

On three dimensional locally ϕ -semisymmetric trans-Sasakian manifolds with respect to generalized Tanaka Webster Okumura Connection

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Abstract. The object of the present paper is to characterize the locally ϕ - semisymmetry in three dimensional trans-Sasakian manifolds with respect to generalized Tanaka Webster Okumura connection. Locally Ricci semisymmetric, Locally projectively ϕ - semisymmetric and Locally conformally ϕ - semisymmetric trans-Sasakian manifolds of dimension three have also been studied with respect to generalized Tanaka Webster Okumura connection.

1 Introduction

Let M be an n -dimensional, $n \geq 3$, connected smooth Riemannian manifold endowed with the Riemannian metric g . Let ∇ , R , S and r be the Levi-Civita connection, curvature tensor, Ricci tensor and the scalar curvature of M respectively. The manifold M is called locally symmetric due to cartan ([6], [7]) if the global geodesic symmetry at $p \in M$ is an isometry, which is equivalent to the fact that $\nabla R = 0$. Generalizing the concept of local symmetry, the notion of semisymmetry was introduced by Cartan [8] and fully classified by Szabo ([27], [28], [29]). The manifold M is said to be semisymmetric if

$$(R(U, V).R)(X, Y)Z = 0, \quad (1.1)$$

for all vector fields X, Y, Z, U, V on M , where $R(U, V)$ is considered as the derivation of the tensor algebra at each point of M .

In 1977 Takahashi [30] introduced the notion of local ϕ - symmetry on Sasakian manifolds. A Sasakian manifold is said to be locally ϕ -symmetric if

$$\phi^2((\nabla_W R)(X, Y)Z) = 0, \quad (1.2)$$

for any vector fields X, Y, Z, W on M , where ϕ is the structure tensor of the manifold M . The concept of locally ϕ - symmetry on various structures and their generalizations or extension are studied in ([10], [22], [23], [24], [25]). By extending the notion of semisymmetry and generalizing the concept of local ϕ - symmetry of Takahashi [30] the authors in the paper [26] introduced the notion of locally ϕ -semisymmetric Sasakian manifolds. A Sasakian manifold M , $n \geq 3$, is said to be locally ϕ -semisymmetric if,

$$\phi^2\{(R(U, V).R)(X, Y)Z\} = 0, \quad (1.3)$$

for all horizontal vector fields X, Y, Z, U, V on M .

In 1985 J. A. Oubina [18] introduced a new class of almost contact metric manifolds, called trans-Sasakian manifold, which includes Sasakian, Kenmotsu and Cosymplectic structures. The authors in the papers [1], [2], [5] and [9] have studied such manifolds and obtained some interesting results. It is known that [14] trans-Sasakian structure of type $(0, 0)$, $(0, \beta)$ and $(\alpha, 0)$ are Cosymplectic, β -Kenmotsu and α -Sasakian respectively. The local classification of trans-Sasakian manifold is given by J. C. Marrero [17] and it is proved that a trans-Sasakian manifold of dimension $n \geq 5$ is either Cosymplectic or α -Sasakian or β -Kenmotsu manifold. In the present paper we have studied three dimensional trans-Sasakian manifolds with respect to

generalized Tanaka Webster Okumura connection. The notion of generalized Tanaka Webster Okumura connection was introduced and studied by the authors in the paper [13]. In the present paper we have studied three dimensional locally ϕ -semisymmetric trans-Sasakian manifolds with respect to generalized Tanaka Webster Okumura connection. The present paper is organized as follows:

After introduction in Section 1 we give some preliminaries in Section 2. Section 3 is devoted to the study of locally Ricci semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection. In Sections 4 and 5 we have studied respectively locally ϕ - semisymmetric and locally projectively ϕ - semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection. Section 6 is completed with the study of locally conformally ϕ - semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection.

2 Preliminaries

Let M be a $(2n + 1)$ -dimensional connected differentiable manifold endowed with an almost contact metric structure (ϕ, ξ, η, g) , where ϕ is a tensor field of type $(1, 1)$, ξ is a vector field, η is a 1-form and g is a Riemannian metric on M such that [4]

$$\phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1. \quad (2.1)$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y). \quad X, Y \in T(M) \quad (2.2)$$

Then also

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \quad \eta(X) = g(X, \xi). \quad (2.3)$$

$$g(\phi X, X) = 0. \quad (2.4)$$

$$g(\phi X, Y) = -g(X, \phi Y) \quad (2.5)$$

An almost contact metric manifold $M^{2n+1}(\phi, \xi, \eta, g)$ is said to be a trans-Sasakian manifold [18] if $(M^{2n+1} \times R, J, G)$ belongs to the class W_4 [12] of the Hermitian manifolds, where J is the almost complex structure on $M^{2n+1} \times R$ defined by [11]

$$J(Z, f \frac{d}{dt}) = (\phi Z - f\xi, \eta(Z) \frac{d}{dt}), \quad (2.6)$$

for any vector field Z on M^{2n+1} and smooth function f on $M^{2n+1} \times R$ and G is the Hermitian metric on the product $M^{2n+1} \times R$. This may be expressed by the condition [18]

$$(\nabla_X \phi)Y = \alpha(g(X, Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X), \quad (2.7)$$

for some smooth functions α and β on M^{2n+1} , and we say that the trans-Sasakian structure is of type (α, β) . From equation (2.7), it follows that

$$\nabla_X \xi = -\alpha\phi X + \beta(X - \eta(Y)\xi), \quad (2.8)$$

$$(\nabla_X \eta)Y = -\alpha g(\phi X, Y)\xi + \beta g(\phi X, \phi Y). \quad (2.9)$$

