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A NOTE ON RIEMANN LIOUVILLE FRACTIONAL DERIVATIVE AND CONIC DOMAINS

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Abstract. In this paper, we introduce a class $k-SP_\alpha[A,B]$ of analytic function in geometrical starlike of oval and petal type regions $\Delta_k(A,B)$ which unifies a number of classes studied earlier by Janowski, Kanas, Wisnowska, Shams etc. Thus our class includes k-uniformly Janowskl convex functions, k-uniformly Janowski starlike functions, k-uniformly convex functions, k-uniformly starlike functions, Janowski starlike and Janowski convex functions etc. We deduce sufficient condition for a function to be in $k-SP_\alpha[A,B]$ and also coefficient bound for functions of $k-SP_\alpha[A,B]$.

1 Introduction

Kanas and Wisniowska [5,15] generalized the parabolic region $\Delta=\{w:\Re\{w\}>|w-1|\}$ introduced by Goodman [4] introducing Δ_k $k\geq 0$ by

$$\Delta_k=\{u+iv:u>k\sqrt{(u-1)^2+v^2}\}.$$

This domain represents the right half plane for $k=0$, hyperbola for $0<k<1$, a parabola for $k=1$ and ellipse for $k>1$.
The functions $p_k(z)$ play the role of extremal functions for these conic regions where

$$p_k(z)=\begin{cases}\frac{1+z}{1-z}, & k=0\\ 1+\frac{2}{\pi^2}\left(\log\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2, & k=1.\\ 1+\frac{2}{1-k^2}\sinh^2\left[\left(\frac{2}{\pi}\arccos\ k\right)\arctan h\sqrt{z}\right], & 0<k<1.\\ 1+\frac{2}{k^2-1}\sin\left[\frac{\pi}{2R(t)}\int_0^{\frac{u(z)}{\sqrt{t}}}\frac{1}{\sqrt{1-x^2}\sqrt{1-(tx)^2}}dx\right]+\frac{1}{k^2-1}, & k>1,\end{cases} \tag{1.1}$$

where $u(z)=\frac{z-\sqrt{t}}{1-\sqrt{tx}}$, $t\in(0,1)$, $z\in\mathcal{U}$ and z is chosen such that $k=\cosh\left(\frac{\pi R'(t)}{4R(t)}\right)$, $R(t)$ is the Legendre's complete elliptic integral of the first kind and $R'(t)$ is complementary integral $R(t)$.
$p_k(z)=1+\delta_k z+....$, [14] where

$$\delta_k=\begin{cases}\frac{8(\arccos\ k)^2}{\pi^2(1-k^2)}, & 0\leq k<1\\ \frac{8}{\pi^2}, & k=1.\\ \frac{\pi^2}{4(k^2-1)\sqrt{t}(1+t)R^2(t)}, & k>1.\end{cases} \tag{1.2}$$

It was Janowski [1] who introduced the circular domain by defining the following:

Definition 1.1. Let $P[A,B]$, where $-1\leq B<A\leq 1$, denote the class of analytic function p defined on $\mathcal{U}$ with the representation $p(z)=\frac{1+Aw(z)}{1+Bw(z)}$, $z\in\mathcal{U}$, $w(0)=0$, $|w(z)|<1$. In terms of subordination $p\in P[A,B]$ if and only if $p(z)\prec\frac{1+Az}{1+Bz}$.

Geometrically, a function $p(z)\in P[A,B]$ maps the opine unit onto the disk defined by the domain,

$$\Delta[A,B]=\left\{w:\left|w-\frac{1-AB}{1-B^2}\right|<\frac{A-B}{1-B^2}\right\}.$$

The class $P[A,B]$ is connected the class P of functions with positive real part by the relation,

$$p(z) \in P \Leftrightarrow \frac{(A+1)p(z)-(A-1)}{(B+1)p(z)-(B-1)} \in P[A,B].$$

Now using the concepts of Janowski functions and the conic regions, we define the following.

Definition 1.2. A function p is said to be in the class $k-P[A,B]$, if and only if,

$$p(z) \prec \frac{(A+1)p_k(z)-(A-1)}{(B+1)p_k(z)-(B-1)}, \quad k \geq 0,$$

where $p_k(z)$ is defined by (1.1) and $-1 \leq B < A \leq 1$.

