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CORRIGENDUM TO MODULES THAT HAVE A SUPPLEMENT IN EVERY TORSION EXTENSION

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1 Corrigendum

Modules that have a supplement in every torsion extension

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In this erratum to the paper "Modules that have a supplement in every torsion extension" [1], we present revised Theorem 2.8 and Corollary 2.9.

Unfortunately, we have found an error of Theorem 2.8 and Corollary 2.9 in [1]. The following example shows that Theorem 2.8 in [1] is not true, in general.

Example 1.1. Let R be a ring as in [1, Example 2.4]. Put $M =_R R$. Since R is incomplete, by [2, Corollary 7.9], the extension group $Ext_R(K, M) \neq 0$ where K is the quotient fields of R. It follows from [3, Corollary 1 of Proposition 3.4] that M is a TE-module. Consider the following short exact sequence with rows:

$$\mathbb{E} : 0 \longrightarrow M \xrightarrow{\iota} M \oplus M \xrightarrow{\pi} M \longrightarrow 0.$$

where ι is the canonical injection and π is the canonical projection. Therefore, $Ext_R(\frac{K}{R}, M \oplus M) \neq 0$ according to [4, Proposition 5 (d)]. Hence, the direct sum $M \oplus M$ is not a TE-module.

The following are corrected under a certain condition.

Theorem 1.2. *(Theorem 2.8 in [1])Let*

$$0 \longrightarrow K \xrightarrow{f} M \xrightarrow{g} L \longrightarrow 0$$

be a short exact sequence. Suppose that L is a torsion module. If K and L are TE-modules, then M is a TE-module.

Proof. Without restriction of generality we will assume that $K \leq M$. Let N be a torsion extension of M. For $K \leq M \leq N$,

$$\frac{N}{M} \cong \frac{\frac{N}{K}}{\frac{M}{K}}$$

is torsion, and so $\frac{M}{K}$ is a torsion extension of $\frac{N}{K}$. Since $L \cong \frac{M}{K}$ is a TE-module, there exists a submodule $\frac{V}{K}$ of $\frac{N}{K}$ such that $\frac{M}{K} + \frac{V}{K} = \frac{N}{K}$ and $\frac{(M \cap V)}{K} << \frac{V}{K}$. Note that $N = M + V$. Since $\frac{M}{K}$ and $\frac{\frac{N}{K}}{\frac{M}{K}}$, we get $\frac{N}{K}$ is torsion, and so $\frac{V}{K}$ is torsion. By the assumption, K has a supplement K' in V, i.e. $V = K + K'$ and $K \cap K' << K'$ because K is a TE-module. Now we have $N = M + V = M + K'$. Suppose that $M + X = N$ for some submodule X of K'. It follows that $\frac{M}{K} + \frac{(X+K)}{K} = \frac{N}{K}$, hence $\frac{(X+K)}{K} = \frac{V}{K}$ by the minimality of $\frac{V}{K}$. Then we have $V = X + K$ and so $X = K^{'}$ by the minimality of $K^{'}$. Thus $K^{'}$ is a supplement of M in N. Therefore M is a TE-module. □

Corollary 1.3. *(Corollary 2.9 in [1]) Let R be an arbitrary ring and $M = M_1 \oplus M_2$, where M_1 is any TE R-module and M_2 is a torsion TE R-module. Then, M is a TE-module.*

Proof. It follows from Theorem 1.2. □

References

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