The Ricci operator, Ricci tensor and curvature tensor for three dimensional trans-Sasakian manifolds have been studied in the paper [9]. From [9], we have the following for three dimensional trans-Sasakian manifolds:

$$2\alpha\beta + \xi\alpha = 0, \quad (2.10)$$

$$S(X, \xi) = \{2(\alpha^2 - \beta^2)\}\eta(X) - \xi - (\phi X)\alpha\beta, \quad (2.11)$$

$$\begin{aligned}
S(X, Y) &= \left\{ \frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2) \right\} g(X, Y) \\
&- \left\{ \frac{r}{2} + \xi\beta - 3(\alpha^2 - \beta^2) \right\} \eta(X)\eta(Y) \\
&- \{Y\beta + (\phi Y)\alpha\}\eta(X) - \{X\beta + (\phi X)\alpha\}\eta(Y),
\end{aligned} \tag{2.12}$$

and

$$\begin{aligned}
R(X, Y)Z &= \left\{ \frac{r}{2} + 2\xi\beta - 2(\alpha^2 - \beta^2) \right\} \{g(Y, Z)X - g(X, Z)Y\} \\
&- g(Y, Z) \left[\left\{ \frac{r}{2} + \xi\beta - 3(\alpha^2 - \beta^2) \right\} \eta(X)\xi \right. \\
&- \eta(X)(\phi \text{grad}\alpha - \text{grad}\beta) + \{X\beta + (\phi X)\alpha\}\xi \left. \right] \\
&+ g(X, Z) \left[\left\{ \frac{r}{2} + \xi\beta - 3(\alpha^2 - \beta^2) \right\} \eta(Y)\xi \right. \\
&- \eta(Y)(\phi \text{grad}\alpha - \text{grad}\beta) + \{Y\beta + (\phi Y)\alpha\}\xi \left. \right] \\
&- [\{Z\beta + (\phi Z)\alpha\}\eta(Y) + \{Y\beta + (\phi Y)\alpha\}\eta(Z) \\
&+ \left\{ \frac{r}{2} + \xi\beta - 3(\alpha^2 - \beta^2) \right\} \eta(Y)\eta(Z)]X \\
&+ [\{Z\beta + (\phi Z)\alpha\}\eta(X) + \{X\beta + (\phi X)\alpha\}\eta(Z) \\
&+ \left\{ \frac{r}{2} + \xi\beta - 3(\alpha^2 - \beta^2) \right\} \eta(X)\eta(Z)]Y,
\end{aligned} \tag{2.13}$$

where S is the Ricci tensor of type $(0, 2)$, R is the curvature tensor of type $(1, 3)$ and r is the scalar curvature of the manifold M .

Now if X, Y and Z are orthogonal to ξ then relations (2.12) and (2.13) are changes to

$$S(X, Y) = \left\{ \frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2) \right\} g(X, Y), \tag{2.14}$$

and

$$\begin{aligned}
R(X, Y)Z &= \left\{ \frac{r}{2} + 2\xi\beta - 2(\alpha^2 - \beta^2) \right\} \{g(Y, Z)X - g(X, Z)Y\} \\
&- g(Y, Z) \{X\beta + (\phi X)\alpha\}\xi + g(X, Z) \{Y\beta + (\phi Y)\alpha\}\xi.
\end{aligned} \tag{2.15}$$

Again from (2.15) we have

$$R(\xi, Y)Z = \left\{ \frac{r}{2} + 2\xi\beta - 2(\alpha^2 - \beta^2) \right\} g(Y, Z)\xi - g(Y, Z)(X\beta)\xi. \tag{2.16}$$

and

$$R(X, Y)\xi = 0, \tag{2.17}$$

where X, Y and Z are orthogonal to ξ

The generalized Tanaka Webster Okumura connection [13] $\tilde{\nabla}$ and the Levi-Civita connection ∇ are related by

$$\tilde{\nabla}_X Y = \nabla_X Y + A(X, Y), \tag{2.18}$$

for all vectors fields X, Y on M . Here

$$A(X, Y) = \alpha \{g(X, \phi Y)\xi + \eta(Y)\phi X\} + \beta \{g(X, Y)\xi - \eta(Y)X\} - l\eta(X)\phi Y, \tag{2.19}$$

where l is a real constant.

The Torsion \tilde{T} of the gTWO-connection $\tilde{\nabla}$ is given by

$$\tilde{T}(X, Y) = \alpha \{2g(X, \phi Y)\xi - \eta(X)\phi Y + \eta(Y)\phi X\} + \eta(X)(\beta Y - l\phi Y) - \eta(Y)(\beta X - l\phi X). \tag{2.20}$$

Again relation between the curvature tensors \tilde{R} and R with respect to the generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ and the Levi-Civita connection ∇ respectively is given by [21]

