3. Also a normal almost contact metric structure satisfying the condition $d\Phi = 0$ is said to be quasi-Sasakian [3]. In the paper [10], Olszak studied the curvature properties of normal almost contact manifold of dimension three with several examples. Also in [6], U. C. De and A. K. Mondal studied three dimensional normal almost contact metric manifolds satisfying certain curvature conditions.

T. Takahashi and S. Tanno [11] introduced the notion of isometric immersion on $K$-contact manifolds. D. E. Blair, T. Koufogiorgos and R. Sharma [4] studied isometric immersion for three dimensional contact manifolds satisfying $Q\phi = \phi Q$. In [7], U. C. De, A. Yildiz and A. Sarkar studied isometric immersion of three dimensional quasi-Sasakian manifolds. The quasi-Sasakian manifolds are particular types of normal almost contact metric manifolds. So in this paper we like to study isometric immersion on three-dimensional normal almost contact metric manifold.

The object of the present paper is to study a three-dimensional normal almost contact metric manifold which is isometrically immersed in a four-dimensional Riemannian manifold of constant curvature 1.

The present paper is organized as follows:
Section 1 is the introductory section. In section 2 we give some preliminary notion of three dimensional normal almost contact metric manifolds. In section 3 we derive some results of three-dimensional normal almost contact metric manifolds isometrically immersed in a four-dimensional Riemannian manifold of constant curvature 1. In this section we also prove a necessary and sufficient condition for the immersion to be minimal. Finally in section 4 we construct an example of three-dimensional normal almost contact metric manifold which illustrates some results obtained in section 3.

2 Preliminaries

For a normal almost contact metric structure $(\phi, \xi, \eta, g)$ on $M$, we have [10]

$$(\nabla_X \phi)(Y) = g(\phi \nabla_X \xi, Y) - \eta(Y) \phi \nabla_X \xi,$$  \hspace{1cm} (2.1)

$$\nabla_X \xi = \alpha [X - \eta(X) \xi] - \beta \phi X,$$  \hspace{1cm} (2.2)

where $2\alpha = div \xi$ and $2\beta = tr(\phi \nabla \xi)$. $div \xi$ is the divergent of $\xi$ defined by $div \xi = trace\{X \mapsto \nabla_X \xi\}$ and $tr(\phi \nabla \xi) = trace\{X \mapsto \phi \nabla_X \xi\}$. Using (2.2) in (2.1) we get

$$(\nabla_X \phi)(Y) = \alpha [g(\phi X, Y) \xi - \eta(Y) \phi X] + \beta [g(X, Y) \xi - \eta(Y) X].$$  \hspace{1cm} (2.3)

Also in this manifold the following relations hold:
\begin{align}
R(X,Y)\xi &= \left[ Y\alpha + (\alpha^2 - \beta^2)\eta(Y) \right] \phi^2 X \\
&\quad - \left[ X\alpha + (\alpha^2 - \beta^2)\eta(X) \right] \phi^2 Y \\
&\quad + [Y\beta + 2\alpha\beta\eta(Y)]\phi X \\
&\quad - [X\beta + 2\alpha\beta\eta(X)]\phi Y,
\end{align}

\begin{align}
S(X,\xi) &= -X\alpha - (\phi X)\beta \\
&\quad - [\xi \alpha + 2(\alpha^2 - \beta^2)]\eta(X),
\end{align}

where $R$ denotes the curvature tensor and $S$ is the Ricci tensor.

On the other hand, the curvature tensor in three dimensional Riemannian manifold always satisfies

\begin{align}
R(X,Y)Z &= S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY \\
&\quad - \frac{r}{2} [g(Y,Z)X - g(X,Z)Y],
\end{align}

where $r$ is the scalar curvature of the manifold.

By (2.4), (2.5) and (2.8) we can derive

\begin{align}
S(Y,Z) &= \left( \frac{r}{2} + \xi \alpha + \alpha^2 - \beta^2 \right) g(\phi Y, \phi Z) \\
&\quad - \eta(Y)(Z\alpha + (\phi Z)\beta) - \eta(Z)(Y\alpha + (\phi Y)\beta) \\
&\quad - 2(\alpha^2 - \beta^2) \eta(Y)\eta(Z).
\end{align}

From (2.6) it follows that if $\alpha, \beta$ = constant, then the manifold is either $\beta$-Sasakian or $\alpha$-Kenmotsu [9] or cosympletic [1].

