COMMON FIXED POINT THEOREMS FOR WEAKLY COMPATIBLE MAPPINGS IN INTUITIONISTIC GENERALIZED FUZZY METRIC SPACES

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Communicated by Jose Luis Lopez-Bonilla

MSC 2010 Classifications: 47H10, 54H25.

Keywords and phrases: Compatible Mappings, G-Metric Spaces, Intuitionistic Generalized Fuzzy Metric Spaces.

Abstract. In this paper, we obtain unique common fixed point theorems for six weakly compatible mappings in intuitionistic generalized fuzzy metric spaces.

1 Introduction

The theory of fuzzy sets has evolved in many directions after investigation of the notion of fuzzy sets by Zadeh [19]. Many authors have introduced the concept of a fuzzy metric space in different ways [5,8]. George and Veeramani [4] modified the concept of a fuzzy metric space introduced by Kramosil and Michalek [8] and defined a Hausdorff topology on this fuzzy metric space. Alternatively, Mustafa and Sims [9] introduced a new notion of a generalized metric space called G-metric space. Rao et al. [15] proved two unique common coupled fixed point theorems for three mappings in symmetric G-fuzzy metric spaces. Sun and Yang [17] introduced the concept of G-fuzzy metric spaces and proved two common fixed point theorems for four mappings. Some interesting references on G-metric spaces are [9-12]. Park [14] introduced and discussed in a notion of intuitionistic fuzzy metric space which is based both on the idea of intuitionistic fuzzy set due to Atanassov [1], and the concept of a fuzzy metric space given George and Veeramani [5]. We have generalized the result of Rao [15]. Before giving our main results, we obtain unique common fixed point theorems for six weakly compatible mappings in intuitionistic generalized fuzzy metric spaces.

Definition 1.1 A 3-tuple \((X, G, *)\) is called a G-Fuzzy Metric Space (Shortly GFMS) if \(X\) is an arbitrary nonempty set, \(*\) is a continuous t-norm and \(G\) is a fuzzy set on the \(X^3 \times (0, \infty)\) satisfying the following conditions: For each \(t, s > 0\)

(i) \(G(x, x, y, t) > 0\) for all \(x, y \in X\) with \(x \neq y\),
(ii) \(G(x, x, y, t) \geq G(x, y, z, t)\) for all \(x, y, z \in X\) with \(y \neq z\),
(iii) \(G(x, y, z, t) = 1\) if and only if \(x = y = z\),
(iv) \(G(x, y, z, t) = G(p(x, y, z), t)\) where \(p\) is a permutation function,
(v) \(G(x, y, z, t + s) \geq G(a, y, z, t) \ast G(x, a, a, s)\) for all \(x, y, z, a \in X\),
(vi) \(G(x, y, z, t) : (0, \infty) \rightarrow [0,1]\) is continuous.

Definition 1.2 A 5 tuple \((X, G, H, *, \dagger)\) is said to be an Intuitionistic Generalized Fuzzy Metric Space (Shortly GIFMS or IGFMS) if \(X\) is an arbitrary non-empty set, \(*\) is a continuous t-norm, \(\dagger\) is a continuous t-conorm, \(G\) and \(H\) are fuzzy sets on \(X^3 \times (0, \infty)\) satisfying the following conditions: For each \(x, y, z, a \in X\) and \(t, s > 0\)