$$\begin{aligned}
\tilde{R}(X, Y)Z &= R(X, Y)Z + \alpha\{g(Y, \nabla_X \phi Z) - g(X, \nabla_Y \phi Z) + g(X, \phi \nabla_Y Z)\xi \\
&+ \eta(\nabla_Y Z)\phi X - g(Y, \phi \nabla_X Z)\xi - \eta(\nabla_X Z)\phi Y - \eta(Z)\phi[X, Y]\} \\
&+ \beta\{\eta(\nabla_X Z)Y - \eta(\nabla_Y Z)X + \eta(Z)[X, Y]\} \\
&- l\{\eta(X)\phi \nabla_Y Z + \eta(Y)\phi \nabla_X Z + \eta([X, Y])\phi Z\} \\
&+ \{\alpha g(Y, \phi Z) + \beta g(Y, Z)\}\{\nabla_X \xi + \alpha \phi X + \beta(\eta(X)\xi - X)\} \\
&- \{\alpha g(X, \phi Z) + \beta g(X, Z)\}\{\nabla_Y \xi + \alpha \phi Y + \beta(\eta(Y)\xi - Y)\} \\
&+ (\alpha \phi Y - \beta Y)[\nabla_X \eta(Z) + \alpha\{g(X, \phi \eta(Z))\xi + \eta(\eta(Z))\phi X\} \\
&+ \beta\{g(X, \eta(Z))\xi - \eta(\eta(Z))X\} - l\eta(X)\phi \eta(Z)] \\
&- (\alpha \phi X - \beta X)[\nabla_Y \eta(Z) + \alpha\{g(Y, \phi \eta(Z))\xi + \eta(\eta(Z))\phi Y\} \\
&+ \beta\{g(Y, \eta(Z))\xi - \eta(\eta(Z))Y\} - l\eta(Y)\phi \eta(Z)] \\
&+ \alpha \eta(Z)[\nabla_X \phi Y - \nabla_Y \phi X + \beta\{g(X, \phi Y) - g(Y, \phi X)\}\xi + l\{\eta(X)Y - \eta(Y)X\}] \\
&- \beta \eta(Z)[\nabla_X Y - \nabla_Y X + \alpha\{g(X, \phi Y)\xi + \eta(Y)\phi X - g(Y, \phi X)\xi \\
&+ \eta(X)\phi Y\} - \beta\{\eta(Y)X - \eta(X)Y\} + l\{\eta(Y)\phi X - \eta(X)\phi Y\}] \\
&- l\eta(Y)[\nabla_X \phi Z + \alpha\{\eta(X)\eta(Z) - g(X, Z)\}\xi + \beta g(X, \phi Z)\xi \\
&+ l\{\eta(X)Z - \eta(X)\eta(Z)\xi\}] + l\eta(X)[\nabla_Y \phi Z + \alpha\{\eta(Y)\eta(Z) - g(Y, Z)\}\xi \\
&+ \beta g(Y, \phi Z)\xi + l\{\eta(Y)Z - \eta(Y)\eta(Z)\xi\}] \\
&- l\phi Z[\nabla_X \eta(Y) - \nabla_Y \eta(X) + \alpha\{g(X, \phi \eta(Y))\xi + \eta(\eta(Y))\phi X\} - g(Y, \phi \eta(X))\xi \\
&- \eta(\eta(X))\phi Y\} + \beta\{g(X, \eta(Y))\xi - \eta(\eta(Y))X - g(Y, \eta(X))\xi + \eta(\eta(X))Y\} \\
&+ l\{\eta(Y)\phi \eta(X) - \eta(X)\phi \eta(Y)\}].
\end{aligned} \tag{2.21}$$

We suppose that $X, Y, Z, \nabla_X Z$ and $\nabla_Y Z$ are orthogonal to ξ . Then (2.21) becomes

$$\begin{aligned}
\tilde{R}(X, Y)Z &= R(X, Y)Z + \alpha\{g(Y, \nabla_X \phi Z) - g(X, \nabla_Y \phi Z) + g(X, \phi \nabla_Y Z) \\
&- g(Y, \phi \nabla_X Z)\}\xi + \beta g([X, Y], Z)\xi \\
&+ \{\alpha g(Y, \phi Z) + \beta g(Y, Z)\}\{\nabla_X \xi + \alpha \phi X + \beta - X\} \\
&- \{\alpha g(X, \phi Z) + \beta g(X, Z)\}\{\nabla_Y \xi + \alpha \phi Y + \beta - Y\},
\end{aligned} \tag{2.22}$$

3 Locally Ricci semisymmetric three dimensional trans-Sasakian manifolds with generalized Tanaka Webster Okumura connection

In this section we have studied locally Ricci semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection.

Definition 3.1. A trans-Sasakian manifold of dimension three is said to be locally Ricci-semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if

$$(\tilde{R}(U, V) \cdot \tilde{S})(X, Y) = 0, \tag{3.1}$$

for all horizontal vector fields X, Y, U, V on M .

Thus for a trans-Sasakian manifold of dimension three we obtain by using (2.8) in (2.22)

$$\begin{aligned}
\tilde{R}(X, Y)Z &= R(X, Y)Z + \alpha\{g(Y, \nabla_X \phi Z) - g(X, \nabla_Y \phi Z) + g(X, \phi \nabla_Y Z) \\
&- g(Y, \phi \nabla_X Z)\}\xi + \beta g([X, Y], Z)\xi,
\end{aligned} \tag{3.2}$$

where X, Y and Z are orthogonal to ξ .

Applying ϕ^2 on both side of (3.2) we get

$$\phi^2\{\tilde{R}(X, Y)Z\} = \phi^2\{R(X, Y)Z\} \tag{3.3}$$

Now taking inner product on both side of (3.2) by W we obtain

$$g(\tilde{R}(X, Y)Z, W) = g(R(X, Y)Z, W). \tag{3.4}$$

From relation (3.4) we obtain

$$\tilde{S}(X, W) = S(X, W), \quad (3.5)$$

where W is orthogonal to ξ .

Again we know that for a trans-Sasakian manifolds the Ricci tensor S with respect to Levi Civita connection is

$$S(X, Y) = \left(\frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2)\right)g(X, Y), \quad (3.6)$$

where X and Y are orthogonal to ξ .

i.e.

$$S(X, Y) = Hg(X, Y), \quad (3.7)$$

where

$$H = \frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2). \quad (3.8)$$

Therefore by (3.5) and (3.6) we get

$$\tilde{S}(X, Y) = \left(\frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2)\right)g(X, Y), \quad (3.9)$$

Again we know that the Ricci operator \tilde{Q} with generalized Tanaka Webster Okumura connection is given by

$$\tilde{S}(X, Y) = g(\tilde{Q}X, Y). \quad (3.10)$$

Thus combining (3.9) and (3.10) we get

$$\tilde{Q}X = \left(\frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2)\right)X. \quad (3.11)$$

Now we know that

$$(\tilde{R}(U, V).\tilde{S})(X, Y) = (\tilde{R}(U, V).S)(X, Y). \quad (3.12)$$

or

$$(\tilde{R}(U, V).\tilde{S})(X, Y) = -S(\tilde{R}(U, V)X, Y) - S(X, \tilde{R}(U, V)Y) \quad (3.13)$$

Using (2.14) and (3.2) in relation (3.13) we get

$$(\tilde{R}(U, V).\tilde{S})(X, Y) = -H\{g(R(U, V)X, Y) + g(X, R(U, V)Y)\} \quad (3.14)$$