Geometrically, the function $p \in k-P[A,B]$ takes all values from the domain $\Delta_k[A,B]$, $-1 \leq B < A \leq 1,\ k \geq 0$ which is defined as

$$\Delta_k[A,B] = \left\{ w : \Re\left(\frac{(B-1)w(z)-(A-1)}{(B+1)w(z)-(A+1)}\right) > k\left|\frac{(B-1)w(z)-(A-1)}{(B+1)w(z)-(A+1)} - 1\right| \right\}$$

or equivalently

$$\Delta_k[A,B] = \{u+iv : [(B^2-1)(u^2+v^2) - 2(AB-1)u + (A^2-1)]^2$$
$$> k^2[(-2(B+1)(u^2+v^2) + 2(A+B+2)u - 2(A+1))^2 + 4(A-B)^2v^2]\}.$$

The domain $\Delta_k[A,B]$ retains the conic domain Δ_k inside the circular region defined by $\Delta[A,B]$. the impact of $\Delta[A,B]$ on the conic domain Δ_k changes the original shape of the conic regions . The ends of hyperbola and parabola get closer to each other but never meet anywhere and the ellipse gets the oval shape. When $A \to 1,\ B \to -1$, the radius of the circular disk defined by $\Delta[A,B]$ tends to infinity, consequently the arms of hyperbola and parabola expand and the oval turns into ellipse . we see that $\Delta_k[1,-1] = \Delta_k$, the conic domain defined by Kanas and Wisniowska[15]. The authors in [9,...,13] studied classes which are related to conic region.

Let $\mathcal{A}$ denote the class of functions of form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.3}$$

which are analytic in the open unit disk $\mathcal{U} = \{z : z \in \mathbb{C} \ and \ |z| < 1\}$, and $\mathcal{S}$ denote the subclass of $\mathcal{A}$ consisting of all function which are univalent in $\mathcal{U}$.
The fractional derivative of order α in the sense of Riemann Liouville is defined [2] by

$$D_z^\alpha f(z) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{f(\zeta)}{(z-\zeta)^\alpha} d\zeta. \ 0 \leq \alpha < 1,$$

where f is an analytic function in a simply connected domain of the z-plane containing the origin and the multiplicity of $(z-\zeta)^{-\alpha}$ is removed by requiring $\log(z-\zeta)$ to be real when $z-\zeta > 0$. Fractional derivative of higher order are defined by

$$D_z^{\alpha+\beta} f(z) = \frac{d^\beta}{dz^\beta} D_z^\alpha f(z), \ \ \beta \in \mathbb{N}_0.$$

Using the fractional derivative $D_z^\alpha f$ Owa and Srivastava [3] introduced the operator $\Omega^\alpha : \mathcal{A} \to \mathcal{A}$, which is known as an extension of fractional derivative and fractional integral as follows

$$\Omega^\alpha f(z) = \Gamma(2-\alpha) z^\alpha D_z^\alpha f(z), \quad \alpha \neq 2,3,4,... \tag{1.4}$$
$$= z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\alpha)}{\Gamma(n+1-\alpha)} a_n z^k$$

Now using the concepts of the fractional derivative and conic regions we define the following:

Definition 1.3. A function $f \in \mathcal{A}$ is said to be in the class $k - SP_\alpha[A, B]$, $k \geq 0, \alpha \neq 2, 3, 4, ...,\ -1 \leq B < A \leq 1$, if and only if

$$\Re\left(\frac{(B-1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)} - (A-1)}{(B+1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)} - (A+1)}\right) > k\left|\frac{(B-1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)} - (A-1)}{(B+1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)} - (A+1)} - 1\right|, \tag{1.5}$$