Also we have a 3-dimensional normal almost contact metric manifold is quasi-Sasakian if and only if $\alpha = 0$ [10].

Now we prove a Lemma here:

**Lemma 2.1.** A three dimensional compact normal almost contact metric manifold satisfying $\phi^2(\text{grad}\alpha) + \phi(\text{grad}\beta) = 0$ is a quasi-Sasakian manifold, provided $\beta$ is a non-zero constant.

**Proof.** From the relation $\phi^2(\text{grad}\alpha) + \phi(\text{grad}\beta) = 0$, we obtain

\begin{align}
- \text{grad}\alpha + (\xi \alpha)\xi + \phi(\text{grad}\beta) = 0.
\end{align}

Taking inner product with $X$ in (2.9) yields

\begin{align}
g(\text{grad}\beta, \phi X) + d\alpha(X) - (\xi \alpha)\eta(X) = 0.
\end{align}

Differentiating (2.10) covariantly with respect to $Y$, we obtain

\begin{align}
g(\nabla_Y \text{grad}\beta, \phi X) + g(\text{grad}\beta, (\nabla_Y \phi) X) + (\nabla_Y d\alpha) X \\
- g(\nabla_Y \text{grad}\alpha, \xi)\eta(X) - (\xi \alpha)(\nabla_Y \eta)(X) = 0.
\end{align}

Interchanging $X$ and $Y$ in (2.11), we get

\begin{align}
g(\nabla_X \text{grad}\beta, \phi Y) + g(\text{grad}\beta, (\nabla_X \phi) Y) + (\nabla_X d\alpha) Y \\
- g(\nabla_X \text{grad}\alpha, \xi)\eta(Y) - (\xi \alpha)(\nabla_X \eta)(Y) = 0.
\end{align}
Subtracting (2.11) from (2.12), we have
\begin{align}
g(\nabla_X \text{grad} \beta, \phi Y) - g(\nabla_Y \text{grad} \beta, \phi X) + g(\text{grad} \beta, (\nabla_X \phi) Y) \\
- g(\text{grad} \beta, (\nabla_Y \phi) X) - g(\nabla_X \text{grad} \alpha, \xi(Y)) + g(\nabla_Y \text{grad} \alpha, \xi(X)) \\
- (\xi \alpha)[(\nabla_X \eta) Y - (\nabla_Y \eta) X] = 0.
\end{align}
(2.13)

Let \( \{E_0, E_1, E_2\} \) be a \( \phi \)-basis on the manifold where \( E_0 = \xi \) and \( \phi E_1 = E_2 \). Taking \( X = E_1, Y = E_2 \) in (2.13) and using \( (\nabla_{E_1} \phi) E_2 = \alpha \xi, (\nabla_{E_2} \phi) E_1 = -\alpha \xi \) and \( (\nabla_{E_i} \eta) E_2 = -\beta \), we obtain
\[ g(\nabla_{E_1} \text{grad} \beta, E_1) + g(\nabla_{E_2} \text{grad} \beta, E_2) = 2\alpha(\xi \beta) + 2(\xi \alpha) \beta. \]
(2.14)

Differentiating (2.6) covariantly with respect to \( \xi \), we get
\[ g(\nabla_{\xi} \text{grad} \beta, \xi) = -2\beta(\xi \alpha) - 2\alpha(\xi \beta). \]
(2.15)

Adding (2.14) and (2.15), we obtain
\[ \Delta \beta = 0. \]
(2.16)

Since the manifold is compact we have \( \beta \) is constant. If \( \beta \) is a non-zero constant then we easily obtain from (2.6) that \( \alpha = 0 \) and hence the manifold becomes a quasi-Sasakian manifold. This proves the lemma. \( \square \)

3 Isometric immersion of three-dimensional normal almost contact metric manifolds

**Definition 3.1.** Let \( M \) and \( M' \) be smooth manifolds of dimension \( m \) and \( m' \) respectively. If \( f : M \to M' \) is a smooth map and \( f_x : T_x(M) \to T_{f(x)}(M') \) is the tangential map at \( x \in M \) then \( f \) is said to be an immersion if \( f_x \) is injective for each \( x \in M \).

Let \( M \) and \( M' \) be two Riemannian manifolds with Riemannian metric \( g \) and \( g' \) respectively. A map \( f : M \to M' \) is called isometric at a point \( x \) of \( M \) if \( g(X, Y) = g'(f_* X, f_* Y) \), for all \( X, Y \in T_x(M) \).