Again using (2.15) in relation (3.14) we get

$$\begin{aligned} (\tilde{R}(U, V).\tilde{S})(X, Y) &= -H\left\{\frac{r}{2} + 2\xi\beta - 2(\alpha^2 - \beta^2)\right\}[g(V, X)g(U, Y) \\ &\quad - g(U, X)g(V, Y) + g(U, X)g(V, Y) - g(V, X)g(U, Y)] \\ &= 0. \end{aligned} \quad (3.15)$$

Thus we are in a position to state the following:

Theorem 3.2. *Every trans-Sasakian manifolds of dimension three is locally Ricci semisymmetric with respect to generalized Tanaka Webster Okumura connection.*

4 Locally ϕ -semisymmetric three dimensional trans-Sasakian manifolds with respect to generalized Tanaka Webster Okumura connection

In this section we have studied locally ϕ -semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection.

Definition 4.1. A trans-Sasakian manifold of dimension three is said to be locally ϕ -semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if

$$\phi^2\{(\tilde{R}(U, V).\tilde{R})(X, Y)Z\} = 0, \quad (4.1)$$

for all horizontal vector fields X, Y, Z, U, V on M .

Now we know that

$$\begin{aligned} [(\tilde{R}(U, V) \cdot \tilde{R})(X, Y)Z] &= \tilde{R}(U, V)\tilde{R}(X, Y)Z - \tilde{R}(\tilde{R}(U, V)X, Y)Z \\ &\quad - \tilde{R}(X, \tilde{R}(U, V)Y)Z - \tilde{R}(X, Y)\tilde{R}(U, V)Z \end{aligned} \quad (4.2)$$

Now using (3.2) in first and last terms of right hand side of (4.2) we get

$$\begin{aligned} \tilde{R}(U, V)\tilde{R}(X, Y)Z &= R(U, V)R(X, Y)Z \\ &\quad + [\alpha\{g(V, \nabla_U \phi \tilde{R}(X, Y)Z) - g(U, \nabla_V \phi \tilde{R}(X, Y)Z)\} \\ &\quad + g(U, \phi \nabla_V \tilde{R}(X, Y)Z) - g(V, \phi \nabla_U \tilde{R}(X, Y)Z)] \\ &\quad + \beta g([U, V], \tilde{R}(X, Y)Z)]\xi. \end{aligned} \quad (4.3)$$

and

$$\begin{aligned} \tilde{R}(X, Y)\tilde{R}(U, V)Z &= R(X, Y)R(U, V)Z \\ &\quad + [\alpha\{g(Y, \nabla_X \phi \tilde{R}(U, V)Z) - g(X, \nabla_Y \phi \tilde{R}(U, V)Z)\} \\ &\quad + g(X, \phi \nabla_Y \tilde{R}(U, V)Z) - g(Y, \phi \nabla_X \tilde{R}(U, V)Z)] \\ &\quad + \beta g([X, Y], \tilde{R}(U, V)Z)]\xi. \end{aligned} \quad (4.4)$$

Similarly using (3.2) in second and third terms of right hand side of (4.2) we get

$$\begin{aligned} \tilde{R}(\tilde{R}(U, V)X, Y)Z &= R(R(U, V)X, Y)Z \\ &\quad + [\alpha\{g(V, \nabla_U \phi X) - g(U, \nabla_V \phi X)\} \\ &\quad + g(U, \phi \nabla_V X) - g(V, \phi \nabla_U X)] \\ &\quad + \beta g([U, V], X)]R(\xi, Y)Z \\ &\quad + [\alpha\{g(Y, \nabla_{\tilde{R}(U, V)X} \phi Z) - g(\tilde{R}(U, V)X, \nabla_Y \phi Z)\} \\ &\quad + g(\tilde{R}(U, V)X, \phi \nabla_Y Z) - g(Y, \phi \nabla_{\tilde{R}(U, V)X} Z)] \\ &\quad + \beta g([\tilde{R}(U, V)X, Y], Z)]\xi \end{aligned} \quad (4.5)$$

and

$$\begin{aligned} \tilde{R}(X, \tilde{R}(U, V)Y)Z &= -\tilde{R}(\tilde{R}(U, V)Y, X)Z \\ &= -R(R(U, V)Y, X)Z \\ &\quad - [\alpha\{g(V, \nabla_U \phi Y) - g(U, \nabla_V \phi Y)\} \\ &\quad + g(U, \phi \nabla_V Y) - g(V, \phi \nabla_U Y)] \\ &\quad + \beta g([U, V], Y)]R(\xi, X)Z \\ &\quad - [\alpha\{g(X, \nabla_{\tilde{R}(U, V)Y} \phi Z) - g(\tilde{R}(U, V)Y, \nabla_X \phi Z)\} \\ &\quad + g(\tilde{R}(U, V)Y, \phi \nabla_X Z) - g(X, \phi \nabla_{\tilde{R}(U, V)Y} Z)] \\ &\quad + \beta g([\tilde{R}(U, V)Y, X], Z)]\xi \end{aligned} \quad (4.6)$$