where $\Omega^\alpha f(z)$ is defined by (1.4).
Or equivalently,

$$\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)} \in k - P[A, B].$$

The following special cases are of interest
(i)$0 - SP_\alpha[1, -1] = SP_\alpha$, the class introduced by Srivastava and Mishra in [**??**].
(ii) $k - SP_0[A, B] = k - ST[A, B]$ introduced by Khalida Inyat Noor and Sarfraz Nawaz Malik [8].
(iii)$k - SP_1[A, B] = k - UCV[A, B]$ introduced also by Khalida Inyat Noor and Sarfraz Nawaz Malik [8].
(iv)$k - SP_0[[1, -1] = k - ST$ the well-known class of k-uniformly starlike functions introduced by Kanas and Wisniowska [5].
(v)$k - SP_1[[1, -1] = k - UCV$ the well-known class of k-uniformly convex functions introduced by Kanas and Wisniowska [5].
(vi)$k - SP_0[1 - 2\beta, -1] = SD(k, \beta)$, this class introduced by Shams [6].
(vii)$k - SP_1[1 - 2\beta, -1] = KD(k, \beta)$, this class introduced by Shams [6].
(viii)$0 - SP_0[A, B] = S^*[A, B]$, the well-known class of Janowski starlike functions introduced by Janowski [1].
(ix)$0 - SP_1[A, B] = S^*[A, B]$, the well-known class of Janowski convex functions introduced by Janowski [1].
We need the following lemma to prove our main results.

Lemma 1.4. *[8] Let $h(z) = 1 + \sum_{n=1}^{\infty} c_n z^n \in P[A, B]$. Then*

$$|c_n| \leq \frac{|\delta_k|(A - B)}{2},$$

where δ_k is defined by (1.2).

2 Main results

Theorem 2.1. *A function $f \in \mathcal{A}$ and of the form* (1.3) *is in the class $k - SP_\alpha[A, B]$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{2(k+1)(n-1) + |n(B+1) - (A+1)|\}\delta_n(\alpha)|a_n| < |B - A|, \tag{2.1}$$

where

$$\delta_n(\alpha) = \frac{\Gamma(n+1)\Gamma(2-\alpha)}{\Gamma(n+1-\alpha)}. \tag{2.2}$$

and

$$-1 \leq B < A \leq 1, k \geq 0$$

.

Proof. Assuming that (2.1) holds, then it suffices to show that

$$k\left|\frac{(B-1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)} - (A-1)}{(B+1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)} - (A+1)} - 1\right| - \Re\left[\frac{(B-1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)} - (A-1)}{(B+1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)} - (A+1)} - 1\right] < 1,$$

we get

$$
\begin{aligned}
&k\left|\frac{(B-1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)}-(A-1)}{(B+1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)}-(A+1)}-1\right|-\Re\left[\frac{(B-1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)}-(A-1)}{(B+1)\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)}-(A+1)}-1\right]\\
&\qquad\le(k+1)\left|\frac{(B-1)z(\Omega^\alpha f(z))'-(A-1)\Omega^\alpha f(z)}{(B+1)(\Omega^\alpha f(z))'-(A+1)\Omega^\alpha f(z)}-1\right|\\
&\qquad=2(k+1)\left|\frac{\Omega^\alpha f(z)-z(\Omega^\alpha f(z))'}{(B+1)z(\Omega^\alpha f(z))'-(A+1)\Omega^\alpha f(z)}\right|\\
&\qquad=2(k+1)\left|\frac{\sum_{n=2}^\infty(1-n)\delta_n(\alpha)a_nz^n}{(B-A)z+\sum_{n=2}^\infty[n(B+1)-(A+1)]\delta_n(\alpha)a_nz^n}\right|\\
&\qquad\le 2(k+1)\frac{\sum_{n=2}^\infty|1-n|\delta_n(\alpha)|a_n|}{|B-A|-\sum_{n=2}^\infty|n(B+1)-(A+1)|\delta_n(\alpha)|a_n|}.
\end{aligned}
$$

The last expression is bounded above by 1, then

$$\sum_{n=2}^{\infty}\{2(k+1)(n-1)+|n(B+1)-(A+1)|\}\delta_n(\alpha)|a_n|<|B-A|,$$

and this completes the proof. □

When $\alpha=0$, we have the following known result, proved by Khalida Inayat Noor and Sarfraz Nawaz Malik in [8].

Corollary 2.2. *A function $f\in\mathcal{A}$ and form (1.3) in the class $k-ST[A,B]$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{2(k+1)(n-1)+|n(B+1)-(A-1)|\}|a_n|<|B-A|, \tag{2.3}$$

where $-1\le B<A\le 1$ and $k\ge 0$.

For $\alpha=0,\ A=1\ \ and\ B=-1$, we have following result due to Kanas and Wisniowska [5].