(i) \(G(x, y, z, t) + H(x, y, z, t) \leq 1\),
(ii) \(G(x, x, y, t) > 0\) for all \(x, y \in X\) with \(x \neq y\),
(iii) \(G(x, x, y, t) \geq G(x, y, z, t)\) for \(y \neq z\),
(iv) \(G(x, y, z, t) = 1\) if and only if \(x = y = z\),
(v) \( G(x, y, z, t) = G(p(x, y, z), t) \), where \( p \) is a permutation function,
(vi) \( G(x, a, a, t) \star G(a, y, z, s) \leq G(x, y, z, t + s) \),
(vii) \( G(x, y, z, .) : (0, \infty) \to [0, 1] \) is continuous,
(viii) \( G \) is a non-decreasing of \( \mathbb{R}^+ \), \( \lim_{t \to \infty} G(x, y, z, t) = 1 \), \( \lim_{t \to 0} G(x, y, z, t) = 0 \) for all \( x, y, z \in X, t > 0 \),
(ix) \( H(x, x, y, t) < 1 \) for \( x \neq y \),
(x) \( H(x, x, y, t) \leq H(x, y, z, t) \) for \( y \neq z \),
(xi) \( H(x, y, z, t) = 0 \) if and only if \( x = y = z \),
(xii) \( H(x, a, a, t) \star H(a, y, z, s) \geq H(x, y, z, t + s) \),
(xiii) \( H(x, a, a, t) \odot H(a, y, z, s) \geq H(x, y, z, t + s) \),
(xiv) \( H(x, y, z, .) : (0, \infty) \to [0,1] \) is continuous,
(xv) \( H \) is a non-increasing function on \( \mathbb{R}^+ \), \( \lim_{t \to \infty} H(x, y, z, t) = 0 \), \( \lim_{t \to 0} H(x, y, z, t) = 1 \) for all \( x, y, z \in X, t > 0 \).

In this case, the pair \( (G, H) \) is called an intuitionistic generalized fuzzy metric on \( X \).

Lemma 1.3 Let \( (X, G, H, \star, \odot) \) be an intuitionistic generalized fuzzy metric space. Then \( G \) and \( H \) are continuous function on \( X^3 x (0, \infty) \). Now onwards, we assume the following condition:

\[
\lim_{t \to \infty} G(x, y, z, t) = 1 \quad \text{and} \quad \lim_{t \to \infty} H(x, y, z, t) = 0 \quad \text{for all} \quad x, y, z \in X
\]

Lemma 1.4 Let \( (X, G, H, \star, \odot) \) be an intuitionistic generalized fuzzy metric space. If there exists \( k \in (0,1) \) such that

\[
\min \{G(x, y, z, kt), G(u, v, w, kt)\} \geq \min \{G(x, y, z, t), G(u, v, w, t)\},
\]

\[
\max \{H(x, y, z, kt), H(u, v, w, kt)\} \leq \max \{H(x, y, z, t), H(u, v, w, t)\}.
\]

for all \( x, y, z, u, v, w \in X \) and \( t > 0 \), then \( x = y = z \) and \( u = v = w \).

2 Main Result

Let \( \phi \) denote the set of all continuous non decreasing functions \( \phi, \psi : [0, \infty) \to [0, \infty) \) such that \( \phi^n(t) \to 0 \) as \( n \to \infty \) and \( \psi^n(t) \to 1 \) as \( n \to \infty \) for all \( t > 0 \). It is clear that \( \phi(t) < t, \psi(t) > t \) for all \( t > 0 \) and \( \phi(0) = 0 \) and \( \psi(1) = 1 \).

Theorem 2.1

Let \( (X, G, H, \star, \odot) \) be an intuitionistic generalized fuzzy metric space and \( S, T, R, f, g, h : X \to X \) be satisfying

(i) \( S(X) \subseteq g(X), T(X) \subseteq h(X) \) and \( R(X) \subseteq f(X) \),
(ii) One of the \( f(X), g(X) \) and \( h(X) \) is a complete subspace of \( X \),
(iii) The pairs \( (S,f), (T,g) \) and \( (R,h) \) are weakly compatible and

\[
G(Sx, Ty, Rz, t) \geq \phi \left( \min \left\{ G(fx, gy, hz, t), \frac{1}{3}[G(fx, Sx, Ty, t) + G(gy, Ty, Rz, t) + G(hz, Rz, Sx, t)], \frac{1}{4}[G(fx, Ty, hz, t) + G(Sx, gy, hz, t)] + G(fx, gy, Rz, t) \right\} \right)
\]
\[ H(Sx, Ty, Rz, t) \leq \psi \max \left\{ \frac{1}{3} H(fx, Sx, Ty, t), \frac{1}{4} H(gy, Ty, Rz, t) + H(hz, Rz, Sx, t), \frac{1}{2} H(fx, gy, hz, t) + H(Sx, gy, hz, t), +H(fx, gy, Rz, t) \right\} \] (2.1.2)

for all \( x, y, z \in X \), where \( \phi, \psi \in \Phi \). Then either one of the pairs \( (S, f) \), \( (T, g) \) and \( (R, h) \) has a coincidence point or the maps \( S, T, R, f, g \) and \( h \) have a unique common fixed point in \( X \).