Using (4.3), (4.4), (4.5) and (4.6) in relation (4.2) we get

$$\begin{aligned}
 [(\tilde{R}(U, V).\tilde{R})(X, Y)Z] &= [(R(U, V).(X, Y)Z] \\
 &+ [\alpha\{g(V, \nabla_U \phi \tilde{R}(X, Y)Z) - g(U, \nabla_V \phi \tilde{R}(X, Y)Z)\} \\
 &+ g(U, \phi \nabla_V \tilde{R}(X, Y)Z) - g(V, \phi \nabla_U \tilde{R}(X, Y)Z)\} \\
 &+ \beta g([U, V], \tilde{R}(X, Y)Z)]\xi \\
 &+ [\alpha\{g(V, \nabla_U \phi X) - g(U, \nabla_V \phi X)\} \\
 &+ g(U, \phi \nabla_V X) - g(V, \phi \nabla_U X)\} \\
 &+ \beta g([U, V], X)]R(\xi, Y)Z \\
 &+ [\alpha\{g(Y, \nabla_{\tilde{R}(U, V)X} \phi Z) - g(\tilde{R}(U, V)X, \nabla_Y \phi Z)\} \\
 &+ g(\tilde{R}(U, V)X, \phi \nabla_Y Z) - g(Y, \phi \nabla_{\tilde{R}(U, V)X} Z)\} \\
 &+ \beta g([\tilde{R}(U, V)X, Y], Z)]\xi \\
 &- [\alpha\{g(V, \nabla_U \phi Y) - g(U, \nabla_V \phi Y)\} \\
 &+ g(U, \phi \nabla_V Y) - g(V, \phi \nabla_U Y)\} \\
 &+ \beta g([U, V], Y)]R(\xi, X)Z \\
 &- [\alpha\{g(X, \nabla_{\tilde{R}(U, V)Y} \phi Z) - g(\tilde{R}(U, V)Y, \nabla_X \phi Z)\} \\
 &+ g(\tilde{R}(U, V)Y, \phi \nabla_X Z) - g(X, \phi \nabla_{\tilde{R}(U, V)Y} Z)\} \\
 &+ \beta g([\tilde{R}(U, V)Y, X], Z)]\xi \\
 &+ [\alpha\{g(Y, \nabla_X \phi \tilde{R}(U, V)Z) - g(X, \nabla_Y \phi \tilde{R}(U, V)Z)\} \\
 &+ g(X, \phi \nabla_Y \tilde{R}(U, V)Z) - g(Y, \phi \nabla_X \tilde{R}(U, V)Z)\} \\
 &+ \beta g([X, Y], \tilde{R}(U, V)Z)]\xi.
 \end{aligned} \tag{4.7}$$

Now applying ϕ^2 on both side of (4.7) we get

$$\begin{aligned}
 \phi^2[(\tilde{R}(U, V).\tilde{R})(X, Y)Z] &= \phi^2[(R(U, V).(X, Y)Z] \\
 &+ [\alpha\{g(V, \nabla_U \phi X) - g(U, \nabla_V \phi X)\} \\
 &+ g(U, \phi \nabla_V X) - g(V, \phi \nabla_U X)\} \\
 &+ \beta g([U, V], X)]\{\phi^2 R(\xi, Y)Z\} \\
 &- [\alpha\{g(V, \nabla_U \phi Y) - g(U, \nabla_V \phi Y)\} \\
 &+ g(U, \phi \nabla_V Y) - g(V, \phi \nabla_U Y)\} \\
 &+ \beta g([U, V], Y)]\{\phi^2 R(\xi, X)Z\}
 \end{aligned} \tag{4.8}$$

Using (2.16) in (4.8) we get

$$\phi^2[(\tilde{R}(U, V).\tilde{R})(X, Y)Z] = \phi^2[(R(U, V).(X, Y)Z] \tag{4.9}$$

Thus we are in a position to state the following:

Theorem 4.2. *A trans-Sasakian manifold of dimension three is locally ϕ -semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if and only if it is so with respect to Levi-civita connection ∇ .*

5 Locally projectively ϕ -semisymmetric three dimensional trans-Sasakian manifolds with generalized Tanaka Webster Okumura connection

In this section we have studied locally projectively ϕ - semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection.

Definition 5.1. For a $(2n+1)$ - dimensional ($n>1$) manifold the Weyl projective curvature tensor \tilde{P} with respect to generalized Tanaka Webster Okumura connection will be given by

$$\tilde{P}(X, Y)Z = \tilde{R}(X, Y)Z - \frac{1}{2n}[\tilde{S}(Y, Z)X - \tilde{S}(X, Z)Y]. \tag{5.1}$$

In view of (3.2) and (3.9) we get from (5.1)

$$\begin{aligned} \tilde{P}(X, Y)Z &= R(X, Y)Z + \alpha\{g(Y, \nabla_X \phi Z) - g(X, \nabla_Y \phi Z) \\ &+ g(X, \phi \nabla_Y Z) - g(Y, \phi \nabla_X Z)\}\xi + \beta g([X, Y], Z)\xi \\ &- \frac{H}{2n}\{g(Y, Z)X - g(X, Z)Y\}, \end{aligned} \quad (5.2)$$

where

$$H = \frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2). \quad (5.3)$$

Definition 5.2. A $(2n+1)$ -dimensional $(n>1)$ trans-Sasakian manifold is said to be a Locally projectively ϕ -semisymmetric with respect to generalized Tanaka Webster Okumura connection if the relation

$$\phi^2[(\tilde{R}(X, Y).\tilde{P})(Z, U)V] = 0, \quad (5.4)$$

holds for all horizontal vector fields X, Y, Z, U and V on M .

Let M be a 3-dimensional connected trans-Sasakian manifold. Now we know that

$$\begin{aligned} [(\tilde{R}(X, Y).\tilde{P})(Z, U)V] &= \tilde{R}(X, Y)\tilde{P}(Z, U)V - \tilde{P}(\tilde{R}(X, Y)Z, U)V \\ &- \tilde{P}(Z, \tilde{R}(X, Y)U)V - \tilde{P}(Z, U)\tilde{R}(X, Y)V \end{aligned} \quad (5.5)$$

Now by using (3.2) and (5.2) on the first term of right hand side of (5.5) we have

$$\begin{aligned} \tilde{R}(X, Y)\tilde{P}(Z, U)V &= R(X, Y)R(Z, U)V + \alpha\{g(Y, \nabla_X \phi R(Z, U)V) \\ &- g(X, \nabla_Y \phi R(Z, U)V) + g(X, \phi \nabla_Y R(Z, U)V) \\ &- g(Y, \phi \nabla_X R(Z, U)V)\}\xi + \beta g([X, Y], R(Z, U)V)\xi \\ &+ \alpha\{g(U, \nabla_Z \phi V) - g(Z, \nabla_U \phi V) \\ &+ g(Z, \phi \nabla_U V) - g(U, \phi \nabla_Z V)\}\xi + \beta g([Z, U], V)\xi \\ &- \frac{H}{2n}[g(U, V)\tilde{R}(X, Y)Z - g(Z, V)\tilde{R}(X, Y)U] \end{aligned} \quad (5.6)$$