Corollary 2.3. *A function $f\in\mathcal{A}$ and form (1.3) in the class $k-ST$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{n+k(n-1)\}|a_n|<1,\quad k\ge 0. \tag{2.4}$$

For $\alpha=0,\ A=1-2\beta\ \ and\ B=-1$ with $0\le\beta<1$, we arrive at Shams et result in [6].

Corollary 2.4. *A function $f\in\mathcal{A}$ and form (1.3) in the class $SD(k,\beta)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{n(k+1)-(k+\beta)\}|a_n|<1-\beta, \tag{2.5}$$

where $0\le\beta<1$ and $k\ge 0$.

Also for $\alpha=0,\ A=1-2\beta\ \ and\ B=-1,\ \ k=0$ with $0\le\beta<1$, then we get the well-known Silverman's result [7].

Corollary 2.5. *A function $f\in\mathcal{A}$ and form (1.3) in the class $S^*(\beta)$, if it satisfies the condition*

$$\sum_{n=2}^{\infty}\{(n-\beta)\}|a_n|<1-\beta, \tag{2.6}$$

where $0\le\beta<1$.

Theorem 2.6. *Let $f \in k - SP_\alpha[A,B]$ and is of the form (1.3). Then for $n \geq 2$.*

$$|a_n| \leq \frac{1}{\delta_n(\alpha)} \prod_{j=0}^{n-2} \frac{|\delta_k(A-B) - 2jB|}{2(j+1)}, \tag{2.7}$$

where δ_k is defined (1.2) and $\delta_n(\alpha)$ is defined by (2.2).

Proof. By the definition we have

$$\frac{z(\Omega^\alpha f(z))'}{\Omega^\alpha f(z)} = p(z), \tag{2.8}$$

where

$$p(z) \in P[A,B].$$

Since $p(z) = 1 + \sum_{n=1}^{\infty} c_n z^n$, and from (2.8), we have

$$z + \sum_{n=2}^{\infty} n\delta_n(\alpha) a_n z^n = \left[z + \sum_{n=2}^{\infty} \delta_n(\alpha) a_n z^n\right]\left[1 + \sum_{n=1}^{\infty} c_n z^n\right].$$

Equating coefficients of z^n on both sides, we have

$$(n-1)\delta_n(\alpha) a_n = \sum_{j=1}^{n-1} \delta_{n-j}(\alpha) a_{n-j} c_j, \qquad a_1 = \delta_1(\alpha) = 1.$$

This implies that

$$|a_n| \leq \frac{1}{(n-1)\delta_n(\alpha)} \sum_{j=1}^{n-1} \delta_{n-j}(\alpha) a_{n-j} c_j, \qquad a_1 = \delta_1(\alpha) = 1.$$

By Lemma 1.4, we get

$$|a_n| \leq \frac{|\delta_k|(A-B)}{2(n-1)\delta_n(\alpha)} \sum_{j=1}^{n-1} \delta_j(\alpha)|a_j|, \qquad a_1 = \delta_1(\alpha) = 1. \tag{2.9}$$

Now we prove that

$$\frac{|\delta_k|(A-B)}{2(n-1)\delta_n(\alpha)} \sum_{j=1}^{n-1} \delta_j(\alpha)|a_j| \leq \frac{1}{\delta_n(\alpha)} \prod_{j=0}^{n-2} \frac{|\delta_k(A-B) - 2jB|}{2(j+1)}. \tag{2.10}$$

For this, we use the induction method.
For $n = 2$: from (2.9), we have

$$|a_2| \leq \frac{|\delta_k|(A-B)}{2}.$$

From (2.7), we have

$$|a_2| \leq \frac{|\delta_k|(A-B)}{2}.$$

For $n = 3$: from (2.9), we have

$$|a_3| \leq \frac{|\delta_k|(A-B)}{4\delta_3(\alpha))}\left[1 + \frac{|\delta_k|(A-B)}{2}\right].$$

From (2.7), we have

$$|a_3| \leq \frac{1}{\delta_3(\alpha)} \frac{|\delta_k|(A-B)}{2} \frac{|\delta_k(A-B) - 2B|}{4}$$

$$|a_3| \leq \frac{1}{\delta_3(\alpha)} \frac{|\delta_k|(A-B)}{2} \frac{|\delta_k(A-B)+2B|}{4}$$
$$\leq \frac{|\delta_k|(A-B)}{2\delta_3(\alpha)} \left[1 + \frac{|\delta_k|(A-B)}{2}\right].$$