Proof: Choose \( x_0 \in X \). By (i), there exists \( x_1, x_2, x_3 \in X \) such that \( Sx_0 = gx_1 = y_0 \), \( Tx_1 = hx_2 = y_1 \), \( Rx_2 = fx_3 = y_2 \). Inductively, there exist sequences \( \{x_n\} \) and \( \{y_n\} \) in \( X \) such that \( y_{3n} = Sx_{3n} = gx_{3n+1}, y_{3n+1} = Tx_{3n+1} = hx_{3n+2} \) and \( y_{3n+2} = Rx_{3n+2} = fx_{3n+3} \), where \( n = 0, 1, 2, \ldots \). If \( y_{3n} = y_{3n+1} \) then \( x_{3n+1} \) is a coincidence point of \( g \) and \( T \). If \( y_{3n+1} = y_{3n+2} \) then \( x_{3n+2} \) is a coincidence point of \( h \) and \( R \). If \( y_{3n+2} = y_{3n+3} \) then \( x_{3n+3} \) is a coincidence point of \( f \) and \( S \). Now assume that \( y_n \neq y_{n+1} \) for all \( n \). Denote \( d_n = G(y_n, y_{n+1}, y_{n+2}, t) \) and \( \rho_n = H(y_n, y_{n+1}, y_{n+2}, t) \). Putting \( x = x_{3n}, y = x_{3n+1}, z = x_{3n+2} \) in (2.1.1), we get

\[ d_n = G(y_{3n}, y_{3n+1}, y_{3n+2}, t) = G(Sx_{3n}, Tx_{3n+1}, Rx_{3n+2}, t) \]

If \( d_3 \leq d_3 \) then from lemma (1.4), we have \( d_n \geq \phi(d_{n-1}) > d_n \).

It is a contradiction. Hence \( d_3 \geq d_{3n-1} \). Now from lemma (1.4), \( d_3 \geq \phi(d_{3n-1}) > d_3 \).

\[ \rho_3 = H(y_{3n}, y_{3n+1}, y_{3n+2}, t) = H(Sx_{3n}, Tx_{3n+1}, Rx_{3n+2}, t) \]
If \( \rho_n \geq \rho_{n-1} \) then from lemma (1.4), we have \( \rho_n \leq \psi(\rho_n) < \rho_n \). It is a contradiction. Hence \( \rho_n \leq \rho_{n-1} \). Now from lemma (1.4), \( \rho_n \leq \psi(\rho_{n-1}) \). Similarly, by putting \( x = x_{n+3}, y = x_{n+1}, z = x_{n+2} \) and \( x = x_{n+3}, y = x_{n+4}, z = x_{n+2} \) in (2.1.1) and (2.1.2) we get \( d_{3n+1} \geq \phi(d_{3n}), d_{3n+2} \geq \phi(d_{3n+1}) \) and \( \rho_{n+1} \leq \psi(\rho_n), \rho_{n+2} \leq \psi(\rho_{n+1}) \). Thus, from lemma (1.4), Equations (2.1.1) and (2.1.2), we have

\[
G(y_n, y_{n+1}, y_{n+2}, t) \geq \phi(G(y_{n-1}, y_n, y_{n+1}, t)) \geq \ldots \geq \phi^n(G(y_0, y_1, y_2, t))
\]

We have \( G(y_n, y_{n+1}, y_{n+2}, t) \geq G(y_n, y_{n+1}, y_{n+2}, t) \geq \phi^n(G(y_0, y_1, y_2, t)) \) and

\[
H(y_n, y_{n+1}, y_{n+2}, t) \leq \psi(H(y_{n-1}, y_n, y_{n+1}, t)) \leq \ldots \leq \psi^n(H(y_0, y_1, y_2, t))
\]