Using (3.2), (5.1) and (5.2) in the second term of right hand side of (5.5) we get

$$\begin{aligned} \tilde{P}(\tilde{R}(X, Y)Z, U)V &= R(R(X, Y)Z, U)V \\ &+ [\alpha\{g(U, \nabla_{R(X, Y)Z} \phi V) - g(R(X, Y)Z, \nabla_U \phi V) \\ &+ g(R(X, Y)Z, \phi \nabla_U V) - g(U, \phi \nabla_{R(X, Y)Z} V)\} \\ &+ \beta g([R(X, Y)Z, U], V)]\xi \\ &+ [\alpha\{g(Y, \nabla_X \phi Z) - g(X, \nabla_Y \phi Z) + g(X, \phi \nabla_Y Z) \\ &- g(Y, \phi \nabla_X Z)\} + \beta g([X, Y], Z)]\tilde{R}(\xi, U)V \\ &- \frac{H}{2n}[g(U, V)\tilde{R}(X, Y)Z - g(R(X, Y)Z, V)U]. \end{aligned} \quad (5.7)$$

Similarly for third and fourth terms of right hand side of (5.5), using (3.2), (5.1) and (5.2) we have the following:

$$\begin{aligned} \tilde{P}(Z, \tilde{R}(X, Y)U)V &= R(Z, R(X, Y)U)V \\ &+ [\alpha\{g(R(X, Y)Z, \nabla_Z \phi V) - g(Z, \nabla_{R(X, Y)Z} \phi V) \\ &- g(R(X, Y)Z, \phi \nabla_Z V) + g(Z, \phi \nabla_{R(X, Y)Z} V)\} \\ &+ \beta g([R(X, Y)Z, U], V)]\xi \\ &+ [\alpha\{g(Y, \nabla_X \phi U) - g(X, \nabla_Y \phi U) + g(X, \phi \nabla_Y U) \\ &- g(Y, \phi \nabla_X U)\} + \beta g([X, Y], U)]\tilde{R}(Z, \xi)V \\ &- \frac{H}{2n}[g(R(X, Y)U, V)Z - g(Z, V)\tilde{R}(X, Y)U] \end{aligned} \quad (5.8)$$

and

$$\begin{aligned} \tilde{P}(Z, U)\tilde{R}(X, Y)V &= R(Z, U)R(X, Y)V \\ &+ \alpha\{g(U, \nabla_Z \phi R(X, Y)V) - g(Z, \nabla_U \phi R(X, Y)V) \\ &+ g(Z, \phi \nabla_U R(X, Y)V) - g(U, \phi \nabla_Z R(X, Y)V)\}\xi \\ &+ \beta g([Z, U], R(X, Y)V)\xi \\ &- \frac{H}{2n}\{g(U, R(X, Y)V)Z - g(Z, R(X, Y)V)U\}. \end{aligned} \quad (5.9)$$

Using (5.6), (5.7), (5.8) and (5.9) in (5.5) we get,

$$\begin{aligned}
 [(\tilde{R}(X, Y) \cdot \tilde{P})(Z, U)V] &= [R(X, Y) \cdot R](Z, U)V \\
 &+ \alpha\{g(Y, \nabla_X \phi R(Z, U)V) \\
 &- g(X, \nabla_Y \phi R(Z, U)V) + g(X, \phi \nabla_Y R(Z, U)V) \\
 &- g(Y, \phi \nabla_X R(Z, U)V)\} \xi + \beta g([X, Y], R(Z, U)V) \xi \\
 &+ \alpha\{g(U, \nabla_Z \phi V) - g(Z, \nabla_U \phi V) \\
 &+ g(Z, \phi \nabla_U V) - g(U, \phi \nabla_Z V)\} \xi + \beta g([Z, U], V) \xi \\
 &+ \alpha\{g(U, \nabla_{R(X, Y)Z} \phi V) - g(R(X, Y)Z, \nabla_U \phi V) \\
 &+ g(R(X, Y)Z, \phi \nabla_U V) - g(U, \phi \nabla_{R(X, Y)Z} V)\} \\
 &+ \beta g([R(X, Y)Z, U], V) \xi \\
 &+ [\alpha\{g(Y, \nabla_X \phi Z) - g(X, \nabla_Y \phi Z) + g(X, \phi \nabla_Y Z) \\
 &- g(Y, \phi \nabla_X Z)\} + \beta g([X, Y], Z)] \tilde{R}(\xi, U)V \\
 &+ [\alpha\{g(R(X, Y)Z, \nabla_Z \phi V) - g(Z, \nabla_{R(X, Y)Z} \phi V) \\
 &- g(R(X, Y)Z, \phi \nabla_Z V) + g(Z, \phi \nabla_{R(X, Y)Z} V)\} \\
 &+ \beta g([R(X, Y)Z, U], V)] \xi \\
 &+ [\alpha\{g(Y, \nabla_X \phi U) - g(X, \nabla_Y \phi U) + g(X, \phi \nabla_Y U) \\
 &- g(Y, \phi \nabla_X U)\} + \beta g([X, Y], U)] \tilde{R}(Z, \xi)V \\
 &+ \alpha\{g(U, \nabla_Z \phi R(X, Y)V) - g(Z, \nabla_U \phi R(X, Y)V) \\
 &+ g(Z, \phi \nabla_U R(X, Y)V) - g(U, \phi \nabla_Z R(X, Y)V)\} \xi \\
 &+ \beta g([Z, U], R(X, Y)V) \xi
 \end{aligned} \tag{5.10}$$