Let the hypothesis be true for $n = m$. From (2.9), we have

$$|a_m| \leq \frac{|\delta_k|(A-B)}{2(m-1)\delta_m(\alpha)} \sum_{j=1}^{m-1} \delta_j(\alpha)|a_j|, \qquad a_1 = \delta_1(\alpha) = 1.$$

From (2.7), we have

$$|a_m| \leq \frac{1}{\delta_m(\alpha)} \prod_{j=0}^{m-2} \frac{|\delta_k(A-B) - 2jB|}{2(j+1)}.$$
$$\leq \frac{1}{\delta_m(\alpha)} \prod_{j=0}^{m-2} \frac{|\delta_k|(A-B) + 2j}{2(j+1)}.$$

By the induction hypothesis, we have

$$\frac{|\delta_k|(A-B)}{2(m-1)\delta_m(\alpha)} \sum_{j=1}^{m-1} \delta_j(\alpha)|a_j| \leq \frac{1}{\delta_m(\alpha)} \prod_{j=0}^{m-2} \frac{|\delta_k|(A-B) + 2j}{2(j+1)}.$$

Multiplying both sides by $\frac{\delta_m(\alpha)}{\delta_{m+1}(\alpha)} \frac{|\delta_k|(A-B)+2(m-1)}{2m}$, we have

$$\frac{1}{\delta_{m+1}(\alpha)} \prod_{j=0}^{m-1} \frac{|\delta_k|(A-B) + 2j}{2(j+1)} \geq \frac{|\delta_k|(A-B)}{2(m-1)\delta_m(\alpha)} . \frac{\delta_m(\alpha)}{\delta_{m+1}(\alpha)} \frac{|\delta_k|(A-B) + 2(m-1)}{2m} \sum_{j=1}^{m-1} \delta_j(\alpha)|a_j|,$$
$$= \frac{|\delta_k|(A-B)}{2m\delta_{m+1}(\alpha)} \left[\frac{|\delta_k|(A-B)}{2(m-1)} \sum_{j=1}^{m-1} \delta_j(\alpha)|a_j| + \sum_{j=1}^{m-1} \delta_j(\alpha)|a_j|\right],$$
$$\geq \frac{|\delta_k|(A-B)}{2m\delta_{m+1}(\alpha)} \left[\delta_m(\alpha)|a_m| + \sum_{j=1}^{m-1} \delta_j(\alpha)|a_j|\right],$$
$$= \frac{|\delta_k|(A-B)}{2m\delta_{m+1}(\alpha)} \sum_{j=1}^{m} \delta_j(\alpha)|a_j|.$$

That is

$$\frac{|\delta_k|(A-B)}{2m\delta_{m+1}(\alpha)} \sum_{j=1}^{m} \delta_m(\alpha)|a_j| \leq \frac{1}{\delta_{m+1}(\alpha)} \prod_{j=0}^{m-1} \frac{|\delta_k|(A-B) + 2j}{2(j+1)}.$$

Which shows that inequality (2.10) is true for $n = m + 1$. Hence the required result. □

When $\alpha = 0$ we get result introduced by Khalida Inayat Noor and Sarfraz Nawaz Malik in [8].

Corollary 2.7. *Let $f \in k - ST[A, B]$ and is of the form (1.3). Then*

$$|a_n| \leq \prod_{j=0}^{n-2} \frac{|\delta_k(A-B) - 2jB|}{2(j+1)}, \quad -1 \leq B < A \leq 1, \quad n \geq 2.$$

For $\alpha = 0,\ A = 1\ B = -1$ we arrive at Kanas and Wisniowska result in [5].

Corollary 2.8. *Let $f \in k - ST$ and is of the form (1.3). Then*

$$|a_n| \leq \prod_{j=0}^{n-2} \frac{|\delta_k + j|}{(j+1)}, \qquad n \geq 2.$$

Also for $\alpha = 0,\ k = 0\ \delta_k = 2$, we get result due to Janowski in [1].

Corollary 2.9. *Let $f \in S^*[A,B]$ and is of the form (1.3). Then*

$$|a_n| \leq \prod_{j=0}^{n-2} \frac{|(A-B) - jB|}{(j+1)}, \quad -1 \leq B < A \leq 1, \quad n \geq 2.$$

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