An immersion \( f \) which is isometric at every point of \( M \) is called an isometric immersion [12].

If \( X \) and \( Y \) are two vector fields on a manifold \( M \) which is immersed in a Riemannian manifold \( M' \) then we know that [12] \( B(X, Y) = \nabla_X Y - \nabla_Y X \), where \( B \) is the second fundamental form and \( \nabla \) and \( \nabla \) denote the covariant differentiation with respect to the Levi-Civita connection in \( M \) and \( M' \) respectively.

We consider a three-dimensional normal almost contact metric manifold which is isometrically immersed in a four-dimensional Riemannian manifold of constant curvature \( 1 \). Then we can write the Gauss and Codazzi equations as [5]
\[ R(X, Y) = XAY + AXAY, \]
(3.1)
\[ R(X, Y) Z = g(Y, Z) X - g(X, Z) Y + g(AY, Z) AX - g(A, Z) AY, \]
(3.2)
\[ (\nabla_X A)(Y) = (\nabla_Y A)(X), \]
(3.3)

where \( A \) is a \( (1, 1) \) tensor field associated with second fundamental form \( B \) given by \( B(X, Y) = g(A X, Y) \). \( A \) is symmetric with respect to \( g \). If the trace of \( A \) vanishes then the immersion is called minimal. The type number of the immersion is equal to the rank of \( A \). From (3.2) it follows that
\[ g(R(X, Y) Z, U) = g(Y, Z) g(X, U) - g(X, Z) g(Y, U) \]
\[ + g(AY, Z) g(A, U) - g(A, Z) g(AY, U). \]
In the above equation putting \( X = U = e_i \), where \( \{e_i\} \), \( i = 1, 2, 3 \), is an orthonormal basis of the tangent space at each point of the manifold \( M \) and taking summation over \( i \), we get
\[
S(Y, Z) = 2g(Y, Z) + g(AY, Z)\theta - g(AAY, Z),
\]  
where \( \theta \) is the trace of \( A \). Replacing \( Z \) by \( \xi \) we get from (3.4)
\[
S(Y, \xi) = 2g(Y, \xi) + g(AY, \xi)\theta - g(AAY, \xi).
\]  
In view of (2.5) we get from (3.5)
\[
-\langle Ya \rangle - \langle \phi Y \rangle\beta - \{\xi\alpha + 2(\alpha^2 - \beta^2) + 2\}\eta(Y) = g(AY, \xi) - g(AAY, \xi).
\]  
For \( g(\text{grad}f, X) = df(X) \), symmetry of \( A \) and skew-symmetry of \( \phi \), the equation (3.6) implies
\[
-g\text{grad} \alpha + \phi(\text{grad} \beta) = \{\xi\alpha + 2(\alpha^2 - \beta^2) + 2\}\xi + AA\xi = 0.
\]  
If \( \theta = 0 \) the equation (3.7) reduces to
\[
-g\text{grad} \alpha + \phi(\text{grad} \beta) = \{\xi\alpha + 2(\alpha^2 - \beta^2) + 2\} \xi + AA\xi = 0.
\]  
Thus we can state the following:

**Theorem 3.1.** If a three dimensional normal almost contact metric manifold is isometrically immersed in a four dimensional Riemannian manifold of constant curvature 1 and if the immersion is minimal then (3.8) holds.

We now suppose that the relation (3.8) holds. Then in view of (3.7) we have \( \theta A\xi = 0 \). Therefore either \( \theta = 0 \) or \( A\xi = 0 \). If \( A\xi = 0 \), then from (3.7) we get
\[
-g\text{grad} \alpha + \phi(\text{grad} \beta) - \{\xi\alpha + 2(\alpha^2 - \beta^2) + 2\} \xi = 0.
\]  
Applying \( \phi \) on both sides of (3.9), we obtain
\[
\phi^2(\text{grad} \beta) - \phi(\text{grad} \alpha) = 0.
\]  
In view of the Lemma 2.1. we state the following:

**Theorem 3.2.** If a three dimensional compact normal almost contact metric manifold is isometrically immersed in a four dimensional Riemannian manifold of constant curvature 1 and if (3.8) holds then either the immersion is minimal or the manifold is a quasi-Sasakian manifold, \( \beta \) being a non-zero constant.