Now applying ϕ^2 on both side of (5.10) we get

$$\begin{aligned}
 \phi^2[(\tilde{R}(X, Y) \cdot \tilde{P})(Z, U)V] &= \phi^2[R(X, Y) \cdot R](Z, U)V \\
 &+ [\alpha\{g(Y, \nabla_X \phi Z) - g(X, \nabla_Y \phi Z) + g(X, \phi \nabla_Y Z) \\
 &- g(Y, \phi \nabla_X Z)\} + \beta g([X, Y], Z)] \{\phi^2 \tilde{R}(\xi, U)V\} \\
 &+ [\alpha\{g(Y, \nabla_X \phi U) - g(X, \nabla_Y \phi U) + g(X, \phi \nabla_Y U) \\
 &- g(Y, \phi \nabla_X U)\} + \beta g([X, Y], U)] \{\phi^2 \tilde{R}(Z, \xi)V\}
 \end{aligned} \tag{5.11}$$

In view of (3.2) and (2.16) we get from (5.11)

$$\phi^2[(\tilde{R}(X, Y) \cdot \tilde{P})(Z, U)V] = \phi^2[R(X, Y) \cdot R](Z, U)V \tag{5.12}$$

Thus we are in a position to state the following:

Theorem 5.3. *A trans-Sasakian manifold of dimension three is locally projectively ϕ -semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if and only if it is locally ϕ -semisymmetric with respect to Levi-civita connection ∇ .*

6 Locally conformally ϕ -semisymmetric three dimensional trans-Sasakian manifolds with respect to generalized Tanaka Webster Okumura connection

In this section we have studied locally conformally ϕ - semisymmetric trans-Sasakian manifolds of dimension three with respect to generalized Tanaka Webster Okumura connection.

Definition 6.1. For a $(2n + 1)$ dimensional Riemannian manifold the Weyl conformal curvature tensor with respect to generalized Tanaka Webster Okumura connection is defined by

$$\begin{aligned}
 \tilde{C}(X, Y)Z &= \tilde{R}(X, Y)Z - \frac{1}{2n-1} \{\tilde{S}(Y, Z)X - \tilde{S}(X, Z)Y \\
 &+ g(Y, Z)\tilde{Q}X - g(X, Z)\tilde{Q}Y\} \\
 &+ \frac{\tilde{r}}{2n(2n-1)} \{g(Y, Z)X - g(X, Z)Y\}.
 \end{aligned} \tag{6.1}$$

Using (3.9) and (3.11) in (6.1) we get

$$\tilde{C}(X, Y)Z = \tilde{R}(X, Y)Z + \left\{ \frac{r}{2n(2n-1)} - \frac{2H}{2n-1} \right\} \{g(Y, Z)X - g(X, Z)Y\}, \quad (6.2)$$

where $H = \frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2)$.
i.e.

$$\tilde{C}(X, Y)Z = \tilde{R}(X, Y)Z + N\{g(Y, Z)X - g(X, Z)Y\}, \quad (6.3)$$

where $N = \left\{ \frac{r}{2n(2n-1)} - \frac{2H}{2n-1} \right\}$.
Using (3.2) in (6.3) we obtain,

$$\begin{aligned} \tilde{C}(X, Y)Z &= R(X, Y)Z + [\alpha\{g(Y, \nabla_X \phi Z) - g(X, \nabla_Y \phi Z) \\ &+ g(X, \phi \nabla_Y Z) - g(Y, \phi \nabla_X Z)\} + \beta g([X, Y], Z)]\xi \\ &+ N\{g(Y, Z)X - g(X, Z)Y\}, \end{aligned} \quad (6.4)$$

Definition 6.2. A trans-Sasakian manifold of dimension three is said to be locally conformally ϕ -semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if

$$\phi^2\{(\tilde{R}(U, V).\tilde{C})(X, Y)Z\} = 0, \quad (6.5)$$

for all horizontal vector fields X, Y, Z, U, V on M .

Now we know that

$$\begin{aligned} [(\tilde{R}(U, V).\tilde{C})(X, Y)Z] &= \tilde{R}(U, V)\tilde{C}(X, Y)Z - \tilde{C}(\tilde{R}(U, V)X, Y)Z \\ &- \tilde{C}(X, \tilde{R}(U, V)Y)Z - \tilde{C}(X, Y)\tilde{R}(U, V)Z \end{aligned} \quad (6.6)$$

Using (3.2), (2.17) and (6.3) in each terms of right hand side of (6.6) and by straightforward calculation we obtain the followings:

$$\begin{aligned} \tilde{R}(U, V)\tilde{C}(X, Y)Z &= R(U, V)C(X, Y)Z \\ &+ [\alpha\{g(Y, \nabla_X \phi Z) - g(X, \nabla_Y \phi Z) + g(X, \phi \nabla_Y Z) \\ &- g(Y, \phi \nabla_X Z)\} + \beta g([X, Y], Z)]\xi \\ &+ [\alpha\{g(V, \nabla_U \phi \tilde{C}(X, Y)Z) - g(U, \nabla_V \phi \tilde{C}(X, Y)Z) \\ &+ g(U, \phi \nabla_V \tilde{C}(X, Y)Z) - g(V, \phi \nabla_U \tilde{C}(X, Y)Z)\} \\ &+ \beta g([U, V], \tilde{C}(X, Y)Z)]\xi \\ &+ N\{g(Y, Z)R(U, V)X - g(X, Z)R(U, V)Y\}, \end{aligned} \quad (6.7)$$

$$\begin{aligned} \tilde{C}(\tilde{R}(U, V)X, Y)Z &= C(R(U, V)X, Y)Z \\ &+ [\alpha\{g(V, \nabla_U \phi X) - g(U, \nabla_V \phi X) + g(U, \phi \nabla_V X) \\ &- g(V, \phi \nabla_U X)\} + \beta g([U, V], X)]R(\xi, Y)Z \\ &+ [\alpha\{g(Y, \nabla_{\tilde{R}(U, V)X} \phi Z) - g(\tilde{R}(U, V)X, \nabla_Y \phi Z) \\ &+ g(\tilde{R}(U, V)X, \phi \nabla_Y Z) - g(Y, \phi \nabla_{\tilde{R}(U, V)X} Z) \\ &+ \beta g([\tilde{R}(U, V)X, Y], Z)]\xi \\ &+ N\{g(Y, Z)\tilde{R}(U, V)X - g(R(U, V)X, Z)Y\}, \end{aligned} \quad (6.8)$$