We have \( H(y_n, y_{n+1}, y_{n+2}, t) \leq H(y_n, y_{n+1}, y_{n+2}, t) \leq \psi^n(H(y_0, y_1, y_2, t)). \) Now for \( m > n \), we have

\[
G(y_n, y_n, y_{m}, t) \geq G(y_n, y_n, y_{m-1}, t) + G(y_{m-1}, y_n, y_{m-1}, t) \ldots \phi(y_{m-1}, y_{m-1}, y_{m-1}, y_{m-1}, t) + \phi^{n-1}(y_{m-1}, y_{m-1}, y_{m-1}, y_{m-1}, y_{m-1}, t) \rightarrow 1 \text{ as } n \rightarrow \infty
\]

\[
H(y_n, y_n, y_{m}, t) \leq H(y_n, y_n, y_{m-1}, t) + H(y_{m-1}, y_n, y_{m-1}, t) \ldots \psi(y_{m-1}, y_{m-1}, y_{m-1}, y_{m-1}, y_{m-1}, t) \rightarrow 0 \text{ as } n \rightarrow \infty.
\]

Since \( \phi^n(t) \rightarrow 1 \) and \( \psi^n(t) \rightarrow 0 \) as \( n \rightarrow \infty \) for \( t > 0 \), hence \( \{y_n\} \) is \( G \)-Cauchy. Suppose \( f(X) \) is \( G \)-complete. Then there exists \( p.t \in X \) such that \( y_{3n+2} \rightarrow p = ft \). Since \( \{y_n\} \) is \( G \)-Cauchy, it follows that \( y_{3n} \rightarrow p \) and \( y_{3n+1} \rightarrow p \) as \( n \rightarrow \infty \). And \( G(St, Tx_{3n+1}, Rx_{3n+2}, t) \)
\[
\begin{align*}
\geq \phi & \left( \min \left\{ \begin{array}{c}
G(f, x_{3n}, h_{x_{3n+2}}, t), \frac{1}{3}G(f, s, x_{3n+1}, t) \\
+G(gx_{3n+1}, Tx_{3n+1}, Rx_{3n+2}, t) + G(hx_{3n+2}, Rx_{3n+2}, s, t), \\
\frac{1}{4}G(f, Tx_{3n+1}, hx_{3n+2}, t) + G(s, gx_{3n+1}, hx_{3n+2}, t) \\
+G(f, gx_{3n+1}, Rx_{3n+2}, t)
\end{array} \right\} \right) \\
\leq \psi & \left( \max \left\{ \begin{array}{c}
H(f, x_{3n}, h_{x_{3n+2}}, t), \frac{1}{3}H(f, s, x_{3n+1}, t) \\
+H(gx_{3n+1}, Tx_{3n+1}, Rx_{3n+2}, t) + H(hx_{3n+2}, Rx_{3n+2}, s, t), \\
\frac{1}{4}H(f, Tx_{3n+1}, hx_{3n+2}, t) + H(s, gx_{3n+1}, hx_{3n+2}, t) \\
+H(f, gx_{3n+1}, Rx_{3n+2}, t)
\end{array} \right\} \right)
\end{align*}
\]

Letting \( n \to \infty \), we get

\[
G(Sp, p, p, t) \geq \phi \left( \min \left\{ \frac{1}{3}[G(p, St, p, t) + 1 + G(p, p, St, t)], \frac{1}{4}[1 + G(St, p, p, t)] \right\} \right)
\]

\[
G(St, p, p, t) \geq \phi(G(St, p, p, t)) \text{ and } H(St, Tx_{3n+1}, Rx_{3n+2}, t)
\]

\[
\leq \psi \left( \max \left\{ \frac{1}{3}[H(p, St, p, t) + 0 + H(p, p, St, t)], \frac{1}{4}[0 + H(St, p, p, t) + 0] \right\} \right)
\]

\[
H(St, p, p, t) \leq \psi(H(St, p, p, t), \text{ since } \phi \text{ is non decreasing and } \psi \text{ is non increasing.}
\]

Hence \( St = p \). Thus \( p = ft = St \). Since the pairs \((S, f)\) is weakly compatible, we have \( fp = Sp \).