By virtue of (1.1) and (2.6) we obtain from (2.8)
\[
S(\phi Y, \phi Z) = (\frac{r}{2} + \xi\alpha + \alpha^2 - \beta^2)g(\phi Y, \phi Z).
\]  
From (3.4) we also have
\[
S(\phi Y, \phi Z) = 2g(\phi Y, \phi Z) + g(A\phi Y, \phi Z)\theta - g(AA\phi Y, \phi Z).
\]  
From (3.11) and (3.12), we get
\[
(\frac{r}{2} + \xi\alpha + \alpha^2 - \beta^2 - 2)g(\phi^2 Y, Z) - g(\phi A\phi Y, Z)\theta + g(\phi AA\phi Y, Z) = 0.
\]  
We obtain from (3.13)
\[
(\frac{r}{2} + \xi\alpha + \alpha^2 - \beta^2 - 2)\phi^2 - \theta\phi A\phi + \phi AA\phi = 0.
\]
If $\theta = 0$, then (3.14) reduces to
\[
\left(\frac{r}{2} + \xi \alpha + \alpha^2 - \beta^2 - 2\right) \phi^2 + \phi A \phi = 0. 
\] (3.15)

Thus we can state the following:

**Theorem 3.3.** If a three dimensional normal almost contact metric manifold is isometrically im-
mersed in a four dimensional Riemannian manifold of constant curvature 1 and if the immersion
is minimal then (3.15) holds.

Next let (3.15) holds. Then from (3.14) we get $\theta \phi A = 0$. Hence either $\theta = 0$ or $\phi A \phi = 0$. Hence we can state the following:

**Theorem 3.4.** If a three dimensional normal almost contact metric manifold is isometrically im-
mersed in a four dimensional Riemannian manifold of constant curvature 1 and if (3.15) holds
then either the immersion is minimal or $\phi A \phi = 0$.

Combining Theorem 3.3 and Theorem 3.4 we get a necessary and sufficient condition for the
immersion is minimal as the following:

**Theorem 3.5.** If a three dimensional normal almost contact metric manifold is isometrically im-
mersed in a four dimensional Riemannian manifold of constant curvature 1, then the immersion
is minimal if and only if (3.13) holds, provided $\phi A \phi \neq 0$.

Putting $Z = \xi$ in (3.2) and using (2.4), we obtain
\[
[Y \alpha + (\alpha^2 - \beta^2) \eta(Y)] \phi^2 X 
\]
\[-[X \alpha + (\alpha^2 - \beta^2) \eta(X)] \phi^2 Y 
\]
\[+[Y \beta + 2 \alpha \beta \eta(Y)] \phi X 
\]
\[-[X \beta + 2 \alpha \beta \eta(X)] \phi Y 
\]
\[= \eta(Y) X - \eta(X) Y + \eta(A Y) AX - \eta(A X) A Y. 
\]

Putting $Y = \xi$ in (3.16) and using (1.1), (2.6) yields
\[
(\xi \alpha + \alpha^2 - \beta^2 + 1)[X - \eta(X) \xi] + \eta(A \xi) AX - \eta(A X) A \xi = 0. 
\] (3.17)

Now $g(A X, Y) = B(X, Y)$ and we know that $B(X, Y) = \bar{\nabla} X Y - \nabla X Y$. Hence
\[g(A \xi, \xi) = B(\xi, \xi) = \bar{\nabla} \xi \xi - \nabla \xi \xi, \] (3.18)

and
\[g(A X, \xi) = B(X, \xi) = \bar{\nabla} X \xi - \nabla X \xi. \] (3.19)

Using (3.18), (3.19) in (3.17), we obtain
\[
(\xi \alpha + \alpha^2 - \beta^2 + 1)[X - \eta(X) \xi] - (\bar{\nabla} \xi \xi - \nabla \xi \xi) A \xi = 0. 
\] (3.20)

From [8] we know that $2 \nabla X X = \text{grad} f$, where $f = g(X, X)$ is a smooth function on a Riemann-
ian manifold endowed with a metric $g$. Then for $X = \xi$ and $g(\xi, \xi) = 1$, we get $\bar{\nabla} \xi \xi = 0$, since
\text{grad} 1 = 0. Also from (2.1) it follows that $\nabla \xi \xi = 0$. Hence applying $\phi$ on both sides of (4.20)
we obtain
\[
(\xi \alpha + \alpha^2 - \beta^2 + 1) \phi X = 0. 
\] (3.21)

Since $\phi X \neq 0$, unless $X = \xi$, we have
\[
(\xi \alpha + \alpha^2 - \beta^2 + 1) = 0. 
\] (3.22)

Therefore we can state the following:
Theorem 3.6. If a three dimensional normal almost contact metric manifold is isometrically immersed in a four dimensional Riemannian manifold of constant curvature 1, then the manifold satisfies the relation (3.22).