$$\begin{aligned} \tilde{C}(X, \tilde{R}(U, V)Y)Z &= C(X, R(U, V)Y)Z \\ &+ [\alpha\{g(V, \nabla_U \phi Y) - g(U, \nabla_V \phi Y) + g(U, \phi \nabla_V Y) \\ &- g(V, \phi \nabla_U Y)\} + \beta g([U, V], Y)]R(X, \xi)Z \\ &+ [\alpha\{g(\tilde{R}(U, V)Y, \nabla_X \phi Z) - g(X, \nabla_{\tilde{R}(U, V)Y} \phi Z) \\ &+ g(X, \phi \nabla_{\tilde{R}(U, V)Y} Z) - g(\tilde{R}(U, V)Y, \phi \nabla_X Z) \\ &+ \beta g([X, \tilde{R}(U, V)Y], Z)]\xi \\ &+ N\{g(R(U, V)Y, Z)X - g(X, Z)\tilde{R}(U, V)Y\}, \end{aligned} \quad (6.9)$$

and

$$\begin{aligned}
 \tilde{C}(X, Y)\tilde{R}(U, V)Z &= R(X, Y)R(U, V)Z \\
 &+ [\alpha\{g(Y, \nabla_X \phi\tilde{R}(U, V)Z) - g(X, \nabla_Y \phi\tilde{R}(U, V)Z)\} \\
 &+ g(X, \phi\nabla_Y \tilde{R}(U, V)Z) - g(Y, \phi\nabla_X \tilde{R}(U, V)Z)] \\
 &+ \beta g([X, Y], \tilde{R}(U, V)Z)]\xi \\
 &+ N\{g(R(U, V)Z, Y)X - g(R(U, V)Z, X)Y\}.
 \end{aligned}
 \tag{6.10}$$

Now using (6.7), (6.8), (6.9) and (6.10) in (6.6) we get

$$\begin{aligned}
 [(\tilde{R}(U, V).\tilde{C})(X, Y)Z] &= [(R(U, V).C)(X, Y)Z] \\
 &+ [\alpha\{g(Y, \nabla_X \phi Z) - g(X, \nabla_Y \phi Z) + g(X, \phi\nabla_Y Z) \\
 &- g(Y, \phi\nabla_X Z)\} + \beta g([X, Y], Z)]\xi \\
 &+ [\alpha\{g(V, \nabla_U \phi\tilde{C}(X, Y)Z) - g(U, \nabla_V \phi\tilde{C}(X, Y)Z) \\
 &+ g(U, \phi\nabla_V \tilde{C}(X, Y)Z) - g(V, \phi\nabla_U \tilde{C}(X, Y)Z)\} \\
 &+ \beta g([U, V], \tilde{C}(X, Y)Z)]\xi \\
 &+ N\{g(Y, Z)R(U, V)X - g(X, Z)R(U, V)Y\} \\
 &+ [\alpha\{g(V, \nabla_U \phi X) - g(U, \nabla_V \phi X) + g(U, \phi\nabla_V X) \\
 &- g(V, \phi\nabla_U X)\} + \beta g([U, V], X)]R(\xi, Y)Z \\
 &+ [\alpha\{g(Y, \nabla_{\tilde{R}(U, V)X} \phi Z) - g(\tilde{R}(U, V)X, \nabla_Y \phi Z) \\
 &+ g(\tilde{R}(U, V)X, \phi\nabla_Y Z) - g(Y, \phi\nabla_{\tilde{R}(U, V)X} Z)\} \\
 &+ \beta g([\tilde{R}(U, V)X, Y], Z)]\xi + Ng(Y, Z)\tilde{R}(U, V)X \\
 &+ [\alpha\{g(V, \nabla_U \phi Y) - g(U, \nabla_V \phi Y) + g(U, \phi\nabla_V Y) \\
 &- g(V, \phi\nabla_U Y)\} + \beta g([U, V], Y)]R(X, \xi)Z \\
 &+ [\alpha\{g(\tilde{R}(U, V)Y, \nabla_X \phi Z) - g(X, \nabla_{\tilde{R}(U, V)Y} \phi Z) \\
 &+ g(X, \phi\nabla_{\tilde{R}(U, V)Y} Z) - g(\tilde{R}(U, V)Y, \phi\nabla_X Z)\} \\
 &+ \beta g([X, \tilde{R}(U, V)Y], Z)]\xi - Ng(X, Z)\tilde{R}(U, V)Y \\
 &+ [\alpha\{g(Y, \nabla_X \phi\tilde{R}(U, V)Z) - g(X, \nabla_Y \phi\tilde{R}(U, V)Z) \\
 &+ g(X, \phi\nabla_Y \tilde{R}(U, V)Z) - g(Y, \phi\nabla_X \tilde{R}(U, V)Z)\} \\
 &+ \beta g([X, Y], \tilde{R}(U, V)Z)]\xi.
 \end{aligned}
 \tag{6.11}$$

Applying ϕ^2 on both side of (6.11) and using (2.3), (3.3) and (2.16) we obtain

$$\phi^2[(\tilde{R}(U, V).\tilde{C})(X, Y)Z] = \phi^2[(R(U, V).C)(X, Y)Z].
 \tag{6.12}$$

Thus we are in a position to state the following:

Theorem 6.3. *A trans-Sasakian manifold of dimension three is locally conformally ϕ -semisymmetric with respect to generalized Tanaka Webster Okumura connection $\tilde{\nabla}$ if and only if it is locally ϕ -semisymmetric with respect to Levi-civita connection ∇ .*

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