Putting \( x = p, y = x_{3n+1}, z = x_{3n+2} \) in (2.1), we get

\[
G(St, Tx_{3n+1}, Rx_{3n+2}, t)
\]

\[
\geq \phi \left( \min \left\{ \begin{array}{c}
G(fp, gx_{3n+1}, hx_{3n+2}, t), \frac{1}{3}G(fp, Sp, Tx_{3n+1}, t) \\
+G(gx_{3n+1}, Tx_{3n+1}, Rx_{3n+2}, t) + G(hx_{3n+2}, Rx_{3n+2}, Sp, t), \\
\frac{1}{4}G(fp, Tx_{3n+1}, hx_{3n+2}, t) + G(s, gx_{3n+1}, hx_{3n+2}, t) \\
+G(fp, gx_{3n+1}, Rx_{3n+2}, t)
\end{array} \right\} \right)
\]

Letting \( n \to \infty \), we have

\[
G(Sp, p, p, t) \geq \phi \left( \min \left\{ \frac{1}{3}[G(Sp, Sp, p, t) + 1 + G(p, p, Sp, t)], \frac{1}{4}[G(Sp, p, p, t) + G(Sp, p, p, t) + G(Sp, p, p, t)] \right\} \right)
\]

Since \( G(Sp, Sp, p, t) \geq 2G(Sp, p, p, t) \), we have \( G(Sp, p, p, t) \geq \phi(G(Sp, p, p, t)) \) and

\[
H(Sp, Tx_{3n+1}, Rx_{3n+2}, t)
\]
Putting \( x = p, y = v, z = x_{n+2} \),

\[
H(f_p, g_{x_{n+1}}, h_{x_{n+2}}, t), \frac{1}{3}[H(f_p, Sp, T_{x_{n+1}}), t]
\]

Thus \( Sp = p \). Hence \( fp = Sp = p \). Since \( p = Sp \in H(Sp, Sp, p, t) \leq G \)

Letting \( n \to \infty \), we have

\[
H(Sp, p, p, t) \leq \psi \left( \max \left\{ H(Sp, p, p, t), \frac{1}{3}[H(Sp, Sp, p, t) + 0 + H(p, p, Sp, t)], \right. \right.
\]

\[
\left. \frac{1}{4}[H(f_p, Sp, Tx_{n+1}, h_{x_{n+2}}, t) + H(Sp, g_{x_{n+1}}, h_{x_{n+2}}, t), t] \right) + H(f_p, gx_{n+1}, Rx_{n+2}, t) \]

Since \( H(Sp, p, p, t) \leq 2H(Sp, p, p, t) \), we have \( H(Sp, p, p, t) \leq \psi(H(Sp, p, p, t)) \).

Thus \( Sp = p \). Hence \( fp = Sp = p \). Since \( p = Sp \in g(X) \), there exists \( su \in X \) such that \( p = gv \).

Putting \( x = p, y = v, z = x_{n+2} \) in (2.1.1) we get

\[
G(Sp, Tv, Rx_{n+2}, t) \geq \phi \left( \min \left\{ \frac{1}{3}[G(p, p, Tv, t) + G(p, p, p, t) + 1], \right. \right.
\]

\[
\left. \frac{1}{4}[G(p, p, p, t) + 1 + 1] \right) \geq \phi(G(p, Tv, t) and
\]

\[
H(Sp, Tv, Rx_{n+2}, t) \leq \psi \left( \max \left\{ \frac{1}{3}[H(p, p, Tv, t) + H(p, Tv, p, t) + 0], \right. \right.
\]

\[
\left. \frac{1}{4}[H(p, p, p, t) + 0 + 0] \right) \leq \psi(H(p, Tv, p, t))
\]

Since \( \phi \) is non-decreasing and \( \psi \) is non-increasing. Thus \( Tv = p \), so that \( p = Tv = gv \).

Since the pair \((T, g)\) is weakly compatible, we have \( Tp = gp \).

\[
G(Sp, Tp, Rx_{n+2}, t) \geq \phi \left( \min \left\{ \frac{1}{3}[G(p, gp, h_{x_{n+2}}, t) + G(p, Sp, Sp, t)], \right. \right.
\]

\[
\left. \frac{1}{4}[G(p, gp, h_{x_{n+2}}, t) + G(Sp, gp, h_{x_{n+2}}, t), t] \right)
\]
Putting \( x = p, y = p, z = w \) in (2.1.1) we get

Thus \( Tp = p \). Hence \( gp = Tp = p \). Since \( p = Tp \)

Since \( H(Tp, Tp, p, t) \)

Since \( G(Tp, Tp, p, t) \)

Letting \( n \to \infty \), we have

Since \( G(Tp, Tp, p, t) \geq 2G(Tp, p, p, t) \), we have, \( G(p, Tp, p, t) \geq \phi(G(p, Tp, p, t)) \).