4 Example

We consider the three dimensional manifold $M = \{(x, y, z) \in \mathbb{R}^3, z \neq 0\}$, where $(x, y, z)$ are standard coordinate of $\mathbb{R}^3$.

The vector fields

$$e_1 = z \frac{\partial}{\partial x}, \quad e_2 = z \frac{\partial}{\partial y}, \quad e_3 = z \frac{\partial}{\partial z}$$

are linearly independent at each point of $M$.

Let $g$ be a Riemannian metric defined by

$$g(e_1, e_3) = g(e_1, e_2) = g(e_2, e_3) = 0,$$

$$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = 1$$

that is, the form of the matrix becomes

$$g = \frac{dx^2 + dy^2 + dz^2}{z^2}.$$  

Let $\eta$ be the 1-form defined by $\eta(Z) = g(Z, e_3)$ for any $Z \in \chi(M)$. Let $\phi$ be the $(1, 1)$ tensor field defined by

$$\phi(e_1) = -e_2, \quad \phi(e_2) = e_1, \quad \phi(e_3) = 0.$$ 

Then using the identity of $\phi$ and $g$, we have

$$\eta(e_3) = 1, \quad \phi^2 Z = -Z + \eta(Z)e_3,$$

$$g(\phi Z, \phi W) = g(Z, W) - \eta(Z)\eta(W)$$

for any $Z, W \in \chi(M)$.

Then for $e_3 = \xi$, the structure $(\phi, \xi, \eta, g)$ defines an almost contact metric structure on $M$.

Let $\nabla$ be the Levi-Civita connection with respect to the metric $g$. Then we have

$$[e_1, e_3] = e_1 e_3 - e_3 e_1 = z \frac{\partial}{\partial x}(z \frac{\partial}{\partial z}) - z \frac{\partial}{\partial z}(z \frac{\partial}{\partial x}) = -e_1,$$

Similarly

$$[e_1, e_2] = 0 \quad and \quad [e_2, e_3] = -e_2.$$ 

The Riemannian connection $\nabla$ of the metric $g$ is given by

$$2g(\nabla_X Y, Z) = X g(Y, Z) + Y g(Z, X) - Z g(X, Y)$$

$$-g(X, [Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y]),$$

(4.1)
which is known as Koszul’s formula. Using (4.1) we have
\[ 2g(\nabla_{e_1}e_3, e_1) = -2g(e_1, e_1) \]
\[ = 2g(-e_1, e_1). \]  
(4.2)

Again by (4.1)
\[ 2g(\nabla_{e_1}e_3, e_2) = 0 = 2g(-e_1, e_2) \]
and
\[ 2g(\nabla_{e_1}e_3, e_3) = 0 = 2g(-e_1, e_3). \]  
(4.3)

From (4.2), (4.3) and (4.4) we obtain
\[ 2g(\nabla_{e_1}e_3, X) = 2g(-e_1, X) \]
for all \( X \in \chi(M) \).

Thus
\[ \nabla_{e_1}e_3 = -e_1. \]

Therefore, (4.1) futher yields
\[ \nabla_{e_1}e_3 = -e_1, \quad \nabla_{e_1}e_2 = 0, \quad \nabla_{e_1}e_1 = -e_3 \]
\[ \nabla_{e_2}e_3 = -e_2, \quad \nabla_{e_3}e_2 = e_3, \quad \nabla_{e_3}e_1 = 0, \]  
(4.5)
\[ \nabla_{e_3}e_3 = 0, \quad \nabla_{e_3}e_2 = 0, \quad \nabla_{e_3}e_1 = 0. \]

(4.5) tells us that the manifold satisfies (2.2) for \( \alpha = -1 \) and \( \beta = 0 \) and \( \xi = e_3 \). Hence the manifold is a normal almost contact metric manifold with \( \alpha, \beta = \)constants.

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