Thus \( Tp = p \). Hence \( gp = Tp = p \). Since \( p = Tp \in h(X) \), there exists \( w \in X \) such that \( p = hw \).

Putting \( x = p, y = p, z = w \) in (2.1.1) we get
Since $\phi$ is non decreasing and $\psi$ is non increasing. Thus $Rw = p$, so that $p = hw = Rw$. Since the pair $(R, h)$ is weakly compatible, we have $Rp = hp$.

Putting $x = p, y = p, z = p$ in (2.1.1) we get,

$$G(p, p, Rp, t) = G(Sp, Tp, Rp, t) \geq \phi \left( \min \left\{ G(fp, gp, Rp, t), \frac{1}{3}[1 + G(p, p, Rp, t)] G(Rp, Rp, p, t), \frac{1}{4}[G(p, p, Rp, t)] \right\} \right)$$

$$H(p, p, Rp, t) = H(Sp, Tp, Rp, t) \leq \psi \left( \max \left\{ H(fp, gp, Rp, t), \frac{1}{3}[0 + H(p, p, Rp, t)] H(Rp, Rp, p, t), \frac{1}{4}[H(p, p, Rp, t)] \right\} \right)$$

Since $G(Rp, Rp, p, t) \geq 2G(p, p, Rp, t)$ and $H(Rp, Rp, p, t) \leq 2H(p, p, Rp, t)$.

We have $G(p, p, Rp, t) \geq \phi(G(p, p, Rp, t))$ and $H(p, p, Rp, t) \leq \psi(H(p, p, Rp, t))$.

Thus $Rp = p$, so that $Rp = hp = p$. It follows that $p$ is a common fixed point of $S, T, R, f, g$ and $h$.

Uniqueness of common fixed point follows easily from (4) and (5). Similarly, we can prove the theorem when $g(X)$ or $h(X)$ is a complete subspace of $X$.

**Corollary 2.2** Let $(X, G, H, *, \circ)$ be an intuitionistic generalized fuzzy metric space and $S, T, R, f, g, h : X \to X$ be satisfying

(i) $S(X) \subseteq g(X), T(X)$ and $R(X) \subseteq f(X)$,

(ii) one of $f(X), g(X)$ and $h(X)$ is a complete subspace of $X$,

(iii) The pairs $(S, f), (T, g)$ and $(R, h)$ are weakly compatible and

(iv) $G(Sx, Ty, Rz, t) \geq \phi(G(fx, gy, hz, t))$ and $H(Sx, Ty, Rz, t) \leq \psi(H(fx, gy, hz, t))$ for all $x, y, z \in X$, where $\phi \in \Phi, \psi \in \Psi$.

Then the maps $S, T, R, f, g$ and $h$ have a unique fixed point in $X$.

**Corollary 2.3** Let $(X,G,H,* , \circ)$ be an intuitionistic generalized fuzzy metric space and $S, T, R : X \to X$ be satisfying $G(Sx, Ty, Rz, t) \geq \phi(G(x, y, z, t))$ and $H(Sx, Ty, Rz, t) \leq \psi(H(x, y, z, t))$ for all $x, y, z \in X$, where $\phi \in \Phi, \psi \in \Psi$. Then the maps $S$ and $T$ have a unique common fixed point in $p \in X$ and $S, T, R$ are $G, H$ continuous at $p$.

**Proof:** There exists $p \in X$ such that $p$ is the unique common fixed point of $S, T$ and $R$ as in Theorem 2.1. Let $\{y_n\}$ be any sequence in $X$ which $G, H$ converges to $p$. Then $G(Sy_n, Sp, Sp, t) = G(Sy_n, Ty_n, Rp, t) \leq \phi(G(y_n, p, p, t)) \to 1$ as $n \to \infty$ and $H(Sy_n, Sp, Sp, t) = H(Sy_n, Ty_n, Rp, t) \geq \psi(H(y_n, p, p, t)) \to 0$ as $n \to \infty$.

Hence $S$ is $G, H$- continuous at $p$. Similarly, we can show that $T$ and $R$ are also $G, H$- continuous at $p$.

**References**


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Received: January 10, 2018.
Accepted: July 29, 2018...